Basic Trigonometry, Significant Figures, and Rounding

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A Quick Review

(Free of Charge and Not for Credit)

by

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Basic Trigonometry, Significant Figures, and Rounding – A Quick Review is a review of some fundamental principles of basic math for use by engineers. This course is prepared for those who might find themselves a bit rusty and would like a quick refresher.

The information in the course is useful for application to the solution of structural problems especially in the fields of statics and strength of materials.

The trigonometry review includes demonstrating - through the use of several example problems – the use of the basic trigonometric functions including:

- the sine (and its inverse, $\sin^{-1}$)
- the cosine (and its inverse, $\cos^{-1}$)
- the tangent (and its inverse, $\tan^{-1}$)
- the Pythagorean Theorem
- the Sum of the Angles
- the Law of Sines; and
- the Law of Cosines

The significant figures and rounding review includes a discussion of the precision and validity of an answer, along with rules and guidelines for using the appropriate number of significant figures, and for rounding answers appropriately.

Trigonometry Review
All triangles have six values – three side lengths and three interior angles. And **all** triangle problems must have at least three known values and at most three unknown values. The solution to a triangle problem – or to solve a triangle problem – is to find the unknown values.

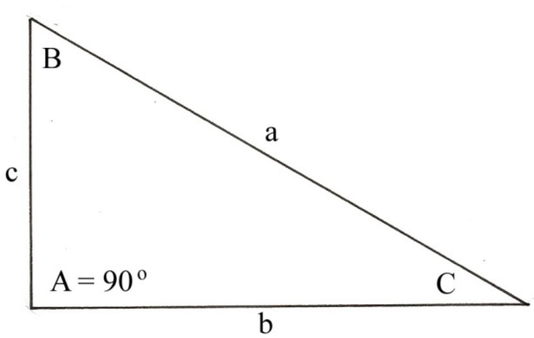
Right Triangle – A right triangle is simply a triangle where one of the angles is 90°. Since one angle is known, 90°, only two other values must be known to solve the triangle problem. And of those two other known values, at least one must be the length of a side. A triangle with three known angles – and not a length of a side – can't be solved because there are an infinite number of triangles that can be drawn when only the angles are known – from very small triangles to very big triangles. They will all be shaped the same.

Right-Triangle Trigonometric Relationships – For the right triangle shown below, angle A is the right angle, the 90° angle. The side opposite the 90° angle (identified by the corresponding lower case letter "a") is called the hypotenuse (hyp) of the triangle.

The other two angles of the triangle ("B" and "C") also each have an opposite side. The opposite side of an angle is usually identified with the same letter of the angle only in the opposite case – in this case "b", and "c". The angle and the side opposite are usually the same letter – one is a capital letter and the other is a lower case letter. In the triangle below, we used the upper case as the angle and, therefore, the lower case as the side opposite.

In a right triangle, the two angles that are not 90° have an adjacent (adj) and an opposite (opp) side. For angle B, the opposite side is "b", and the adjacent side is "c". For angle C, the opposite side is "c" and the adjacent side is "b".

Trigonometric Relationships



Right Triangle

$$\sin B = \frac{opp}{hyp} = \frac{b}{a}$$

$$\cos B = \frac{adj}{hyp} = \frac{c}{a}$$

$$\tan B = \frac{opp}{adj} = \frac{b}{c}$$

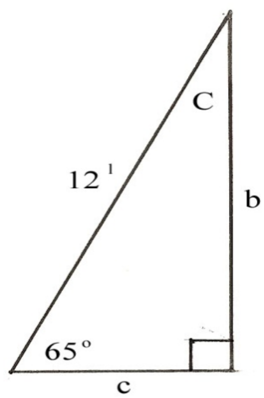
Pythagorean Theorem
$a^2 = b^2 + c^2$

Sum of the Angles
$A + B + C = 180°$
$B + C = 90°$

To solve a right triangle, the right angle trigonometric relationships, the Pythagorean Theorem, and the Sum of the Angles are needed. These are very important relationships and are shown above. They apply to the right triangle shown above.

Often the right angle is not noted as "A = 90°" as shown above. It is common practice to simply draw a small "box" in the corner of the right angle as shown in the triangle below. It is understood that the angle is 90°.

Example: Find the unknown values of the right triangle if the known values are as shown below.



A + B + C = 180°
C = 180° - A – B
C = 180 – 90 – 65
C = 25°

Sin 65° = opp / hyp = b/12
b = (12') x (Sin 65°)
b = (12) x (0.9063)
b = 10.9'

$a^2 = b^2 + c^2$
$c^2 = a^2 - b^2$
$c^2 = (12)^2 - (10.9)^2$
$c^2 = 144 - 119$
$c^2 = 25$
c = 5'

Scalene Triangle – A scalene triangle is a triangle that does NOT have a 90° angle. All three sides are usually different lengths (but don't have to be). Of the three known values, at least one must be the length of a side because, again, a triangle with only three known angles cannot be solved.

Scalene Triangle Trigonometric Relationships – In addition to the trigonometric relationships for the right triangle, frequently the law of sines and the law of cosines will be required to solve for the three unknowns.

Law of Sines – The law of sines states that the ratio of the length of a side to the sine of the opposite angle is a constant.

$$\frac{a}{sin\ A} = \frac{b}{sinB} = \frac{c}{sinC}$$

Law of Cosines – The law of cosines is stated below. Notice that if C = 90°, then Cos C = 0, and the equation becomes the Pythagorean Theorem.

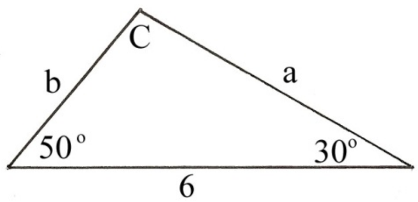
$$c^2 = a^2 + b^2 - 2\ a\ b\ \text{Cos}\ C$$

The equation can also be written as:

$$a^2 = b^2 + c^2 - 2\ b\ c\ \text{Cos}\ A$$
$$b^2 = a^2 + c^2 - 2\ a\ c\ \text{Cos}\ B$$

The following examples are triangle solutions for different combinations of known values.

Example: Find the unknown values of the triangle if the known values are in an **angle-side-angle** sequence.



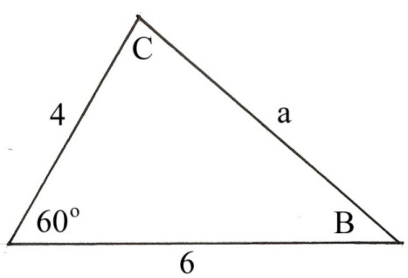
C = 180° - 50° - 30° = <u>100°</u>

$$\frac{6}{\sin 100} = \frac{a}{\sin 50} = \frac{b}{\sin 30}$$

$$a = \frac{\sin 50}{\sin 100}\, x\,(6) = \frac{0.7660}{0.9848}\, x\,(6) = 4.67$$

$$b = \frac{\sin 30}{\sin 100}\, x\,(6) = \frac{0.500}{0.9848}\, x\,(6) = 3.05$$

Example: Find the unknown values of the triangle if the known values are in a **side-angle-side** sequence.



$$a^2 = b^2 + c^2 - 2bc\, cosA$$
$$a^2 = 4^2 + 6^2 - (2)(4)(6)(\,cos60)$$
$$a^2 = 16 + 36 - 24 = 28$$
$$a = \sqrt{28}$$
$\underline{a = 5.29}$

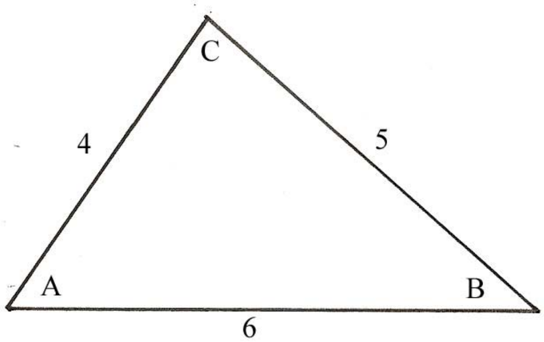
$$\frac{sin60}{5.29} = \frac{0.866}{5.29} = 0.1637 = \frac{sinC}{6} = \frac{sinB}{4}$$

$$sinC = 6\, x\, 0.1637 = 0.9822$$
$$C = sin^{-1}(0.9822)$$
$\underline{C = 79.1°}$

$$sinB = 4\, x\, 0.1637 = 0.6548$$
$$B = sin^{-1}(0.6548)$$
$\underline{B = 40.9°}$

60° + 79.1° + 40.9° = 180° check ✓

Example: Find the unknown values of the triangle if the known values are a **side-side-side** sequence.



$2abcosC = a^2 + b^2 - c^2$
$2\,x\,5\,x\,4\,x\,cosC = 5^2 + 4^2 - 6^2$
$40cosC = 5$
$C = cos^{-1}\left(\frac{5}{40}\right)$
<u>C = 82.8°</u>

$\frac{sin82.8}{6} = \frac{0.992}{6} = 0.1654$

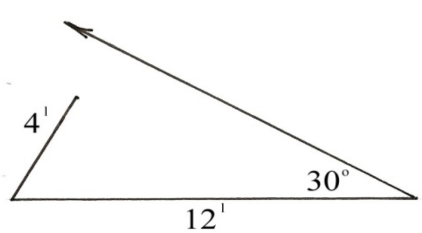
$\frac{sinA}{5} = 0.1654$
$A = sin^{-1}(5\,x\,0.1654)$
<u>A = 55.8°</u>

$B = sin^{-1}(4\,x\,0.1654)$
<u>B = 41.4°</u>

82.8° + 55.8° + 41.4° = 180° check ✓

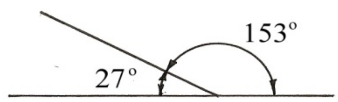
Example: Find the unknown values of the triangle if the known values are a **side-side-angle** sequence. With these three known values there can be <u>three different outcomes</u> when solving for the unknown values

<u>Outcome 1</u> – there can be no solution. In the example below, the leg opposite the 30° angle is too short to reach the leg of unknown length and therefore there is no solution.



<u>Outcome 2</u> – there can be only one solution. If the triangle is a right triangle, there is only one solution. And it is easy to solve using the trig functions.

Outcome 3 – there can be two solutions. For angles between 0° and 180°, the value of the sine of the angle will always be positive. When the angle is 0° the sine is zero. As the angle increases, the value of the sine also increases up to a maximum value of 1.000 at 90°. As the angle continues to increase past 90°, the value of the sine of the angle **decreases** back down to zero at 180°. So, when using the value of the sine to get the value of the angle (when using the Law of Sines), the calculator will not tell you if the angle is less than 90° or more than 90°. The calculator will return identical values for two angles whose sum is 180° as shown below.



sin 27° = 0.4540   sin 153° = 0.4540

When solving a scalene triangle using the Law of Sines, be careful to use the correct value of the angle. When you solve for an unknown angle using the sine, for example sin A = 0.4540, the calculator will probably return a value of 27°. Use your common sense to determine if that is the correct angle – or not.

Here is an example showing two solutions to a scalene triangle.

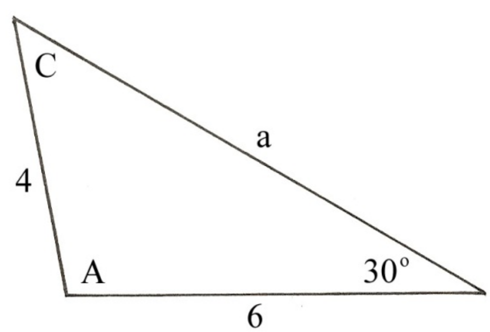
Solution No. 1

$$\frac{sin30}{4} = 0.1250$$

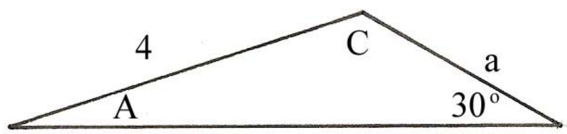
$$C = sin^{-1}(6\ x\ 0.1250)$$
$\underline{C = 48.6°}$

$A = 180° - 48.6° - 30°$
$\underline{A = 101.4°}$

$$a = \frac{sin101.4}{0.1250}$$
$\underline{a = 7.84}$



Solution No. 2



$$\frac{sin30}{4} = 0.1250$$

$C = sin^{-1} (6 \; x \; 0.1250)$
$\underline{C = 131.4°}$

$A = 180° - 131.4° - 30°$
$\underline{A = 18.6°}$

$$a = \frac{sin18.6}{0.1250}$$
$\underline{a = 2.55}$

That concludes the trigonometric review. Now let's do a quick review of significant figures and rounding.

Significant Figures Review
All answers to arithmetic problems are expected to have a precise answer. The difficulty in applying this simple principle lies in being able to tell the difference between arithmetic precision and the validity of an answer.

In most real-world problems such as statics and strength of materials, only the first few digits of an answer are valid or "significant". This is because dimensions are commonly rounded, loads are only approximate (often specified in the building codes as maximum or minimums), and real world connections, bearings, and geometric configurations are usually simplified. Simplification allows relatively simple calculations to closely approximate the solutions to very complex mathematical equations that represent the real boundary conditions.

With the hand held calculators we have today, it is tempting to write many figures in the answer to an arithmetic problem. However, just because the calculator shows eight figures, doesn't mean that they are all valid. Here's the rule:

> **An answer cannot be more accurate than the least accurate number used in the statement of the problem.**

This principle can be illustrated by computing the area of a 2" diameter circle. Obviously the answer is approximately 3.1415926536 in^2 as shown below.

$$A = \pi r^2 = \pi(\frac{2"}{2})^2 = 3.1415926536\, in^2$$

Now, let's say that the diameter of the circle is accurate to three figures (2.00"). That means that the answer cannot be more accurate than three significant figures – which is 3.14 in^2.

Here's why. Because the diameter is accurate to three significant figures – 2.00" – that means that the actual value of the diameter is in the range of 1.995" to 2.005". Therefore, the actual area is somewhere between 3.12595 in^2 and 3.15732 in^2 as shown in the following two calculations.

$$\text{Minimum area} = \pi r^2 = \pi\left(\frac{1.995}{2}\right)^2 = 3.12595 in^2 \rightarrow 3.13\ in^2$$

$$\text{Maximum area} = \pi r^2 = \pi\left(\frac{2.005}{2}\right)^2 = 3.15732 in^2 \rightarrow 3.16\ in^2$$

Rounding these two values to three significant figures means that the actual area of the circle is between 3.13 in^2 and 3.16 in^2. In fact, the area of the circle could actually be 3.13 in^2, 3.14 in^2, 3.15 in^2, or 3.16 in^2. Originally, we said that the answer was approximately 3.14 in^2 which is still true – we have not contradicted the principle that the answer cannot be more accurate than the least accurate number used in the calculation.

Rounding Review

Most values in structural mechanics problems are known with two or three figures of accuracy. It follows that most answers in statics and strength of materials should have two or three figures of accuracy. The widely accepted practice is that all answers have **three significant figures**.

There are a couple of simple exceptions to the three significant figure rule.

- Intermediate values of a calculation can have any number of figures. You are encouraged to carry extra figures through the calculation process, then round the answer.
- Whole number answers need not have the added zeros. An answer of 5 ft is preferred to 5.00 ft, but both are acceptable.
- When two numbers must add to a fixed total, one number may have an added significant figure or the other may have one less significant figure. The fixed total has three significant figures.
  - **Example:** 6.67 + 13.33 = 20.0

    OR

    6.7 + 13.3 = 20.0

When rounding an answer to three significant figures, one occasionally meets the dilemma of a fourth digit that is exactly equal to 5. Should you round up or round down? Both answers are acceptable, but the widely accepted rule is to round to the even number.

Example: Round the answer 1225 to three significant figures.

> Using the widely accepted rule, the answer should be rounded to 1220 instead of 1230.

