A SunCam online continuing education course

WHAT EVERY ENGINEER SHOULD KNOW ABOUT RELIABILITY ENGINEERING II

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Contents

Introduction

Reliability is the probability that a system will perform its required function without failure under specified environmental conditions and for a finite time period. A system's reliability is a measure of stability and overall performance of such system collated during an extended period of time under various and specific sets of test conditions. This type of testing incorporates the results from non-functional testing such as stress testing, security testing, network testing, along with functional testing. It is a combined metric to define a system's overall reliability metric. A measure of reliability should be defined by business requirements in the form of service levels. These requirements should then be used to measure test results and the overall reliability metric of a system under test.

System reliability goals and the attendant component reliability requirements needed to achieve those goals are given prominence during the initial conception design and definition of the system. The **MIL-HDBK-217** is a document that is recognized world-wide as the pre-eminent reliability prediction document used to estimate reliability from standard components. However during prototyping and detailed design phase, prototypes may be built, tested, and analyzed for failure types and modes in order to improve reliability through redesign.

During the manufacturing and construction phases, qualification tests and acceptance tests become important to ensure that the delivered product meets the standards for which it is designed. Through improvement in quality control, defects in the manufacturing process can be eliminated. Finally, the collection of reliability data throughout the operational life of a system is an important task, not just for defect reduction which can only become apparent with extensive field service, but also for the setting and optimization of maintenance schedules, parts replacement, and warranty policies. In almost all cases of reliability testing, the severity and length of tests is limited by both time and cost.

1.1 Types of test

- Accelerated life testing
- Reliability Enhancement Testing
- Reliability growth testing
- Test, analyze and fix
- FRACAS (Failure Reporting and Corrective Action System)

1.1.1 Accelerated life testing-- Types of Accelerated Test

i). **High Usage Rate.**

a). Cycle or run the product faster rate than normal by increasing the products duty cycle.

b). Care should be taken that the increased usage rate does not increase the other stresses thereby resulting in common cause failure. In other words, the failure produced should be the same as those seen under normal usage.

ii). **Over stress testing**

a). This consists of running the product at a higher than normal stress level to shorten product life or to degrade product performance in order to collect needed performance data.

b). Typical accelerating stresses are temperature, voltage, mechanical load, thermal cycling, humidity and vibration.

c). Over stress testing is the most common form of accelerated testing.

iii). Censoring

Censoring is a major and frequent accelerated testing technique in which the tests are terminated before all the specimens have failed **either** because the predetermined test time has elapsed **or** because of the occurrence of specific number of failures. There two type of censoring, namely singly censored or multiply censored.

a). Singly Censored Test

Under singly censored data, we have two type of censoring, namely, Type I, and Type II. In this type of test, the test is terminated at a predetermined time τ (Type I) or the test is terminated when a given number of items have failed (Type II). Items may or may not be replaced during the test.

b). Multiply Censored Test

In this type of test, items are removed at various times during the test. Such removal are necessary **either** because a mechanism that is not under study failed **or** because the unit is no longer available for testing.

iv). Degradation

Accelerated degradation testing involves over stress testing but instead of life, product performance is observed as it degrades over time. Statistical methods similar to those used for the censoring tests are used to calculate the reliability.

v). Specimen Design

The life of some products can be accelerated through the size, geometry and finish of specimens. For instance large specimens fail sooner than small ones, e.g. high capacitance capacitors fail sooner than lower capacitance ones of the same design, this is because the large capacitors have more dielectric area

1.1.2 Reliability Enhancement Testing (RET)

These are tests that are carried out in the design and development phase of a product to remove defects.

The standard approach is

- TEST
- ANALYSE
- FIX

The purpose RET is to improve design and increase durability. It uses step stress testing which is a combination of stresses. It includes stresses in excess of those seen in service. It also includes the use of HALT (highly accelerated life testing). Various methods exist for analysis of the data.

1.1.3 Reliability Growth Testing

For Reliability Growth, prototypes are constructed and tested for failure. The prototypes are fixed and the testing continued. The objective of a reliability growth testing program is to; find problem failure modes, incorporate corrective actions, and therefore increase the reliability of the system. This process is repeated for the duration of the test time T. If the corrective actions are effective then the MTBF or mean trials between failures (MTrBF) will increase from a low initial value to a higher value toward the desired reliability goal or mission requirement. One of the biggest challenges is to determine how much test time is needed for a particular system. If we define the following:

Let: T= total operating time accumulated on all prototypes

and n(T)= number of failures from the beginning of the testing through time T

Then if we assume that as failures occur the system is modified to eliminate the failure modes, it has been shown using the Duane plots that if [n(T)/T] is plotted versus T on a log-log paper, the result tends to be a straight line regardless of the type of electromechanical equipment under consideration.

The rate of change of n(T) is related to the failure rate as follows, namely: $\frac{d}{dT}n(T) = \lambda(T)$

We note that if the Duane plots are straight lines then we can estimate the parameters of the line as follows:

$$\ln[n(T)/T] = A - \alpha T \Rightarrow \frac{n(T)}{T} = e^{a}T^{-\alpha}$$

$$\Rightarrow n(T) = e^{a}T^{-\alpha}T = e^{a}T^{1-\alpha}$$

$$Given: \frac{d}{dT}n(T) = \lambda(T)$$

$$\Rightarrow \lambda(T) = (1-\alpha)KT^{-\alpha}$$

$$Thus\ \ MTBF = \frac{1}{\lambda(T)} = \frac{1}{(1-\alpha)K}T^{\alpha}, \alpha\ is\ a\ positive\ value\ (usually\ 0.5)$$

Hence this demonstrates growth of MTBF with accumulated test time T

1.1.4 Test, Analyze and Fix (TAAF)

These are the types of tests that are carried out in the design and development phase of a product to remove defects and to enhance reliability growth.

Test Analyze and Fix (TAAF) process or more appropriately Test Analyze Fix re-Test (TAFT) is an engineering activity that is incorporated into the Reliability Growth process during the product development stages. TAAF refers to the sequence of activities by which the failure modes are identified, analyzed and corrected, and the corrective action finally validated. It should be noted that 'fix' refers only to correction through re-design and modification to eliminate the cause of failure and does not imply repair.

The TAAF process needs to be a closed loop methodology of test, analyze, fix, re-test and where necessary analyze, fix, re-test and so on until the required objective have been obtained. The process consists of the following steps:

Testing: Testing is conducted under the same operating and environmental condition, whether practical or simulated, as the item or system would experience while in use. Systems designed for use in harsh environments are not likely to show their failure modes when tested under a benign environment. Conversely if a harsh testing program is used in a benign environment then it is unlikely that the test would yield useful results.

Analyze: There needs for detailed and effective analysis of both the usage (human factor) and the resulting failure modes for a failure occurrence based on an effective Defect Report and Corrective Action System (DRACAS). There is little point in resolving a failure mode using design and engineering best practices if failure was caused by human error and could have been avoided through proper training. Effective analysis can only be achieved with a sound and appropriate data-set obtained from well trained and motivated staff.

Fix: Any fix activity should resolve the failure occurrence and related issues including all aspects of the failure mode, engineering, human factors, and related interfaces. Care needs to be taken to ensure that resolving one shortfall does not result in common cause failure mode. Where human error is likely to reoccur an engineered option may be used as a long term solution.

Re-test: Re-test must be carried out under similar conditions and duration as those when the original failure occurred. This is to ensure that the problem has been taken care of without introducing additional failures occurrences or failure modes.

1.1.5 FRACAS (Failure Reporting and Corrective Action System)

This is a closed-loop system for identifying, assessing, and correcting failure related problems in a timely manner. It is implemented at the start of the project and is used by all personnel including the review team.

Testing

The purposes of testing are numerous, including;

- To verify or authenticate the efficacy and feasibility of certain system or configuration.
- To determine which option is the optimum with respect to performance, reliability, cost, modes of behavior under varying conditions, etc.
- To make informed and sensitive comparisons and to further improve economy, maintenance, use of standard parts, and so on.
- To demonstrate whether the item is adequate to meet the requirements of performance and reliability.
- To thoroughly investigate the latent capabilities of the item under severer or more diverse conditions than those immediately anticipated.
- To define what requires testing bearing in mind that the objective is to minimize the number of tests required for cost reasons

2.1 Testing Based on Categories of Failure Types

- Mandatory Tests
  - ✓ Tests based on regulatory requirements e.g. Road worthiness
  - ✓ Tests based on safety requirements

  - ✓ Tests based on customer requirements
  - ✓ Tests based on competitor benchmark
- Testing based on pedigree or experience
  - ✓ A historical product with similar physical and performance attributes and similar testing may be referred to as a guide.
  - ✓ Manufacturing and development engineers may have a 'feel' for the type of testing required based on previous experiences.

2.2 Frequency of testing

- It is important to determine the frequency of testing such as;
  - ✓ Is it for each and every product?
  - ✓ Is it for each and every product for a limited period?
  - ✓ Is the test periodic?
  - ✓ Is there regulatory requirement for testing at a specified interval/frequency?

2.3 Environmental testing

- As its name implies, this form of testing represents a survey of the reaction of the item to the various environments.
- It is usually required in qualification tests and is frequently introduced in the development stage, usually at less numerous or less severe environmental levels.

MIL-HDBK-217: Reliability Prediction

MIL-HDBK-217 also known as **MIL-217** is one of the most used reliability prediction document world-wide. It is the document of reference by both commercial and defense companies. The most recent version or revision is "Military Handbook, Reliability Prediction of Electronic Equipment", MIL-HDBK-217, Revision F, Notice 2, released in February of 1995. It contains failure rate models for numerous electronic components such as integrated circuits, transistors, diodes, resistors, capacitors, relays, switches, and connectors, to name a few. In general, MIL-217 will show a higher failure rate than other standards for the same system. This is because the original intended use of the MIL-217 standard is for aerospace, military, or mission critical applications.

Maintaining reliability and providing essential reliability engineering tools is an essential need with modern electronic systems. Reliability engineering for electronic equipment requires a quantitative baseline, or a reliability prediction analysis platform. The MIL-217 standard was developed for military and aerospace applications; however, it has become widely used for industrial and commercial electronic equipment applications throughout the world. Using the MIL-217 standard for reliability prediction produces calculated Failure Rate and mean time between failures (MTBF) numbers for the individual components, equipment and the overall system. The final calculated prediction results are based on the roll-up, or summation, of all the individual component failure rates. Please note that this is possible under the assumption of constant failure rate f components.

The most current document is MIL-HDBK-217F Notice 2 dated December 2, 1991 was developed by Rome Laboratories, and the US Department of Defense. The purpose was to establish and maintain consistent and uniform methods for estimating the inherent reliability of military electronic equipment and systems. The handbook is intended as a guideline, not a specific requirement, to increase the reliability of equipment being designed.

The handbook contains two methods of reliability prediction, namely; Part Stress Analysis and Parts Count Analysis. The two methods vary in the degree of information required to be provided. The Part Stress Analysis Method requires a greater amount of detailed information and is usually more applicable in the later design phase. The Parts Count Method requires less information such as part quantities, quality level and application environment. It is most applicable during early design or proposal phases of a project. The Parts Count Method will usually result in a higher failure rate or lower system reliability which is a more conservative result than the Parts Stress Method.

3.1 Part Stress Analysis

The Part Stress Analysis method is used the majority of time and is applicable when the design is near completion and a detailed parts list, or BOM (Bill of Materials), plus component stresses are available. By component stresses, the standard is refers to the actual operating conditions such as environment, temperature, voltage, current and power levels that the component will see or undergo. The MIL-217 standard groups components or parts by major categories and then has subgroups within the categories. An example is a "fixed electrolytic (dry) aluminum capacitor" is a subcategory of the "capacitor" group. Each component or part category and it's subgroups have a unique formula or model applied to it for calculating the failure rate for that component or part.

3.2 Failure Rate and Π Factors

The failure rate formulas referred to in category include a base failure rate, λ_b, for the category and subgroup selected. The base failure rates apply to components and parts operating under normal environmental conditions, with power applied, performing the intended function(s), using base component quality levels, and operating at the design stress levels. The standard then applies many Π factors, or multiplying factors, to the base failure rates in order to factor in the actual operating conditions, environment and stress levels referred to above. Base failure rates are adjusted by applying the Π factors, which range from 0 to 1.0, to the underlying equation or model provided for each component category.

The procedure calculates the predicted failure rate at the actual operating conditions for each component in the project. The procedure to determine the overall system level or equipment failure rate is to sum, or roll up the individually calculated failure rates for each component. Most manufactures of electronic equipment assemble a majority of the components on various types of printed circuit boards (PCBs) or as part of a hybrid construction. A failure rate is determined for the PCB or hybrid device by the summation of the failure rates for the numerous components, solder joint connections and other types of construction involved. The failure rate for each connection made to the PCB by electrical connectors is also included. The failure rate for the wire between electrical connections is assumed to be zero. MIL-217 does not utilize a basic circuit board model. It sums the failure rates for each individual connection type times the quantity and adds that to the overall PCB (Block) connection rate to the sum of the attached component failure rates. However, MIL-217 does have a model for PCBs with plated through holes (PTH), surface mount technology (SMT) as well as a model for a hybrid configuration.

3.3 Component Quality

The design quality or "as purchased" quality of the component utilized has a direct effect on the part failure rate and appears in the models as a Π factor, namely, Π_Q. Many of the components covered by MIL-217 specification are available in several quality levels and each has an associated factor, Π_Q. It is especially important to note of microcircuits and integrated circuits (ICs) quality specifications and the resultant pi factors. Parts purchased under older specifications are referred to as "Non-established Reliability" (Non-ER) or they can be broken down into two additional quality levels labeled, "MIL-SPEC" or "Lower". Non-ER parts purchased in complete accordance with a particular MIL specification should be entered for the applicable MIL specification. If some of the quality requirements are waved for the purchased component or if it is a commercial component, the "Commercial", "Lower" or Non-ER rating should be used. Each quality designation has an associated pi factor, Π_Q.

3.4 Environment

Environmental stress is of major concern in establishing the failure rate for components and parts included in a system per the MIL-217 model. Environmental stresses can be quite different from one application environment to another and can subject the equipment to a controlled environment with constant temperature and humidity, or an environment with rapid temperature changes, high humidity, high vibration and high acceleration. The environmental designations included within MIL-217 are included in the formulas as Π_E.

3.5 Thermal Environment

Ambient and operating temperatures have a major impact on the failure rate prediction results of electronic equipment, especially equipment involving semiconductors and integrated circuits. The MIL-217 standard requires an input of ambient temperatures and more definitive data for the calculation of junction temperatures in semiconductors and microcircuits. A thermal analysis should be a part of the design and reliability analysis process for electronic equipment. Ambient temperatures for overall equipment should be the ambient temperature close to the equipment involved. Individual component ambient temperatures should be the operating ambient temperature inside the equipment where such equipment resides. The ambient temperature for components or parts located within the area of hot spots should be adjusted for the higher ambient temperature in the area.

3.6 Typical MIL-217 Failure Rate Model

A sample MIL-217 failure rate model for a simple semiconductor component is shown below. Many components, especially microcircuits, have significantly different and more complex models. A typical example of the type of model used for most other part types is the following for discrete semiconductors.

$$\lambda_p = \lambda_b \Pi_T \Pi_A \Pi_R \Pi_S \Pi_C \Pi_Q \Pi_E$$

Where:

Π_T = Temperature factor

Π_A = Application factor (linear, switching, etc)

Π_R = Power rating factor

Π_S = Electrical (voltage) Stress factor

Π_C = Construction factor

Π_Q = Quality factor

Π_E = Operating environment factor

The above listed Π factors are based on a simple component. There are also Π factors for items such as learning factor, die complexity factor, manufacturing process factor, device complexity factor, programming cycle's factor, package type factor, etc. Each component or part group and its associated subgroup has a base failure rate plus numerous Π factor tables, unique to that component or part, that list factors that are used in the model to adjust the base failure rate.

A non-solid tantalum fixed electrolytic capacitor with specification (Mll-C-3965 & MIL-C-39006) and style CL or CLR for example, has a MIL-217 model as follows:

$$\lambda_p = \lambda_b \Pi_{CV} \Pi_C \Pi_Q \Pi_E \text{ Failures}/10^6 \text{ Hours}$$

Where:

Π_{CV} = Base failure rate for capacitance factor

Π_C = Construction factor

Π_Q = Quality factor (quality levels of D, C, S, B, R, P, M, L, Lower)

Π_E = Operating Environment factor

3.7 MIL-217 Parts Count Analysis

The MIL-217 Parts Count Reliability Prediction is normally used when accurate design data and component specifications are not available. Typically, this will happen during the proposal and bid process or early in the design process. However, this stage in the design process is where design decisions and project specifications, allocations, etc. can be determined with help from preliminary reliability prediction data.

Minimal information is required for a Parts Count Reliability Prediction. The formula for a parts count analysis is simply the summation of the base failure rate of all components in the system. (Refer to the MIL-217 standard for the specific equation) Equipment operating in multiple environments will have the calculations applied to a portion of the equipment in each environment. The MIL-217 standard provides tables for the component groups (same groups as the Parts Stress analysis) listing generic failure rates and quality factors for the different MIL-217 environments. The predicted failure rate results will normally be harsher using the Parts Count method than using the Part Stress analysis. The Parts Count analysis does not factor in the numerous variables and typically uses worst case generic or base failure rates and Π factors in its formulation.

Mean Time to Failure MTTF (θ) and the Chi-square Distribution

4.1 Estimation of mean life MTTF/MTBF (θ)

$\hat{\theta}$ is a maximum likelihood estimator for MTBF. It has the following properties

i). Unbiasedness, ii)Minimum variance, iii) Efficiency, iv).Sufficiency

Example: Assume total test time T= 245 hours. Total failures r =20

Note that the measurements are Time Between Failures, thus:

$$\hat{\theta} = MTBF = \frac{\sum x_i}{r} = \frac{T}{r} = \frac{245}{20} = 12.25$$

Let (x_1, x_2,…x_r) be a sequence of r independent and identically distributed exponential random variables. The sum of these random variables, namely, T is the total test time for those items that failed during the test and those that did not fail.

The degrees of freedom for these life tests are based on or are determined **by the number of failures observed during the test.**

For example in vehicle testing, this could be the kilometers between failures for the first r failures. In such tests the total test time T is very important. In general there is a parametric relationship between the exponential distribution and the Chi-Square distribution, that is, the ratio of 2T to θ is the chi-square distribution as follows:

$$\frac{2x_i}{\theta} \text{ is approximately } \chi^2 (chi - Square) \text{ with } 2 \text{ degrees of freedom} (df)$$

$$\frac{2(x_1 + x_2)}{\theta} \approx \chi^2 \text{ with } 2(2) = 4 \text{ degrees of freedom } (4\,df)$$

$$For: x_1 + x_2 + ... + x_r = T, \Rightarrow\Rightarrow \frac{2T}{\theta} \approx \chi^2 \text{ with } 2r \text{ degrees of freedom} (2r\,df)$$

If the failure rate for the process is constant, then the **underlying assumption** as we estimate the mean time to failure (MTBF) is that the **process distribution** is the **exponential distribution**.

4.2 Singly Censored Test

There are two types of **Singly Censored Tests,** namely Type I and Type II.
Type I censoring is a Time Truncated Test, that is, the test is terminated when a specified time interval T has been accumulated on the test. Type II censoring or the other hand is an Item Truncated Test, that is, the test is terminated when a specified number of items have failed.

In the case of Type I censoring, items may or may not be replaced during the test. Also, the time T is the total time on the test for those that failed and those who survived up to time T.

In the case of Type II censoring, as items fail they are replaced with new ones or the failed ones are repaired and the test continued until the specified number is r fail. In case also, the time T is the total time on the test for those that failed and those who survived up to time T.

4.2.1 Type I Censoring

In this type of test, the test is terminated at a predetermined time. Items may or may not be replaced during the test. If items are replaced then; T=Total test time = nτ, where τ is the specified test duration and n is the number on test.

If items are not replaced then; $T = \sum_{i=1}^{r} x_i + (n-r)\,\tau$, $\Rightarrow MTBF\,(\theta) = \frac{T}{r}$

If say have a vehicle which is tested for k miles. As electrical switches failed, they are replaced until T simulated (or actual miles) is reached. This is a **distance or time truncated test** or Type I test.

$$\hat{\theta} == \frac{T}{r}$$

If put several (n) items on test for a specified time and failed items are not replaced, then this is also Type I

Then: $T = \sum_{i=1}^{r} x_i + (n-r)\,\tau, \Rightarrow \hat{\theta} = \frac{T}{r}$, where τ is the specified test duration

Note that in Type I censoring, we count the number of failures which represents the d*egrees of Freedom (df)*. There are two possibilities with regard to the degrees of freedom.

i) All items on test could have failed by τ; If i). Then df=2r **OR**

ii) Some failed and some survived. If ii). Then df =2r+2 , where 2r would represent the degrees of freedom for the r units that failed and 2 would represent the degree of freedom of all those that did not fail (all lumped into a single non failed unit). A way to look at this second case with respect to those that did not fail is as follows. The degree of freedom for a single unit in this exponential/Chi-square relationship is 2. In the case of the units that did not fail, we lump then into a single unit which as we know has a degree of freedom of 2. So the degrees of freedom for Type I censoring is between 2r and (2r+2), with 2r for the lower confidence limit and 2r+2 for the upper.

4.1.2 Type II Censoring

The test is terminated when the first r-out-of-n failures occur (r<=n). It is also referred to as item truncated test. In this case we are interested in recording failure times. **Note that the degree of freedom for the Type II test is 2r for both the upper and lower confidence intervals**.

i) If failed items not replaced

$$T = \sum_{i=1}^{r} x_i + (n-r)\tau \,, \qquad \hat{\theta} = \frac{T}{r} = \frac{\sum x_i + (n-r)x_r}{r}$$

x_i= time for the i^{th} component failure and x_r= time for the failure of the r^{th} component

ii) If failed items are replaced

If no replacement, then: $T = nx_r$ where x_r is the time the r^{th} unit failed. Since the test is such that a specified number (r) would have to fail for the test to terminate then the degrees of freedom is 2r.

$$\hat{\theta} = \frac{nx_r}{r}$$

4.2 Computation of Confidence Intervals

The confidence interval is based on the test statistic: $\frac{2T}{\theta} \sim \chi^2$

4.2.1 Confidence Interval for Type I Censoring

Test Statistic: $\frac{2T}{\theta} \sim \chi^2$, with degrees of freedom 2r for lower limit and 2r+2 for upper limit

For the upper and lower limits

$$\chi^2_{(1-\alpha/2,2r)} \le \frac{2T}{\theta} \le \chi^2_{(\alpha/2,2r+2)}$$

$$\frac{1}{\chi^2_{(\alpha/2,2r+2)}} \le \frac{\theta}{2T} \le \frac{1}{\chi^2_{(1-\alpha/2,2r)}}$$

$$\frac{2T}{\chi^2_{(\alpha/2,2r+2)}} \le \theta \le \frac{2T}{\chi^2_{(1-\alpha/2,2r)}}$$

Example: The failures for a prototype test vehicle occurred at the following kilometers.
28,820; 36,707; 46,128; 68,345. If the test was scheduled for 72,000 km, then:
i). Estimate θ , ii). Show the reliability function, iii). Establish a 95% confidence interval for θ.

Solution: T=72,000 km. Since this is a time (or distance) terminated test,

i) θ = T/r = 72000/4 = 18,000 km=MTBF

ii) $R(x) = e^{-x/\theta} = e^{-x/18{,}000}$

iii) $$\frac{2T}{\chi^2_{0.025,2r+2}} \le \theta \le \frac{2T}{\chi^2_{0.975,2r}}$$

For Type I Test, df=2r, and 2r+2: that is; 2r=8, and 2r+2= 8 + 2 =10, T = 72,000

$\chi^2_{0.025,\,10} = 20.483$, $\chi^2_{0.975,\,8} = 2.18$

$2(72000)/20.483 \le \theta \le 2(72000)/2.18$

$7030.2 \le \theta \le 66055$

$$P[7030.20 \le \theta \le 66055] = 0.95$$

Example: Suppose we wish to establish a 90% lower confidence limits for reliability at 12,000 km, what is the value of reliability at this lower confidence limit?

Solution:

$$\theta_L = \frac{2T}{\chi^2_{\alpha,2r+2}} = \frac{2(72000)}{\chi^2_{0.1,10}} = 2(72000)/15.987 = 9007.3$$

The confidence interval for reliability is:

$$e^{-\left(\frac{x}{\theta_L}\right)} \le R(x) \le e^{-\left(\frac{x}{\theta_U}\right)}$$

Note: Computation of Reliability at time t.

Given: θ, where θ=T/r; $R(t) = e^{-\left(\frac{t}{\theta}\right)}$

Since we are interested in the lower limit

$R(x) \ge e^{-x/\theta L}$

$R(12000) \ge e^{-12000/9007.3} = 0.264$

This means that we are 90% confident that 12,000km reliability is at least 0.264 or at least 26%.

Question: Assuming we are interested in the distance (or time) at which a certain proportion of the population will fail. (Example: At what kilometer will 10% of the population fail given θ=18,000 km)

Note: Probability of failure $F(x) = 1-e^{-x/\theta}$, then Reliability $R(x) = 1-(1-e^{-x/\theta}) = e^{-x/\theta}$

If p is proportion of the population that fails, then 1-p represents the proportion that would survive.

$$R(x_p) = 1 - F(x) = 1 - p$$

$$R(x_p) = e^{-(x_p/\theta)} \Rightarrow 1 - p = e^{-(x_p/\theta)}, Taking\ \log s: \ln(1-p) = -x_p/\theta$$

$$\ln\left(\frac{1}{1-p}\right) = \frac{x_p}{\theta} \Rightarrow x_p = \theta \ln\left(\frac{1}{1-p}\right), with\ \ p = 0.1,\ R(x_p) = 1 - p = 0.9$$

$$x_{0.1} = \theta \ln\left(\frac{1}{0.9}\right),\ \theta = 18{,}000 \Rightarrow x_{0.1} = 1800 \ln\left(\frac{1}{0.9}\right) = 1896\, km$$

The kilometer at which 10% of the population will fail is 1896 km

4.2.2 Confidence Interval for Type II Censoring

Test Statistic: $\frac{2T}{\theta} \sim \chi^2$, with degrees of freedom 2r for lower limit and 2r for upper limit

For the upper and lower limits

$$\chi^2_{1-\alpha/2,2r} \le \frac{2T}{\theta} \le \chi^2_{\alpha/2,2r} \Rightarrow \frac{2T}{\chi^2_{\alpha/2,2r}} \le \theta \le \frac{2T}{\chi^2_{1-\alpha/2,2r}}$$

Example: 8 leaf springs were cycle tested to failure on an accelerated life test as follows,
8712; 39400; 79000; 151208; 21915; 54613; 11020; 204312

Solution: $T = \sum_{i=1}^{r} x_i + (n-r)\tau$

$$T = \sum_{i=1}^{r} x_i = 8{,}712 + 21{,}915 + \ldots + 204{,}312 = 669{,}360$$

θ= T/r = 669,360/8 = 83,670

i) set a 95% two-sided confidence limits on θ

$\chi^2_{0.975,16} = 6.91$, $\chi^2_{0.025,16} = 28.84$, 2T=2(669360)

$2(669{,}360)/28.84 \le \theta \le 2(669360)/6.91$

$46{,}419 \le \theta \le 193{,}736$

Example: Fifteen automotive A/C switches were cycled and observed for failure. The test was suspended when the fifth failure occurred. Failed switches were not replaced. The failure occurred at the following cycles: 1,410 3,138 6,971 1,872 4,218. Estimate MTBF and establish a two-sided 95% confidence interval for the MTBF

$$T = \sum_{i=1}^{r} x_i + (n-r)\tau = 17609 + (15-5)(6971) = 17609 + 69710 = 87319$$

Hence θ = 87319/5 = 17464

$$\frac{2(87319)}{\chi^2_{0.025,10}} \le \theta \le \frac{2(87319)}{\chi^2_{0.975,10}} \Rightarrow P[8527 \le \theta \le 53735] = 0.95$$

Example:

9 test stands were used to cycle heater switches and the failed switches were replaced. Each stand ran for 20,000 cycles and assume 10 failures (r =10)

T = nτ=(9 switches)(20,000 cycles) = 180,000 cycles

$$\hat{\theta} = \frac{180{,}000}{10} = 18{,}000\, cycles$$

4.3 Multiply Censored Tests including Suspensions

In computing the reliability of a component in the presence of suspensions or censoring, it is important to re-compute the order or rank (.i.e. the position for the items that fail) following the suspension. In order to properly position or rank the failures appropriately, an increment I is computed which is then used to determine the actual position of each event following the suspension, where:

$$I = \frac{(n+1) - (\Pr evious \sim order \sim number)}{1 + (number \sim of \sim items \sim following \sim suspension)}$$

Table 5: Data with Suspension		
Hours on Test	Sequence	Status
544	F1	failure
663	F2	failure
802	S1	suspension
827	S2	suspension
897	F3	failure
914	F4	failure
939	S3	suspension
1084	F5	failure
1099	F6	failure
1202	S4	failure

- First two failures (F_1 and F_2) will have positions or order numbers 1 and 2.
- I_3 =[(10+1)-2]/[1+(6)]= 1.29
- Adding 1.29 to 2, gives the order number of 3.29 to the 3rd failure (F_3). The 4th failure (F_4) has order number 3.29+1.29= 4.58.
- The order number for F_5 is given by:
  I_5 = [(10+1)-4.58)]/[1+(3)]= 1.60, hence the order number for F_5 = 4.58+1.60= 6.18.
- The position for F_6 = 6.18+1.60= 7.78

Table 6: Multiply Censored data with Suspension			
Hours on Test	Position (i)	Median Rank	Mean Rank
544	1.0	0.067	0.091
663	2.0	0.163	0.182
897	3.29	0.288	0.299
914	4.58	0.411	0.416
1084	6.18	0.565	0.562
1099	7.78	0.719	0.707

For the median rank, we use the well known formula

$$Median\,Rank = \frac{i-0.3}{n+0.4}, \quad Mean\;Rank = \frac{i}{n+1}$$

4.3.1 Analysis Of Suspended Data

When the total test time or distance for each component is different, we can apply the principle of 'total time to obtain a failure' in order to compute the time between failures.
To illustrate this principle, examine the following figure:

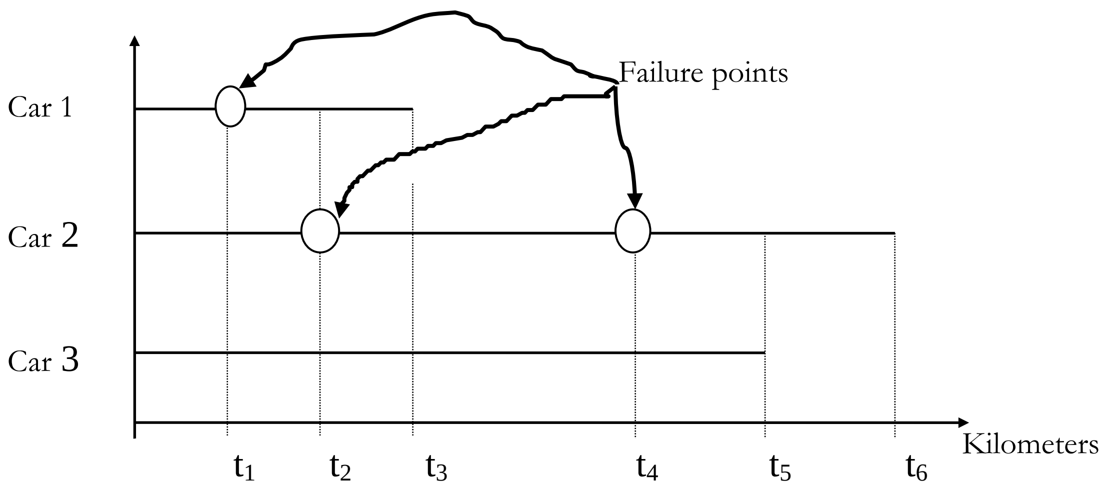


Figure1. Total time to failure

Define the t_i's as the kilometers at which failures occurred. Let τ_i's be the total kilometers up to the i^{th} failure. The basic reasoning is that τ_i total kilometers occurred to produce a failure. In order words, the τ_i represents the total kilometers prior to the i^{th} failure.

1. $\tau_1 = 3t_1$
2. $\tau_2 = 3t_1 + 3(t_2 - t_1) = \tau_1 + 3(t_2 - t_1)$
3. $\tau_3 = 3t_3 + 2(t_4 - t_3) = 2t_4 + t_3$

The basic reasoning is that τ_i total kilometers occurred to produce a failure. In order words, the τ_i represents the total kilometer prior to the i^{th} failure

Table 7a: Odometer Reading (in km) for a Test Car		
car no.	Odometer reading at failure (km)	Total Odometer reading (km)
1	2,467; 3,128; 3,383; 7,988	8,012
2	none	6,147
3	1,870; 6,121; 6,175	9,002
4	3,721; 4,393; 5,848; 6,425; 6,535	11,000
5	498	4,651
6	184; 216; 561; 2,804	5,012
7	2,342; 4,213	12,718

i). For the 1st failure epoch, the total kilometers is t_1 for car #1, t_1 for car #2, and t_1 for car 3 for a total of $3t_1$, hence: $\tau_1=3t_1$

ii). For the 2nd failure epoch, the total kilometers covered is t_1 for car #1, t_1 for car #2, and t_1 for car 3 for a total of $3t_1$. Additionally there is (t_2-t_1) for car #1, is (t_2-t_1) for car #2, and is (t_2-t_1) for care #3, hence $\tau_2=3t_1+3(t_2-t_1)$

iii). For the 3rd failure epoch, the total kilometers covered is t_3 for car #1, t_3 for car #2, and t_3 for car 3 for a total of $3t_3$. Additionally after time t_3, car #1 failed and was put out of service. Also there is time (t_4-t_3) for car #2 and (t_4-t_3) for car #3, hence $\tau_3=3t_3+3(t_4-t_3)$

Table 7b: Odometer reading (in km) for a Test Car-Expanded View

kilometer points	No. of cars operating	Total km	km b/w failures (x_i)
184	7	1,288	1,288
216	7	1,512	224
498	7	3,486	1,974
561	7	3,927	441
1,870	7	13,090	9,163
2,342	7	16,394	3,304
2,467	7	17,269	875
2,804	7	19,628	2,359
3,138	7	21,896	2,268
3,283	7	22,981	1,085
3,721	7	26,047	3,066
4,213	7	29,491	3,444
4,393	7	30,751	1,260
4,651 suspend	7	32,557 **a**	-
5,012 suspend	6	34,465 **b**	-
5,848	5	38,903	8,152
6,121	5	40,268	1,365
6,147 suspend	5	40,398	-
6,175	4	40,510	242
6,425	4	41,510	1,000
6,535	4	41,950	440
7,988	4	47,762	5,812
8,012 suspend	3		
9,002 suspend	2		
11,000 suspend	1		

<u>Total kilometers = (kilometer point) x no. of cars in service at that point</u>

a : 6x5012 + 1x4,651 = 34,465

b : 5x5,848 + 5,012+ 4,651

Note: at point 6,121; total km = (6,121x5) + 5,012+ 4,651 = 40,268
t_r= 47,762 = (1,288+224+1,974+441+….+1,000+440+5,812)
Σ ln x_i = 139.42, B_{19} = 15.89

$\chi^2_{0.95,18} = 9.39$; $\chi^2_{0.05,18} = 28.87$, hence we cannot reject the null hypothesis of the exponential distribution

TESTS TO DETECT INCREASING AND DECREASING FAILURE RATE

The Bartlett's test is useful in detecting either increasing or decreasing failure rates. Specifically it is used to determine whether or not an underlying distribution is the exponential. The test statistic for Bartlett's is the chi-square statistic B_r, where;

$$B_r = \frac{2r\left[\ln(\frac{t_r}{r}) - \frac{1}{r}\left(\sum_{i=1}^{r}\ln x_i\right)\right]}{1+(r+1)/6r}, \text{ approx Chi-Square dist.. and } df = (r-1)$$

with degrees of freedom equal to r-1, where: x_i is the random variable representing time to failure and $t_r = \sum_{i=1}^{r} x_i$. Under the hypothesis of an exponential distribution, the statistic B_r is a two-tailed.

Example: The truck was shaken on a simulator for a total of 245 hours. The time when failure occurred during the testing is as follows:

Table 1: Data for Truck put on Simulator Test			
21.2	74.7	108.6	157.4
47.9	76.8	112.9	164.7
59.2	84.3	127.0	196.8
62.0	91.0	143.9	214.4
74.6	93.3	151.6	218.9

The x_i's represent the time between failures and is calculated as the differences between successive data point. Hence:

Table 2: Time Between Failures for Data in Table 1			
21.2	0.10	15.3	5.8
26.7	2.10	4.30	7.30
11.3	7.50	14.3	32.1
2.80	6.70	16.9	17.6
12.6	2.30	7.70	4.50

Σ ln x_i = ln(21.2) + ln(26.7) + …+ ln(4.5)= 38.80
t_r = 21.2 +26.7+11.3+2.8+..+17.6+4.5= 218.9

Hence $B_{20} = \dfrac{2(20)\left[\ln(\frac{218.9}{20}) - \frac{1}{20}(38.8)\right]}{1+\frac{21}{20}} = 15.42$

Based on α= 0.1 and the critical values of $\chi^2_{0.95,19} = 10.12, \chi^2_{0.05,19} = 30.4$, we cannot reject the null hypothesis that the underlying distribution is the exponential.

5.1 Test for Abnormally Early and Late Failures

We showed that the quantity (2x)/θ is chi-square distributed with 2 degrees of freedom when the random variable x is exponentially distributed, that is: $\frac{2x}{\theta} \sim \chi^2$ has 2 df. Corresponding if we have $(x_1, x_2, \ldots x_r)$ as a sequence of r independent and identically distributed exponential random variables with r failures, then;

$$\left(2\sum_{i=1}^{r} x_i\right)\Big/\theta \sim \chi^2 \Rightarrow \frac{2T}{\theta} \sim \chi^2 \ \ with\ 2r\ df$$

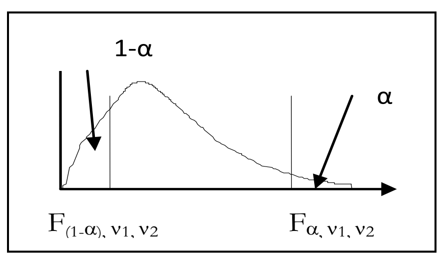
<u>Please Note</u>: Infant mortality failures or early failures are due to manufacturing inconsistencies and inspection errors if there is inspection in the manufacturing process.

Based on introductory statistics, if we take the ratio of two Chi-square random variables with each Chi-square divided by its corresponding degrees of freedom, the result is the F distribution.

$$F_{(df_1, df_2)} = \frac{(Chi-square)/df_1}{(Chi-square)/df_2}$$

Assuming is a sequence of $x_1 \ldots x_r$, x_1 is considered abnormally long or abnormally short. Then we have two distinct groups, namely x_1 in one group and $x_2, x_3, \ldots x_{r-1}$ in the other group. The degree of freedom for the first group df_1 is 2. The degree of freedom for the second group is 2r-2 =2(r-1)

$$F_{2,2(r-1)} = \frac{\left(\frac{2x}{\theta}\right)/2}{\left(\frac{2\sum_{i=2}^{r} x_i}{\theta}\right)/2(r-1)} = \quad F_{2,2r-2} = \frac{(r-1)x_1}{\sum_{i=2}^{r} x_i}$$



5.2 Test for Abnormally Early Failures

Assuming that for a test of time to failure, x_1 (out of x_r) represents an abnormally early failure or very short failure occurrence. Let: $F_{2,2r-2} = \dfrac{(r-1)r_1}{\sum_{i=2}^{r} x_i}, F_{(1-\alpha)2,2r-2} > \dfrac{(r-1)x_1}{\sum_{i=2}^{r} x_i}$

Note: For same degree of freedom, $F_\alpha > F_{(1-\alpha)}$

If the failure time x_1 is significantly small, then this F ratio will be disproportionately small. This means that if the ratio $F_{2,2r-2} = \frac{(r-1)r_1}{\sum_{i=2}^{r} x_i}$ is small, then there is evidence that x_1 represents an abnormally early failure. With respect to significance, this states that we reject the hypothesis that the failure is not too early or is similar to the rest of the failures if and only if, that is

Reject if: $F_{(1-\alpha)2,2r-2} > \frac{(r-1)x_1}{\sum_{i=2}^{r} x_i}$, but $F_{(1-\alpha)2,2r-2} = \frac{1}{F_{\alpha,2r-2,2}}$

Taking the reciprocals changes the direction of the inequality

$\therefore$ Reject if : $F_{\alpha,2r-2,2} < \frac{\sum_{i=2}^{r} X_i}{(r-1)X_1}$

Table 3: Failure times for 20 Turbine Blades			
193	1,793	3,479	5,310
1,582	2,028	4,235	6,809
1,637	2,260	4,264	8,317
1,658	2,272	4,635	9,728
1,786	2,700	4,919	10,700

Example: The following data represents cycles to failure for 20 turbine blades. If the design engineer claims that the first failure occurred too early and hence is not representative of the rest of the data, carry out an analysis to refute or support the engineer's claim at $\alpha=0.05$. Note $x_1 = 193$

H_0: The first failure is representative of the rest of the data;

H_1: The first failure is not representative of the rest of the data

$\sum_{i=2}^{20} x_i = 80{,}112,\ hence\ F_c = \frac{80{,}112}{(19)(193)} = 21.8$, $F_{\alpha,2r-2,2} = F_{0.5,38,2} = 19.47$

Reject if : $F_{0.05,(38,2)} < F_C$, 19.47<21.8 $\Rightarrow$ Reject H_0. Thus the first failure time of 193 is not representative of the rest of the data.

Example for more than one early failure

In the table 3, the first two failures appear to have occurred significantly earlier than the rest. Verify the claim that those two failures occurred rather earlier than normal at $\alpha=0.05$.

We know that $2(X_1+X_2)/\theta$ is chi-square distributed with 4 df also, $2/\theta\ \Sigma X_i$ (i= 3, 20) is chi-square with 36 df , that is df=(40-4=36).

H_0: The first two failures are representative of the rest of the data;

H_1: The first two failures are not representative of the rest of the data

$$F_{0.95,4,36} = \frac{(X_1 + X_2)/4}{\sum_{i=3}^{20} X_i / 36} = \frac{9(X_1 + X_2)}{\sum_{i=3}^{20} X_i} = \frac{9(440)}{468{,}730}$$

$$F_{0.05,36,4} < \frac{\sum_{i=3}^{20} X_i}{9(X_1 + X_2)} = \frac{468{,}730}{9(440)} = 118.37, \quad \mathrm{F}_{0.05,\,36,4} = 5.74, \mathrm{FC} = 118.37$$

Since F(table) is less than F(computed), we must reject the null hypothesis, $F_{table} < F_{computed}$

Hence the two failures occurred early.

5.3 Testing Abnormally Long Failures

Let x_1 could be any failure time among the observations not necessarily the first or last failure time

H_0: Failure was abnormally long

H_1: Failure was not abnormally long

For the F distribution, $F_\alpha > F_{1\text{-}\alpha}$ for the same degrees of freedom (F_α is the right tail)

Reject if: $F_{\alpha,2,2r-2} > \dfrac{(r-1)x_1}{\sum_{i=2}^{r} x_i}$,. *i.e., if :* $F_{table} > F_C$

Example: The data in table 4 represents the times at which the muffler failed. Suppose it is suspected that the first failure occurrence (43,850 km) is abnormally long

Table 4. Failure Times for Muffler			
43,850	65,324	83,541	89,950
47,737	67,105	84,543	100,791
49,111	67,549	84,899	102,431
61,900	69,291	88,191	104,343
64,511	81,154	88,901	105,062

$$\text{Re}\ ject\ if : F_{0.05,2,2r-2} > \frac{(r-1)X_1}{\sum_{i=2}^{r} X_i}, \quad Do\ not\ reject\ if : \quad F_{0.05,2,2r-2} < \frac{(r-1)X_1}{\sum_{i=2}^{r} X_i}$$

$$But : \quad F_{0.05,2,38} = 3.245\ and\ F_C = 0.553, Do\ not\ reject\ H_0$$

Thus the test statistic is not significant; ***hence the failure is NOT abnormally long***

Reliability Data Plotting

6.1 The Weibull Distribution

Advances in technology has made possible the design and manufacture of complex systems whose operation depends on the reliability and availability of the of subsystems and components that comprise such systems. The time to failure of the life of component measured from a specified point or time interval is random variable. In 1951, W. Weibull introduced a distribution that has been found to be very useful in the study of reliability and maintenance of physical systems.

Weibull distribution is widely used in reliability and life data analysis due to its versatility. The Weibull is generally used to model situations where the failure rate is not constant. Depending on the values of the parameters, the Weibull distribution can be used to model a variety of life behaviors. An important aspect of the Weibull distribution is how the values of the shape parameter, β, and the scale parameter, θ, affect such distribution characteristics as the shape of the *pdf* curve, the reliability and the failure rate. The most general form of the Weibull is **the three-parameter** Weibull form where:

$$f(t)=\frac{\beta}{\theta}\left(\frac{x-\delta}{\theta}\right)e^{-\left(\frac{x-\delta}{\theta}\right)^{\beta}},F(t)=1-e^{-\left(\frac{x-\delta}{\theta}\right)^{\beta}}\Rightarrow,R(t)=e^{-\left(\frac{x-\delta}{\theta}\right)^{\beta}}$$

Where: β>0, θ>0, δ>0, and

β = shape parameter or slope

θ = scale parameter or the characteristic life used to locate the distribution on the x or t-axis. It is that value on the time axis where the probability of failure F(t)=0.632

δ = the location or minimum life parameter.

The minimum life δ is important in its own right because it is an indication that the component initial failure time cannot be zero. In order words, the component has a minimum life that is greater than zero hence the notion of minimum life. As a result, the minimum life is often used as a basis for **product warranty specifications**. The minimum life parameter is rarely used and hence its value is frequently set to zero unless there is prior knowledge about its existence and value. When the minimum life δ isset to zero, then the Weibull becomes a two-parameter distribution. In this case, the *pdf* equation reduces to that of the two-parameter Weibull distribution.

$$f(t)=\frac{\beta}{\theta}\left(\frac{x}{\theta}\right)e^{-\left(\frac{x}{\theta}\right)^{\beta}},\quad F(t)=1-e^{-\left(\frac{x}{\theta}\right)^{\beta}}\Rightarrow R(t)=e^{-\left(\frac{x}{\theta}\right)^{\beta}}$$

$$\text{If}\ \ \alpha=(1/\theta)^{\beta},\ f(x)=\alpha\beta^{\beta-1}e^{-(\alpha x)^{\beta}},\ F(x)=1-e^{-(\alpha x)^{\beta}}\Rightarrow R(x)=e^{-(\alpha x)^{\beta}}$$

$$\ln\left[\ln\frac{1}{1-F(t)}\right]=\beta\ln(t-\delta)-\beta\ln(\theta-\delta)\Rightarrow\ln\left[\ln\frac{1}{1-F(t)}\right]=\beta\ln(t)-\beta\ln(\theta)\ \ if\ \delta=0$$

A major benefit of modeling life distributions with the Weibull is that the distribution is robust enough so that for different values of β **(the slope)** it is possible to accommodate a host of different distribution types. **For example, when β equals 1, the Weibull becomes the exponential. Also when β equals 4, the Weibull starts to resemble the normal distribution**.

The Weibull is also the distribution of choice for modeling the different regions of a component's or system's life profile such as the bathtub curve which consists of the early, the constant, and the wearout regions. The Weibull is particularly useful when the failure rate of the system is not constant but increases (IFR = increasing failure rate) or decreases (DFR = decreasing failure rate) over the system or component life profile.

The scale parameter θ is used to locate the distribution on the x-axis. For the Weibull distribution, substituting x = θ in the cumulative distribution gives: F(x = θ) = 1- e^{-1} = 0.632. In other words, the characteristic life is the time (or distance) where the reliability is 0.368 or 37%.
So for any Weibull distribution, the probability of failure prior to θ is 0.632. Thus θ will always divide the area under the probability density function (pdf) into 0.632 and 0.368 for all values of the slope **β**.

In the two-parameter Weibull, the minimum life is assumed to be zero. The mean and variance of the Weibull is as follows:

$$\mu = \theta\Gamma\left(1+\frac{1}{\beta}\right),\ \sigma^2 = \theta^2\left[\Gamma\left(1+\frac{2}{\beta}\right)\right]\ \Gamma^2\left(1+\frac{1}{\beta}\right)\ ,\ where: \Gamma(n) = (n-1)!$$

We can carry out a log-log transformation to linearize the relationship in order to extract the parameters easily via regression analysis. The log-log transformation is as shown.

$$\ln\left[\ln\frac{1}{1-F(t)}\right] = \beta\ln(t-\delta) - \beta\ln(\theta-\delta) \Rightarrow \ln\left[\ln\frac{1}{1-F(t)}\right] = \beta\ln(t) - \beta\ln(\theta)\ \ if\ \delta = 0$$

The Weibull parameters can be estimated in one of two ways, namely; by the graphical approach using the Weibull graph paper or a regular log-log paper) or by regression analysis.
Using the graphical approach and certain assumptions or rule of thumb about the minimum life δ, we can estimate all the parameters of the three-parameter Weibull. Alternatively we can use regression but we have to make some assumptions about minimum life. With regression we can only estimate the two parameters while the third parameter, namely the minimum life, (δ) is assumed or estimated using the graphical approach. The general recommendation for Weibull analysis is to start with a three-parameter model. If the Weibull plot of the initial data shows no obvious curvature, then it is assumed that the minimum life is zero. If there is curvature, then we use trial and error to estimate that value of the minimum life that results in a linear (straight) population line.

6.1.1 Computing the time t for a given Probability of Failure F(t)=p

$$R(t) = e^{-(t/\theta)^\beta}\ ,\ F(t) = 1 - R(t) = 1 - e^{-(t/\theta)^\beta}$$

$$Given\ F(t) = p, R(t) = 1-p,\ find\ t\ \ for\ which\ F(t) = p$$

$$(1-p) = e^{-(t/\theta)^\beta} \Rightarrow \ln(1-p) = -(t/\theta)^\beta \Rightarrow \ln(1/(1-p)) = (t/\theta)^\beta$$

$$\ln(\ln(1/(1-p))) = \beta\ln(t) - \beta\ln\theta \Rightarrow \ln(t) = [\ln(\ln(1/(1-p))) + \beta\ln\theta]/\beta$$

$$\Rightarrow t = \exp\{\ [\ln(\ln(1/(1-p))) + \beta\ln\theta]/\beta\}$$

6.2 Graphical Approach to Extracting Weibull Parameters

We should expect the plot to be linear if plotted on a Weibull paper (which also happens to be a log-log graph paper but with enhanced features to easily extract the slope- β parameter and the scale parameter θ) or on a regular log-log graph paper. If indeed the minimum life is zero, the plot will be a straight line. If it is not then the plot will have some curvature.

If based on the plot we conclude that the minimum life is non-zero, then we employ a rule of thumb which states as follows; If the data has non-zero minimum life, then the minimum life can be estimated in the first instance as $\delta = 0.9x_1$, where $\mathbf{x_1}$ is the data point corresponding to the first failure occurrence. This value can be adjusted up or down from the original data set depending on the nature of the curvature. If the estimate is too large, the graph will curve upwards and if it is too small it will curve downwards. Some trial and error adjustments are necessary to arrive at the value of δ that would produce a straight population line. The adjustment is made by simply subtracting the estimate of δ from all the failure data and then re-plotting. Once a satisfactory estimate of δ has been obtained, then the remaining estimates, namely, the slope and the characteristic life are estimated from the graph.

For the graphical approach, the plot is: t (or km) versus F(t) on a log-log graph paper or a Weibull graph paper. If by inspection, there seems to be minimal or no curvature, then it is assumed that the minimum life is zero. If there is noticeable curvature, then the minimum life is adjusted for by subtract $\mathbf{0.9x_1}$ (or whatever adjustment value seems appropriate) from all the failure data points and then re-plot.

1. After plotting the points on the graph, we then draw the best possible line through all the points. This results in the population line as shown.

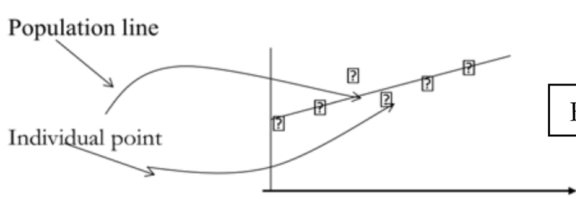


Fig 2: Plotting points on the graph

2). Computing the Characteristic Life θ

Draw a line through the 62.3% (the circle close to the top-half of the graph) point to the population line. At the intersection of these two lines, produce a line to the X-axis. The intersection of these two lines represents the characteristic life θ. Note that for θ, the reason 62.3% is used is obtained as follows. θ is the characteristic life and simply a specific value of t (X-axis). $R(t) = e^{-(t/\theta)^{\beta}}$, $at \;\; t = \theta, \;\; R(t = \theta) = e^{-(\theta/\theta)^{\beta}} = e^{-1} = 0.37$

F(t=θ) = 1-R(t=θ) = 0.623 = 62.3%, So at t=θ, F (t) = 62.3%

Thus given F (t) = 62.3%, then the resulting value of t =θ

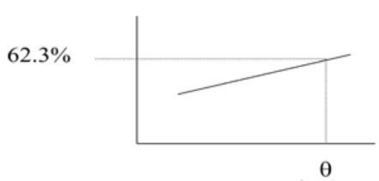


Fig 3: Estimating θ from the graph

3). Computation of β

Two ways of estimating β

i). Slope=Rise/Run : Note that the scales for estimating β using the above approach are on top (as shown) and on the right (as shown)

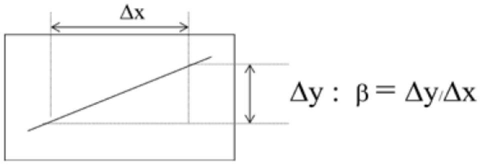


Figure 4: Estimating β using Run vs Rise

ii). At the top of the graph paper is another circle and a scale at the left hand side. Draw a line parallel to the population line through the midpoint of the circle as shown. Produce this line to the scale on the left. The intersection of this line and scale is β (or m).

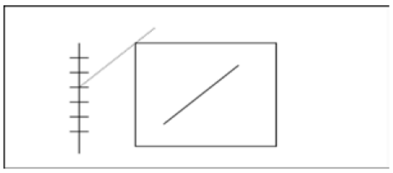


Figure 5. Estimating β using special scale on the Weibull graph

Example: The data of table 8a represents the life of precision grinder wheels measured in number of pieces produced from the grinding process before the wheel expired.

Table 8a: Grinding Precision Wheel Life	
Wheel No. (i)	Pieces Per Wheel
1	22000
2	25000
3	30000
4	33000
5	35000
6	52000
7	63000
8	104000

Table 8b: Grinding Precision Wheel Life with Median Rank and δ=19600			
Wheel No. (i)	Pieces/Wheel	Median Rank	(δ=19600) Adjusted Pieces/Wheel
1	22000	0.08333	2420
2	25000	0.20238	5420
3	30000	0.32143	10420
4	33000	0.44048	13420
5	35000	0.55952	15420
6	52000	0.67857	32420
7	63000	0.79762	43420
8	104000	0.91667	84420

To determine F(t), we use the formula for the median estimate given by: $F_i(t) = \frac{i - 0.3}{n + 0.4}$

Based on the plot (figure 6), it is clear that indeed, the minimum life δ for this Precision Grinding Wheel Data is not zero. Given that information and the fact that we have prior information about the possible location of δ, we use the rule of thumb (0.891^*x_1) rather than the recommended $0.9x_1$, to compute and estimate of the minimum life, that is, δ=0.891(22,000)=19,600. With this value of δ=19,600; we now have the adjusted data as shown in Table 8b. Upon plotting (see figure 7), we observe that the plot resembles a straight line so our estimate of δ =19,600. We also estimate the values of the characteristic life (θ) as (18000+19,600)=37,600 and the slope (β) as 0.94

6.3 Regression Approach to Extracting Weibull Parameters Using EXCEL

$$\ln\left[\ln\frac{1}{1-F(t)}\right] = \beta\ln(t-\delta) - \beta\ln(\theta-\delta) \Rightarrow \ln\left[\ln\frac{1}{1-F(t)}\right] = \beta\ln(t) - \beta\ln(\theta) \quad if\ \delta = 0$$

$$Similar\ to:\ Y = \beta X + C,\quad F(t) = \frac{i-0.3}{n+0.4} = Median\ \ Estimate,\ X = \ln(t),\ C = -\beta\ln(\theta)$$

Table 9a: Summary Output for Regression of Precision Grinding Wheel Data Using EXCEL	
Regression Statistics	
Multiple R	0.989028
R Square	0.978177
Adjusted R Squared	0.974540
Standard Error	0.174318
Observations	8.000000
Intercept	-9.584174
X Variable 1(slope)	0.940157

We employ the Regression Analysis Utility in EXCEL to carry out the analysis. The results are as shown in table 9a and 9b and we notice as follows

$$-\beta\ln(\theta) = C = -9.584174,\ \ The\ Slope\ \beta = 0.9402\,.$$

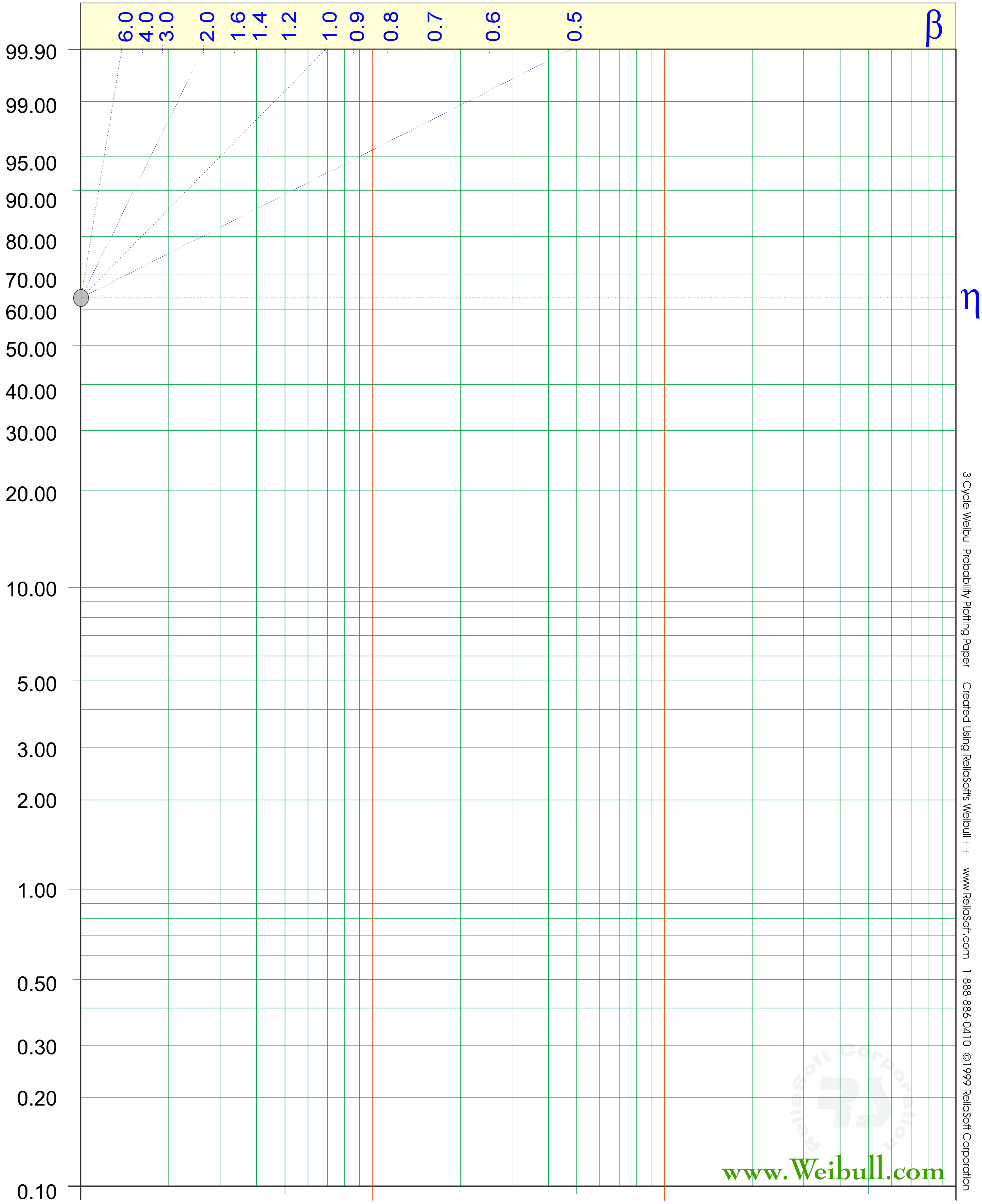

β
6.0
4.0
3.0
2.0
1.6
1.4
1.2
1.0
0.9
0.8
0.7
0.6
0.5
η
99.90
99.00
95.00
90.00
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
5.00
3.00
2.00
1.00
0.50
0.30
0.20
0.10
www.Weibull.com

Figure 7: Plot after Adjusting for the Minimum Life (delta)

{Slope (Beta)=0.94} {Minimum Life (delta)=19,600} {Characteristic life 'Theta' =24000+19,600=43,600}

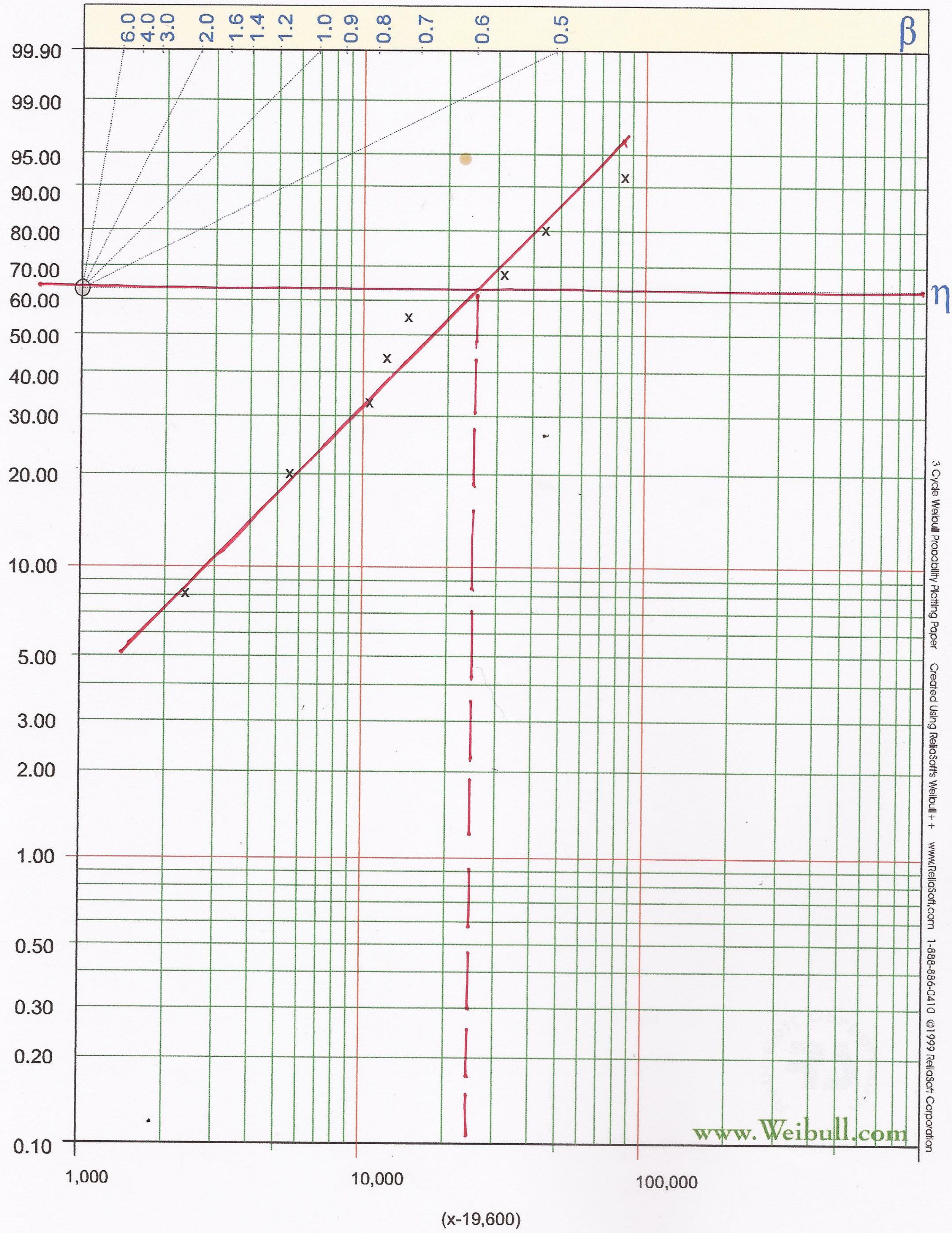


Table 8c: Grinding Precision Wheel Life with Median Rank and δ=19600						
Wheel No (i)	Pieces/Wheel	(δ=19600) Adjusted Pieces/Wheel	Ln(Pieces/Wheel)	Median Rank	1/(1-Median Rank)	Ln(Ln(1/(1-Median Rank)
1	22000	2420	7.791523	0.08333	1.09091	-2.44172
2	25000	5420	8.597851	0.20238	1.25373	-1.48667
3	30000	10420	9.251482	0.32143	1.47368	-0.94735
4	33000	13420	9.504501	0.44048	1.78723	-0.54357
5	35000	15420	9.643421	0.55952	2.27027	-0.19857
6	52000	32420	10.386531	0.67857	3.11111	0.12661
7	63000	43420	10.678675	0.79762	4.94118	0.46850
8	104000	84420	11.343560	0.91667	12.00000	0.91024

$$-\beta\ln(\theta) = C = -9.5842 \Rightarrow \theta = \left(e^{\frac{9.5842}{0.94}}\right) + (19{,}600) = 26795 + 19600 = 46{,}395$$

Table 9b: Summary Output for Regression of Precision Grinding Wheel Data Using EXCEL						
ANOVA						
	df	*SS*	*MS*	*F*	*Sig F*	
Regression	1	8.1740538	8.174054	271.8068965	3.174E-06	
Residual	6	0.1804381	0.030073			
Total	7	8.3544919				
	Coefficients	*Std Error*	*t -Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-9.5841744	0.5535573	-17.3138	2.379E-06	-10.93868	-8.229669
X Variable 1(slope)	0.9401569	0.0570256	16.48657	3.174E-06	0.8006202	1.0796936

6.4 Least Squares Method for Extracting Weibull Parameters

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}} \quad if \quad \alpha = \left(\frac{1}{\theta}\right)^{\beta} = \frac{1}{\theta^{\beta}} \Rightarrow R(t) = e^{-\alpha t^{\beta}}$$

$$F(t) = 1 - e^{-\alpha t^{\beta}}$$

$$\ln(1 - F(t)) = -\alpha t^{\beta} \Rightarrow \ln\left(\frac{1}{1 - F(t)}\right) = \alpha t^{\beta}$$

$$\ln\left(\ln\left(\frac{1}{1 - F(t)}\right)\right) = \ln\alpha + \beta\ln(t)$$

$$\Rightarrow Y = A + \beta X \Rightarrow A = \ln\alpha \Rightarrow \alpha = e^{A}, X = \ln(t)$$

Using Least Square procedure,

$Y = A + \beta X + \varepsilon, \qquad \varepsilon = Y - A - \beta X$

$\varepsilon^2 = (Y - A - \beta X)^2$

$\Sigma\varepsilon^2 = \Sigma(Y_i - A - \beta X_i)^2$

Minimizing $\Sigma\varepsilon^2$ => differentiating with respect to the parameter A, β

$\partial Q/\partial A = -2\Sigma(Y_i - A - \beta X_i)$

$\partial Q/\partial \beta = -2\Sigma(Y_i - A - \beta X_i)(X_i)$

$\sum Y_i = nA + \beta\sum X_i$

$\sum X_i Y_i = A\sum X_i + \beta\sum X_i^2$

$$\begin{bmatrix} \sum Y \\ \sum XY \end{bmatrix} = \begin{bmatrix} n & \sum X \\ \sum X & \sum X^2 \end{bmatrix} \begin{bmatrix} A \\ \beta \end{bmatrix}, \quad \Rightarrow \beta = \frac{\left(n\sum XY - \sum X \sum Y\right)}{\left(n\sum X^2 - \left(\sum X\right)^2\right)}$$

$$A = \overline{Y} + \beta\overline{X}$$

$$\alpha = e^A, \; X = \ln t$$

Table 10a: Failure time (hrs) for Drive Shaft for 100 Shafts (N=100)

Item #	t	F(t)=Median Rank	X=ln(t)	1/(1-F(t))	ln(1/(1-F(t)))	Y=lnln(1/(1-F(t)))	X²	XY
1	6	0.0050	1.79175	1.005025	0.00501	-5.2958	3.21040	-9.4888
2	21	0.0150	3.04452	1.015228	0.01511	-4.1922	9.26911	-12.763
3	50	0.0250	3.91202	1.025641	0.02532	-3.6762	15.3039	-14.381
4	84	0.0350	4.43081	1.036269	0.03563	-3.3346	19.6321	-14.775
5	95	0.0450	4.55387	1.047120	0.04604	-3.0782	20.7377	-14.017
6	130	0.0550	4.86753	1.058201	0.05657	-2.8723	23.6928	-13.980
7	205	0.0650	5.32301	1.069519	0.06721	-2.7000	28.3344	-14.371
8	260	0.0750	5.56068	1.081081	0.07796	-2.5515	30.9211	-14.188
9	270	0.0850	5.59842	1.092896	0.08883	-2.4210	31.3423	-13.553
10	370	0.0950	5.91350	1.104972	0.09982	-2.3044	34.9695	-13.627
11	440	0.1050	6.08677	1.117318	0.11093	-2.1988	37.0488	-13.383
12	480	0.1150	6.17378	1.129944	0.12217	-2.1024	38.1156	-12.979
			57.2567		Sum⇒	-36.727388	292.578	-161.51

Table 10b. Regression Statistic for Failure time For Drive Shaft with n=100	
β	0.70827854
X-bar	4.77139254
Y-bar	-3.0606156
A	-6.4400906
α	0.00159626
$\theta=\alpha^{-1/\beta}$	8889.23264
n	100

$\Sigma X_i=57.24$, $\Sigma X_i^2=292.4392$, $\Sigma Y_i= -6.356$, $\Sigma X_iY_i= -14.008$

The Mean time Between Failure: $\mu = \theta[\Gamma(1+1/\beta)]$

The variance; $\sigma^2 = \theta^2[\Gamma(1+2/\beta) - \Gamma^2(1 + 1/\beta)]$

Gamma Function of x ; $\Gamma(x) = (x-1)!$ If x is integer

For non-integer x

$$\Gamma(x) = (x-1)\Gamma(x-1) = (x-1)(x-2)\Gamma(x-2)$$

$$e.g.\ \Gamma(3.4) = \Gamma(1+2.4) = 2.4\Gamma(2.4) = 2.4\Gamma(1+1.4) = 2.4(1.4)\Gamma(1.4) = 2.4(1.4)(0.887264) = 2.9812$$

$\mu = \theta[\Gamma(1+1/\beta)]=8889.23264[\Gamma(1+1/0.70827854)= 8889.23264\ \Gamma(2.4119)$

$\Gamma(2.4119)= \Gamma(1+1.4119)=1.4119\Gamma(1.4119)$, From the Gamma table $\Gamma(1.4119)=0.8886676$

$\Gamma(2.4119)= 1.4119\Gamma(1.4119)=1.4119(0.8886676)=$**<u>1.2547</u>**

$\mu = \theta[\Gamma(1+1/\beta)]=8889.23264(1.2547)=$<u>11153.41</u>

$\sigma^2 = \theta^2[\Gamma(1+2/\beta) - \Gamma^2(1 + 1/\beta)]$, $\Gamma(1+2/\beta)= \Gamma(3.82374)= \Gamma(1+2.82374)=2.82374\Gamma(2.82374)$

$2.82374(1.82374)\ \Gamma(1.82374)= 2.82374(1.82374)\ 0.937997=$**<u>4.830</u>**

$\sigma^2 = \theta^2[\Gamma(1+2/\beta) - \Gamma^2(1 + 1/\beta)]=8889.23264[4.830-(1.2547)^2]=$<u>28940.92</u>

<u>Note</u>: In the case of the Weibull, θ represents the characteristic life. To avoid confusing it with θ the mean time to failure (MTTF) for the Exponential, some use the notation of η (Neta) for the Weibull characteristic life

6.5 Least Squares Method for Extracting Exponential Parameter (θ)

$$R(x)] = e^{-\left(\frac{x}{\theta}\right)} \Rightarrow F(x) = 1 - e^{-\left(\frac{x}{\theta}\right)}$$

$$Taking\ \log : \ln\left(\frac{1}{1-F(x)}\right) = \frac{x}{\theta} \quad \Rightarrow Y = \beta x,\ where\ the\ slope\ \beta = \frac{1}{\theta}$$

$$i.e.,\ F(x=\theta) = 1 - e^{-1}$$

$$Y = \beta x, \quad where: Y = \ln\left(\frac{1}{1-F(x)}\right)$$

$$\sum e^2 = \sum (Y - \beta x)^2 = Q$$

$$\frac{dQ}{dx} = 2\sum (Y - \beta x)x$$

$$\frac{dQ}{dx} = 0 \Rightarrow 2\sum (Y - \beta x)x = 0 \Rightarrow \sum Yx - \beta \sum x^2 = 0, \Rightarrow \beta = \frac{\sum xY}{\sum x^2}$$

Example: The data in Table 11 is the bottle bursting strength in psi. We want to estimate the Mean Time To Failure (MTTF) θ, assuming the data is has the exponential distribution.

Table 11: Bottle Bursting Strength in psi

Rank (i)	Median Rank=F(x) =(i-0.3)/(20+0.4)	(1/(1-F(x))	y=ln(1/(1-F(x)))	x=Strength	xy	x^2
1	0.034	1.036	0.035	197	6.8785	38809
2	0.083	1.091	0.087	200	17.4023	40000
3	0.132	1.153	0.142	215	30.5236	46225
4	0.181	1.222	0.200	221	44.2279	48841
5	0.230	1.299	0.262	231	60.4929	53361
6	0.279	1.388	0.328	242	79.3004	58564
7	0.328	1.489	0.398	245	97.5441	60025
8	0.377	1.606	0.474	258	122.2747	66564
9	0.426	1.744	0.556	265	147.3257	70225
10	0.475	1.907	0.645	265	171.0022	70225
11	0.525	2.103	0.743	271	201.4638	73441
12	0.574	2.345	0.852	275	234.3583	75625
13	0.623	2.649	0.974	277	269.8851	76729
14	0.672	3.045	1.113	278	309.5328	77284
15	0.721	3.579	1.275	280	357.0192	78400
16	0.770	4.340	1.468	283	415.4362	80089
17	0.819	5.514	1.707	290	495.0886	84100
18	0.868	7.556	2.022	301	608.7072	90601
19	0.917	12.000	2.485	318	790.2003	101124
20	0.966	29.143	3.372	346	1166.785	119716
			Sum	5258	5625.448	1409948

Prob F(x) = (i - 0.3)/(20 + 0.4)

$$\beta = \frac{\sum xY}{\sum x^2} = \frac{5625.448}{1409948} = 0.00399, \ \theta = \frac{1}{\beta} = 250.6374\,hours$$

Dependency/Linked Configuration Analysis

In certain real life situations, we encounter system configurations where the system components are not strictly in series or in parallel. In such a situation, the series/parallel configuration analysis we considered in the previous course will no longer suffice. To solve the problem requires looking at the system or subsystem linkages or component or subsystem dependencies and how the different states of the component or subsystem will make for failure or success of the overall system.

7.1 Dependency Analysis

In the first case we will examine, we will assume that a component success is dependent on the success of a preceding component attached to it. In other words, for two components that are connected, then the success of the forward components depends on whether the components (or component) connected to it directly from the rear or from the side is active or not. A component is considered active if it can send a signal from itself to another component or components that receive signal from it and propagate that signal to the rest of the system resulting in system operation or success. This dependency situation can be readily analyzed by complete enumeration.

In the complete enumeration approach, the idea is to look a subsisting configuration and for each component and that component only, determine the number of ways the system or subsystem will survive or fail. In this approach we analyze either system failure or success but not both at the same time. In figure 8, we have system configuration made up of two subsystems (∝) and (β) where the components are not strictly in series or parallel see (figure 8b).

Figure 8. Dependent System Configuration

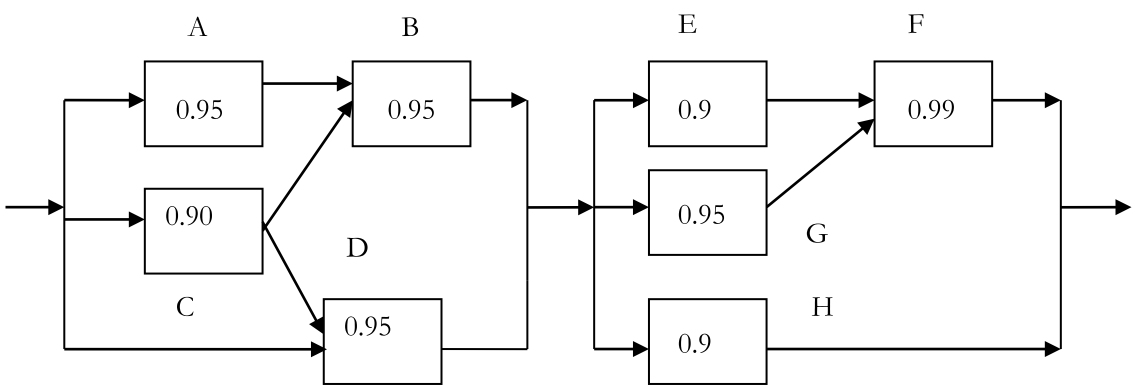
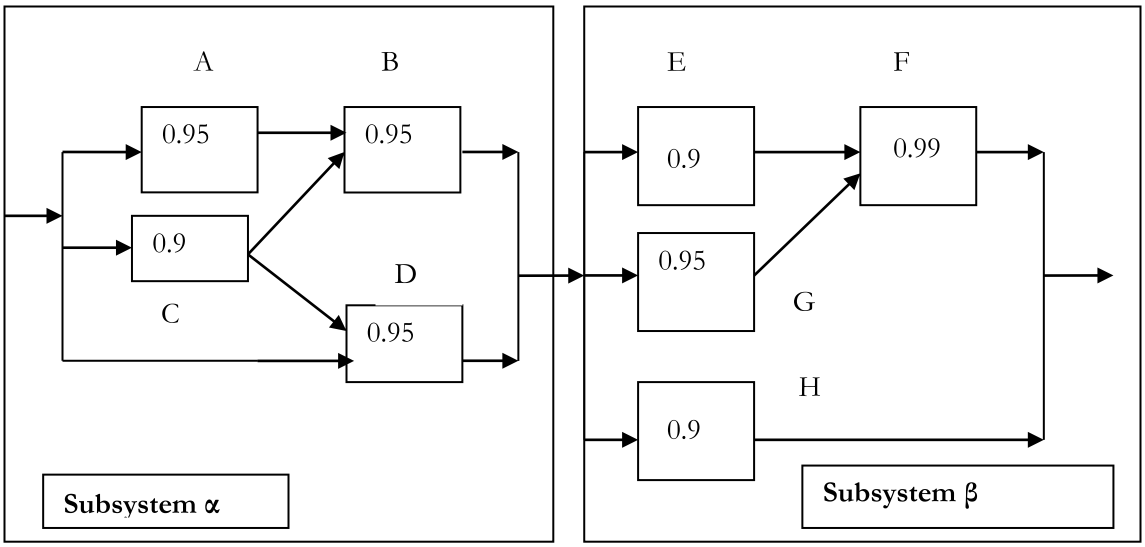


Fig 8b. Dependent System Configuration with subsystems α and β



Consider the following notation for ease of analysis: E_i = Event component i is operative with probability p, and $\overline{E_i}$ = Event component i is not operative with probability q. Using explicit enumeration we can define or determine the mutually exclusive ways a system of n components would work or fail, i.e.

$$(p+q)^n = p^n + {}^nC_1 q^1 p^{n-1} + {}^nC_2 q^2 p^{n-2} + \ldots + {}^nC_n q^n p^{n-n} = p^n + {}^nC_1 q^1 p^{n-1} + {}^nC_2 q^2 p^{n-2} + \ldots + q^n$$

Where ${}^nC_i = \frac{n\,!}{i\,!\,(n-i)\,!}$, $n! = nx(n-1)(n-2)\ldots\ldots(n-(n-1))$

Note: 0! =1, 1! =1

If n=5, n!=5x(4)!=5x4x(3)!=5x4x3x(2)!= 5x4x3x2(1)!= 5x4x3x2x1=120, 4!=4(3)(2)(1)=24

For n=5, and i=2, $\frac{5\,!}{2\,!\,(5-2)\,!} = \frac{5x4x(3)!}{2!(3)!} = \frac{5x4}{2x1} = 10$

While this expansion represents the mutually exclusive ways in which the system would work or fall, the coefficient of each term (the ith component) of the expansion represents the maximum number of ways in which i components out of n would fail or survive. ***Note:*** *The maximum number of ways is: ${}^nC_i q^i p^{n-i}$ for i working components.*

The actual number of ways in which the system will be operative given the failure of i components is determined by the system architecture or configuration but that number would be no more than ${}^nC_i$. Because of the large number of ways the system could work when i components fail, it is easier to evaluate the system failure rather than its success.

Consider the first subsystem α with four components *(n=4, i=1, 2, 3, 4)*

The coefficients may be obtained by using the Pascal triangle as shown:

i	${}^nC_i$ = coefficients for i failed components out-of-n (n=4)
0	1
1	4
2	6
3	4
4	1

Give the configuration of subsystem ∝, we have the following probabilities of failures.

Table 13a: Component Failures Resulting in failure of Subsystem ∝

i	${}^nC_i$	Actual no.	Failure Event	Probabilities of the event occurrence
4	1	1	$\overline{D}\cap\overline{B}\cap\overline{A}\cap\overline{C}$	(.05)(.05)(.05)(.1)=0.0000125
3	4	3	$\overline{D}\cap\overline{B}\cap A\cap\overline{C}$	(.05)(.05)(.95)(.1)=0.0002375
			$\overline{D}\cap\overline{B}\cap\overline{A}\cap C$	(.05)(.05)(.05)(.9)=0.0001125
			$\overline{D}\cap B\cap\overline{A}\cap\overline{C}$	(.05)(.95)(.05)(.1)=0.0002375
2	6	1	$\overline{D}\cap\overline{B}\cap A\cap C$	(.05)(.05)(.95)(.9)=0.0002375
1	4	0	----------None--------------	=0.0000000
			Total probability of Failure ∝	=0.0027375

For example; consider the case where 3-out-of-the 4 components fail. What does this do to the system operation? The possibilities that would result in subsystem failure when 3-out-of 4 components fail are as follows:

$$\overline{D}\cap\overline{B}\cap A\cap\overline{C}$$
$$\overline{D}\cap\overline{B}\cap\overline{A}\cap C$$
$$\overline{D}\cap B\cap\overline{A}\cap\overline{C}$$

These three are the only possibilities that would result in subsystem failure even through the formula (which did not consider the actual system configuration) says it is 4. The others (*i=1, 2 and 4*)are as shown. The resulting total probability of failure is

Total probability of Failure =0.0027375

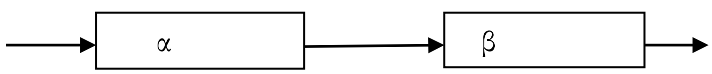
$$R(\overline{\alpha}) = P(\overline{\alpha}) = 0.0027375,$$
$$R(\alpha) = 0.9972625$$

Given the configuration of subsystem β, we have the following probabilities of failures.

Table 13b: Component Failures Resulting in failure of Subsystem β

i	^nC_i	*Actual no.*	*Failure Event*	*Probabilities of the event occurrence*
4	1	1	$\bar{H} \cap \bar{F} \cap \bar{E} \cap \bar{G}$	(.1)(0.01)(.1)(0.05)=0.000005
3	4	3	$\bar{H} \cap \bar{F} \cap \bar{E} \cap G$	(.1)(.01)(.1)(.95)=0.000095
			$\bar{H} \cap \bar{F} \cap E \cap \bar{G}$	(.1)(.01)(.05)(.9)=0.000045
			$\bar{H} \cap F \cap \bar{E} \cap \bar{G}$	(.1)(.1)(.05)(.99)=0.000495
2	6	1	$\bar{H} \cap \bar{F} \cap E \cap G$	(.1)(.01)(.9)(.95)=0.000855
1	4	0	None	=0.000000
0	4	0	None	=0.000000
			Total probability of Failure	=0.001495

$$R(\bar{\beta}) = P(\bar{\beta}) = 0.001495, \text{ and } \quad R(\beta) = 0.998505$$



System Reliability is given by the reliability of

$R(s) = R(\alpha) = R(\alpha)R(\beta) = (0.9972625)(0.998505) = 0.99571159$

Therefore the system reliability is: 0.99571159

6.2 Linked Configuration Analysis

Linkage analysis may also be used to analyze systems that cannot be decomposed into strictly series or strictly parallel configuration. Such systems are also referred to as dependent component system. To analyze such a system, we consider two mutually exclusive states namely the 'linking' component failed or did not fail.

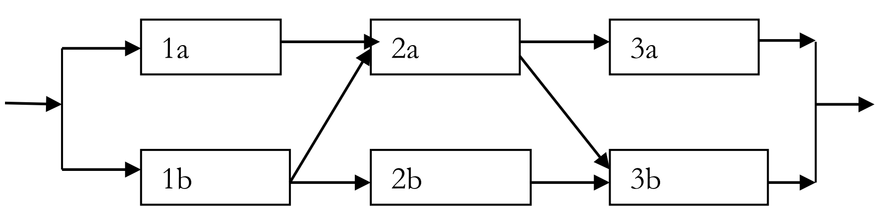


Figure 9: Linked Configuration

Given the system topology, we can consider the link in the system is 2a.

Define R^-, the reliability, given the failure of 2a, and

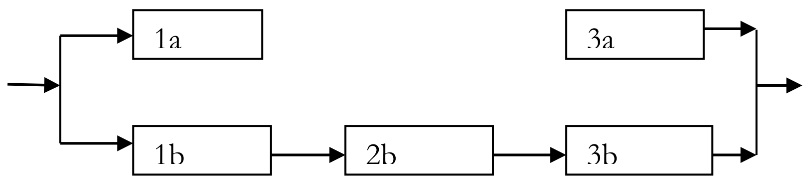
Define R^+, given the successful operation of 2a,

Then since $1-R_2$ is just the probability that it will not, we may write the system reliability as:

$R_S = R^-(1-R_2) + R^+R_2$

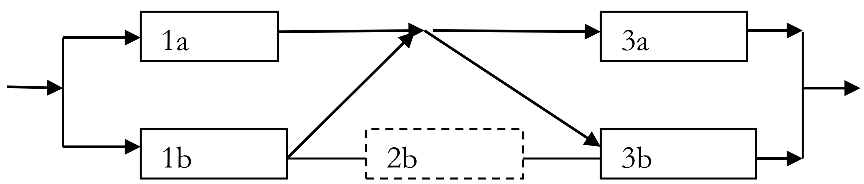
1. <u>Assume 2a Fails</u>, that is R^- :

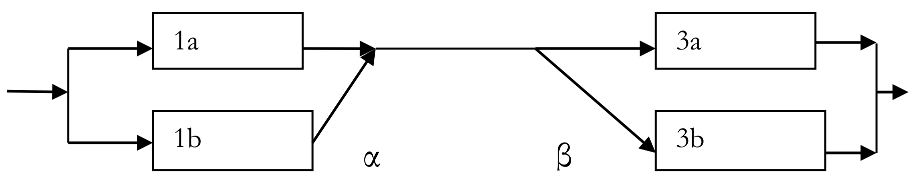
If component 2a fails (the linking component fails), then the system configuration will look like:



$R^-(\text{2a fails}) = R_1R_2R_3$

2. Assume 2a operational, that is R^+ :





$R\alpha = 1-[(1-R_1)(1-R_1)] = 1-[1-2R_1 + R_1^2] = = (2R_1 - R_1^2)$

$R\beta = 1-[(1-R_3)(1-R_3)] = 1-[1-2R_3 + R_3^2] = = (2R_3 - R_3^2)$

Since α and β are in series, the $R_{\alpha\beta} = R\alpha\, R\beta$

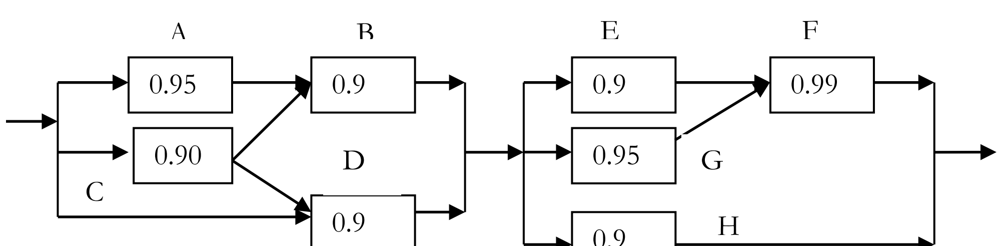
$R^+(\text{2a operational}) = (2R_1 - R_1^2)(2R_3 - R_3^2)$

System Reliability is given as: $R_S = R^-(1-R_2) + R^+R_2$

$R_S = R^-(1-R_2) + R^+R_2 = R_1R_2R_3(1-R_2) + (2R_1 - R_1^2)(2R_3 - R_3^2)\, R_2$

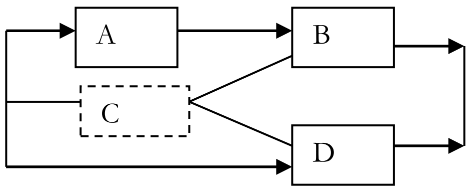
6.2.1 Example of Linked Configuration based on Figure 8:

We will apply the linked configuration approach to figure 8



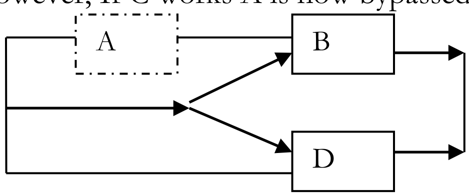
<u>System Reliability</u>: $R = R^-(1-R_2) + R^+R_2$

1. **For α,** link is provided by C. Hence if C fails we have:



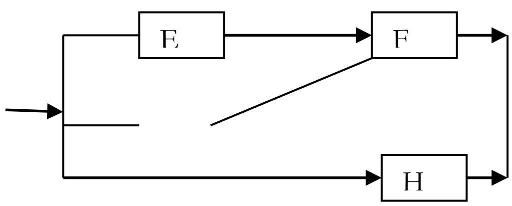
$R^- = 1-[1-P(A)P(B)][1-P(D)] = 0.995125$

However, If C works A is now bypassed because there is no resistance through C



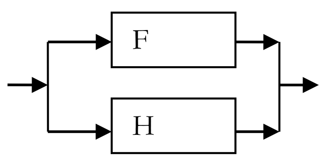
$R^+ = 1-[1-P(B)][1-P(D)] = 0.9975$

2. **For β,** link is provided by G. Hence if G fails, we have



$R^- = 1-[1-(.9)(.99)(1-0.9)] = 0.9891$

However, if G works, both E and H are bypassed



$R^+ = 1-(1-0.9)(1-0.99) = 0.999$

<u>**System Reliability**</u>:

$$R_s = R_\alpha R_\beta = [(0.995125)(0.1) + (0.9975)(0.9)] *[(0.9891)(0.05) + (0.999(0.95)]$$
$$= 0.9972625*0.998505 = 0.9957715925625$$

Summary

A major focus of this course is primarily the rudiments of testing, data handling including plotting. We have looked different types of tests to ensure that we obtain estimates that are both realistic and statistically. System reliability goals and the attendant component reliability requirements needed to achieve those goals are given prominence during the initial conception design and definition of the system. The parts count method as proposed by **MIL-HDBK-217** is recognized world-wide as the pre-eminent reliability prediction document used to estimate reliability from standard components.

The Weibull distribution is perhaps the most important distribution used in reliability analysis because of its robustness. The determination of Weibull parameters therefore is a key issue in the use of the Weibull distribution. We have demonstrated how to estimates Weibull parameters using both regression and Weibull plots using special Weibull graph paper.

Lastly we have also included an important configuration type in reliability analysis, namely dependency/linkage analysis. These types of configurations occur especially in those situations where the system topology cannot be classified as strictly series or parallel. While may seem like an afterthought, it turns that these configurations represents perhaps more than seventy percent of most practical system configurations and topology. We have demonstrated their application with practical examples.

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