Fundamentals of Post-Tensioned Concrete Design for Buildings

Part One

by

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Overview of This Course

This is Part One of a three-part course that covers the fundamentals of post-tensioned concrete design for building structures using unbonded tendons. This course is intended to be an introductory course for structural engineers new to post-tensioned concrete design, and is a good refresher for experienced structural engineers. Part One should be taken before Parts Two and Three. By successfully completing this three-part course, you should be comfortable performing a preliminary design by hand and be able to quickly check a computer generated design or an existing design by hand.

Part One gives a brief historical background and post-tensioned members are differentiated from pre-tensioned members. You will learn about the load balancing concept, hyperstatic moments, pre-stress losses, the basic requirements of the American Concrete Institute's *Building Code Requirements for Structural Concrete*, and nominal flexure and shear capacities of post-tensioned members.

In Part Two, examples of two of the structural systems commonly used in buildings and parking structures are worked, namely a one-way continuous slab and a two-span beam. Part Two is an example-intensive course, with key concepts introduced along the way.

Part Three continues with the study of two-way, post-tensioned slab systems, including a complete design example using the Equivalent Frame concept. Part Three also covers related topics such as punching shear for two-way slabs and moment transfer at the column. Part Three is an example-intensive course, with key concepts introduced along the way.

The user of this course material must recognize that pre-stressed concrete design is a very broad topic and that only certain fundamentals in a specific area are covered here in this course. It is not intended, nor is it possible within the confines of this course, to cover all aspects of pre-stressed concrete design. It is not intended that the material included in this course be used for design of facilities by an engineer who is inexperienced in pre-stressed concrete design without oversight and guidance from someone more experienced in this field. The author of this course has no control or review authority over the subsequent use of this course material, and thus the author accepts no liability for damages that may result from its use.

Pre-Stressed Concrete – Background and Definition

Concrete as a building material has been around for thousands of years. Unlike other isotropic building materials such as steel, wood, and aluminum, concrete and masonry have a high compressive strength as compared to their relatively weak tensile strength. Therefore, until the advent of reinforced concrete in the 1800s, concrete and masonry structures mainly resisted only compressive forces. These structures generally consisted of columns, arches, and domes to take advantage of their compressive capacity while eliminating any tensile demand. Several examples include the following:

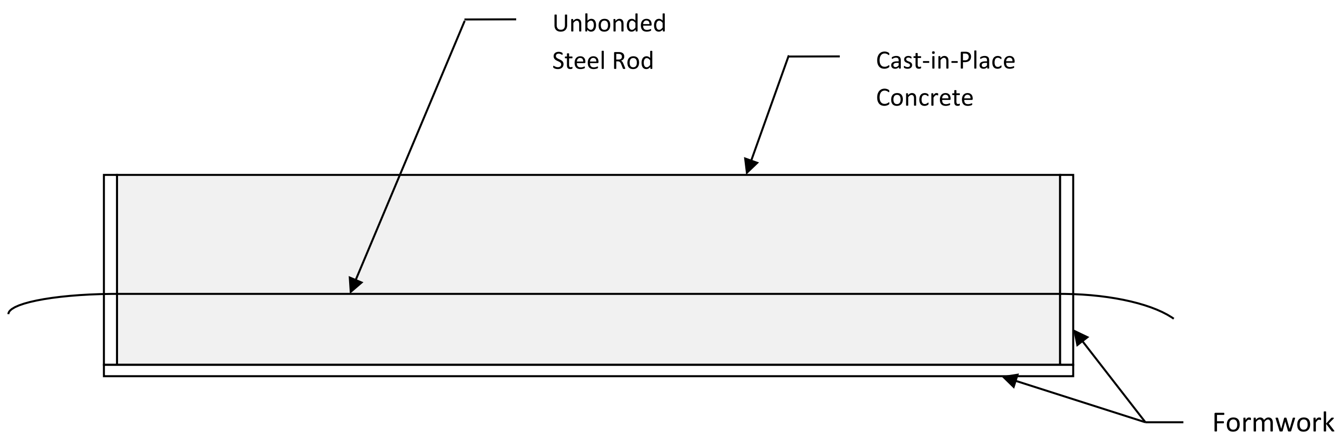
Roman Aqueduct
Segovia, Spain
(1st Century)

Brunelleschi's Dome
Basilica di Santa Maria del Fiore
Florence, Italy (1461)

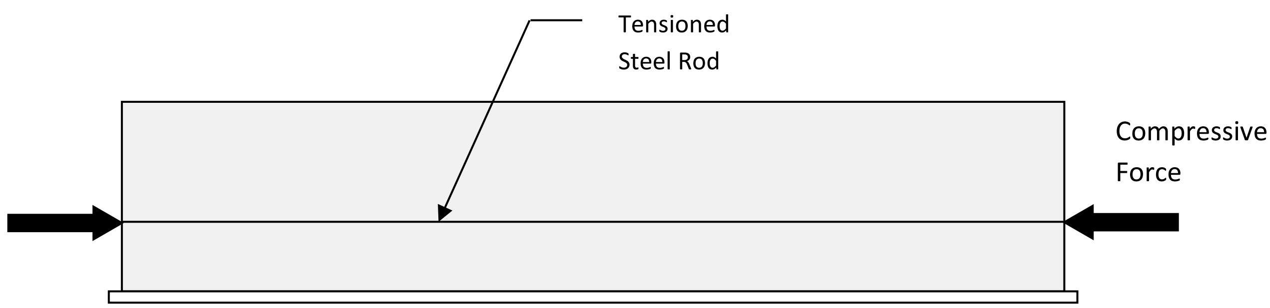
Stari Most Bridge
Mostar, Herzegovina
(1567)

In the middle of the 1800s, the idea of adding iron to concrete to resist tensile stresses was first developed. Joseph Monier exhibited this invention at the Paris Exposition in 1867. With the invention of steel in the later part of the 1800s, the use of steel reinforcing bars to resist tensile forces in concrete structures quickly became widespread. Early reinforced concrete structures included bridges, buildings, retaining walls, culverts, tunnels, docks, and roadways. Thus, "mild" reinforcing steel is strategically placed within, and continuously bonded to, concrete members to resist tensile forces to which they may be subjected. Mild steel reinforcing is also commonly used in combination with concrete to resist compressive and shear forces.

In the early 1900s, the idea of tightening the reinforcing bars to compensate for the shrinkage of the concrete was first suggested. Embedded high strength steel rods were coated to prevent bond with the concrete. After the concrete hardened, the steel rods were tensioned and anchored to the ends of the concrete member by means of threaded nuts. This is known as post-tensioning, since the steel reinforcing is tensioned after the concrete is placed. Post-tensioning can be used with virtually any cast in place concrete member. The following two figures illustrate this concept.

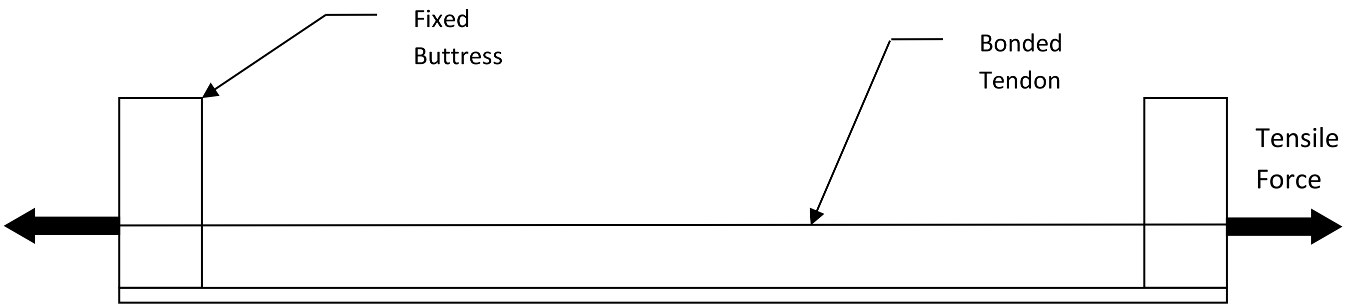


Post-Tensioned Beam Before Force Transfer

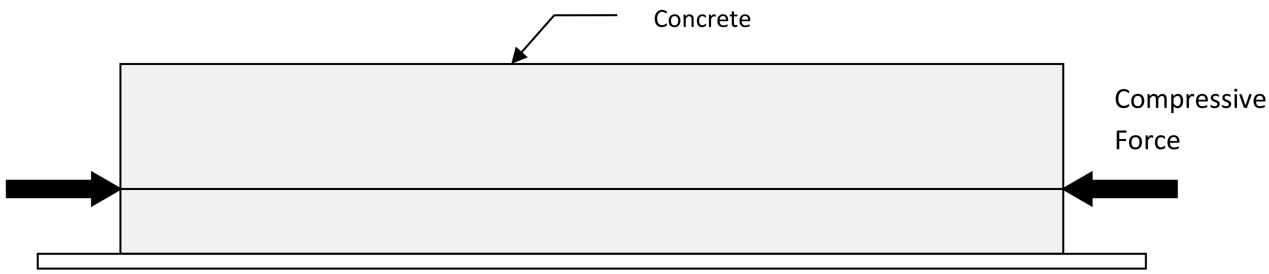


Post-Tensioned Beam After Force Transfer

Another scheme was also developed around the same time whereby the steel rods were initially stretched between two buttresses and then the concrete was placed around the steel rods. After the concrete hardened and bonded to the rods, the ends of the steel rods were released and their tension was thus transferred into the concrete in compression by bond stress between the concrete and the steel rods. This is known as pre-tensioned concrete since the steel is tensioned before the concrete is placed. Refer to the following two figures.

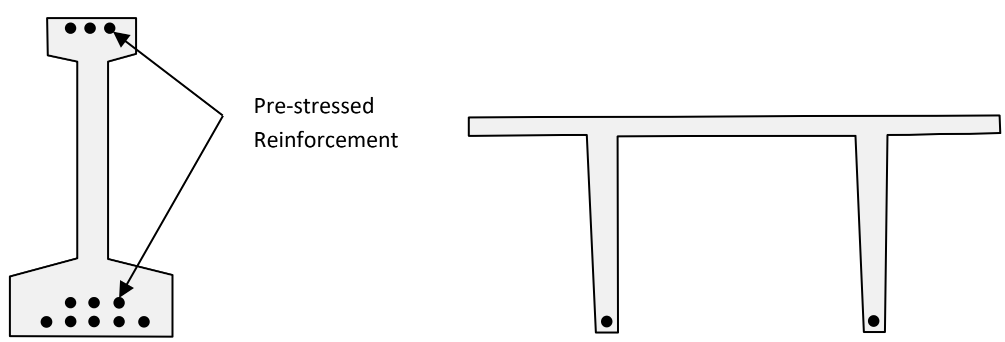


Pre-Tensioned Beam Before Force Transfer



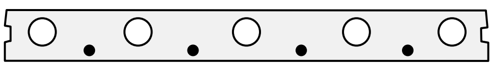
Pre-Tensioned Beam After Force Transfer

Because it is only practical to build buttresses in the shop to resist pre-tensioning forces, most pre-tensioned members are manufactured at a pre-casting facility and shipped to the jobsite. These building components are said to be precast. The fundamentals learned in this course can be applied to the design of precast concrete members. Some common types of pre-cast building elements are shown below.

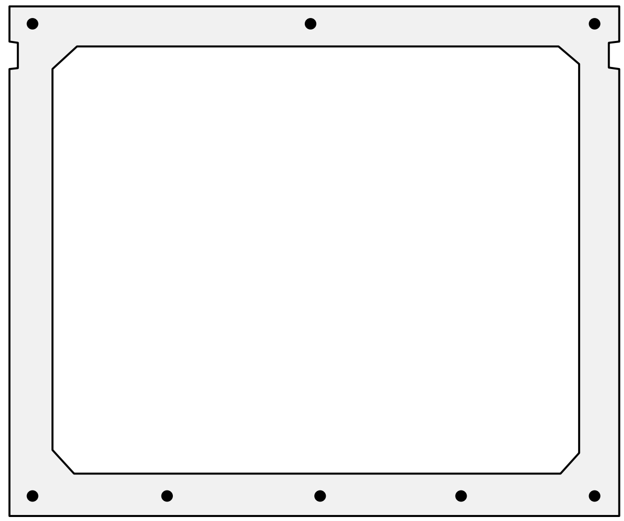


Bridge Girder

Double Tee



Hollow Core Plank



Box Culvert

Pre-stressed concrete is the general term used to describe concrete members that have stresses induced in them before the application of any design loads. Pre-stressed concrete includes both pre-tensioned and post-tensioned concrete.

Post-tensioned concrete is widely used in bridges, shell structures, water tanks, and folded plates. However, the primary focus of this course covers applications of post-tensioning that are typically used in buildings and parking structures, specifically one-way slabs, two way slabs, and continuous beams.

There are two general types of pre-stressed reinforcement in use today; bonded and unbonded. <u>Bonded</u> reinforcing typically consists of high-strength steel wires twisted into a 7-wire strand. These so-called mono-strands are commonly used in pre-tensioned concrete members, although there are other types not as common. Bonded mono-strands are first tensioned, and when concrete is cast around these mono-strands, there is excellent mechanical bond between the concrete and the steel throughout the strand's entire embedded length. After the concrete has cured a sufficient amount, the strands at both ends of the member are released and the force is transferred to the concrete through bond stress. Pre-tensioned strands are can only be straight between two points. Pre-tensioned strands can be "harped" within a form.

Unbonded reinforcing usually consists of a 7-wire strand coated with grease and encapsulated in a plastic sheathing over its entire length to prevent bond to the concrete. This is called a tendon. Tendons are draped in a specific profile and secured within the concrete formwork before concrete is placed. After the concrete has been placed and has cured sufficiently, one or both ends of the unbonded tendon are tensioned by using a hydraulic jack to physically stretch the tendon. The stretched tendon is then locked off against the end of the concrete member thereby transferring the tendon force to the concrete through bearing of the cast-in anchorage. Common anchorage hardware is shown below.

Unbonded Mono-Strand Anchorage

Bonded Multi-Strand Anchorage

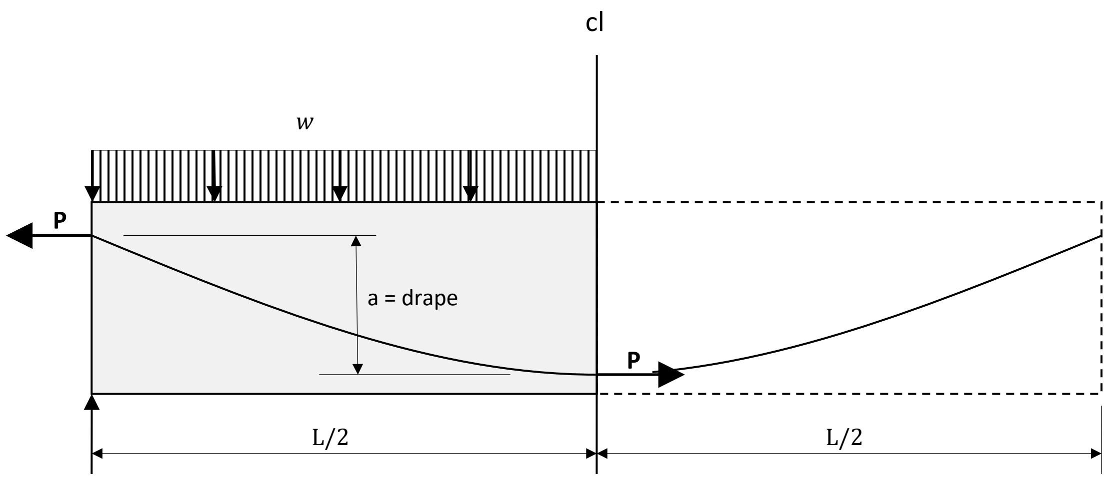
When very large post-tensioning forces require numerous strands, hollow metal tubes, or ducts, are cast into the concrete member in a specific profile. The duct contains uncoated, unstressed strands and has multi-strand anchorages at both ends. After the concrete has been placed in the formwork and cured sufficiently, the strands are tensioned one by one from one or both ends of the member. Once the appropriate amount of tension has been applied to all strands, they are locked off and the remaining air space in the duct is replaced by grout which is pumped in under high pressure. The grout ensures and effective bond between the strands and the concrete and also provides corrosion protection.

Since we will be dealing only with post-tensioned concrete structures in this course, that is, structures that are first cast-in-place and then post-tensioned in place, we will only cover unbonded tendon applications.

The Load Balancing Method

The load balancing method is the most widely used technique to design post-tensioned concrete beams and slabs. This method will be used exclusively in this course. In the load balancing method, a portion of the design load is selected to be "balanced", or carried, by the action of the tendons. The balanced load is commonly taken as 80% of the dead load. The required force in the tendons to carry the balanced load is easily calculated using statics. The concrete member is then analyzed using conventional structural analysis techniques with the equivalent set of tendon loads acting on the member in combination with other externally applied design loads, such as dead load and live load.

Let's consider the free body diagram at mid-span of the following simple span beam with a draped tendon with force P. Note that the common simplifying assumption made in post-tensioned concrete analysis is that the tendon force acts in the horizontal direction at the ends of the member and the small vertical component, if any, can be ignored or is transferred directly to the support. Note that the shear force at the right side of the free body diagram is zero since this occurs at the mid-span of a simple span beam with a uniformly distributed load.



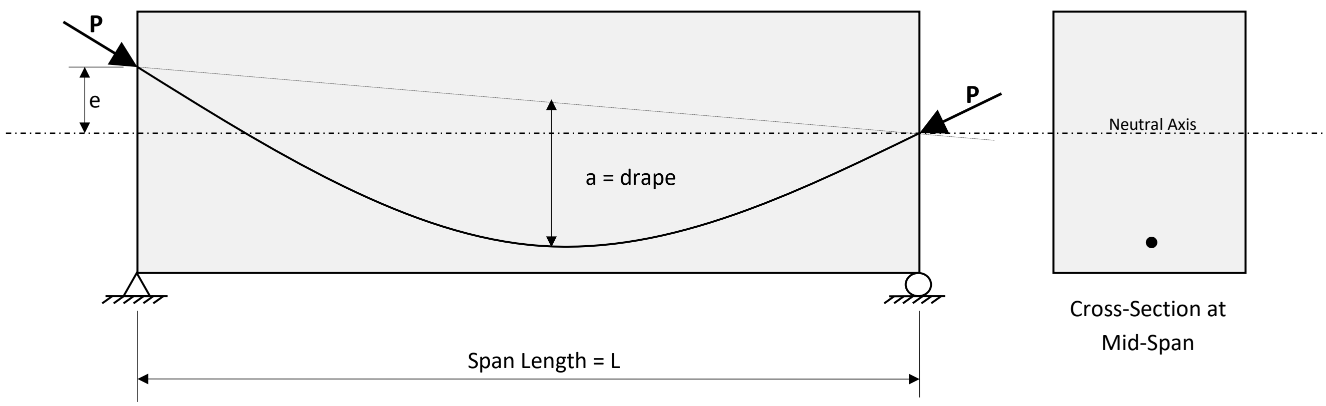
If we sum the moments about the force P at the left support, we get:

$$\sum M = 0$$

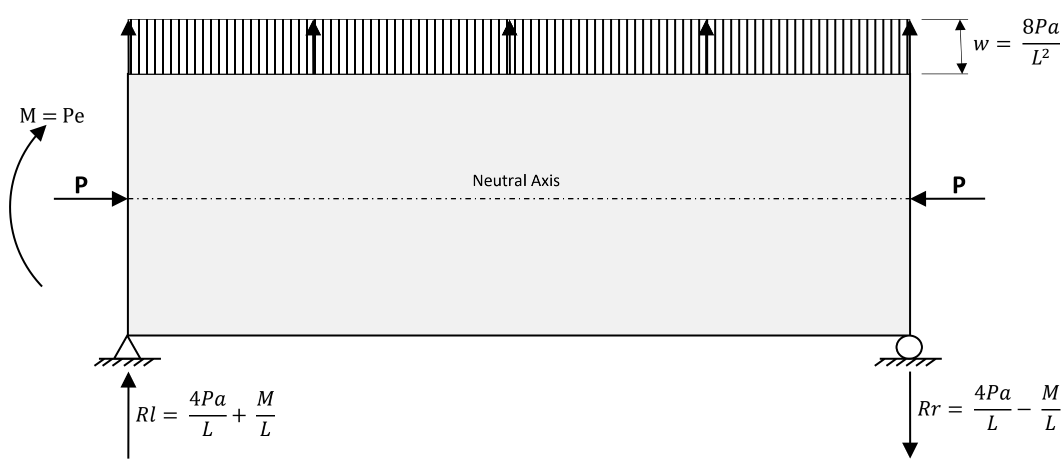
$$\frac{wL}{2} \times \frac{L}{4} = P \times a$$

$$\frac{wL^2}{8a} = P$$

The load balancing concept is further illustrated in the figure below, which shows a simply supported beam and a tendon with a parabolic profile. The beam shown in the first figure below may be analyzed with an equivalent set of tendon loads acting on the member as shown in the second figure. Thus, the equivalent loads acting on the beam consist of the axial force P, an upward uniform load of w, and a clockwise moment M at the left end due to the eccentricity of the tendon with respect to the neutral axis of the beam.



Parabolic Tendon Drape



Equivalent Tendon Loads Applied to Beam

Note that reactions are induced at both ends to keep the system in equilibrium. If we sum the moments about the left support, we get:

$$\sum M = 0$$

$$\frac{8Pa}{L^2} \times L \times \frac{L}{2} - Pe - Rr \times L = 0$$

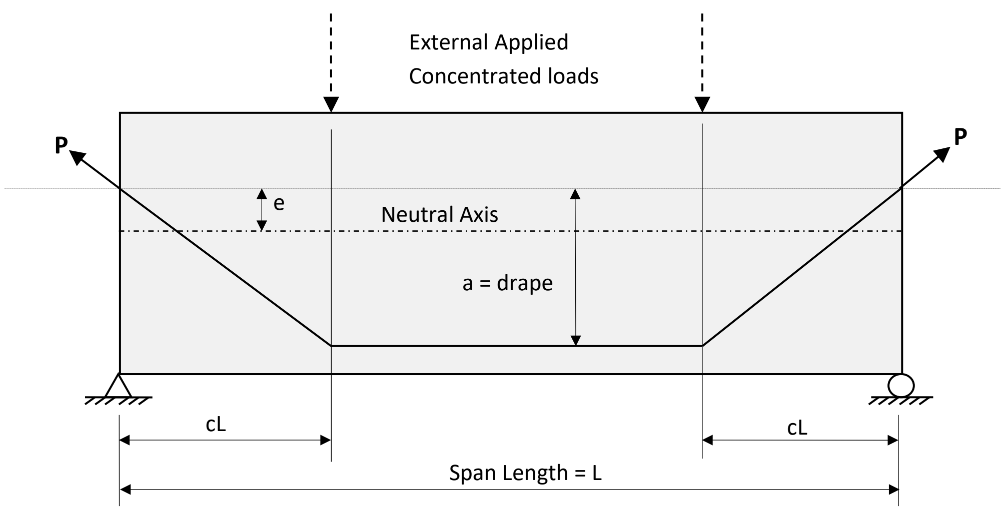
$$\frac{4Pa}{L} - \frac{M}{L} = Rr$$

And if we sum the vertical forces we find the left reaction is:

$$\frac{4Pa}{L} + \frac{M}{L} = Rl$$

Note that the vertical component of the applied pre-stressing force is neglected. This is practical since the tendons are customarily horizontal, or very nearly horizontal, at the end of the members, and the vertical component is usually small.

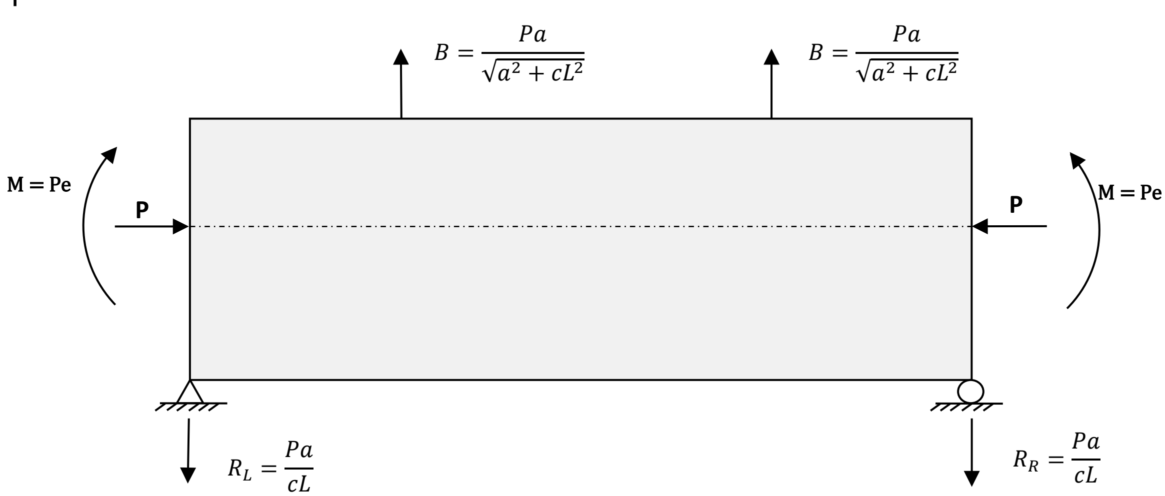
As we have seen, a draped tendon profile supports, or balances, a uniformly distributed load. Now let's consider a beam that is required to support a concentrated load. In the case of a concentrated load on a beam, a concentrated balancing load would be ideal. This can be achieved by placing the pre-stressing tendons in a harped profile. This concept using harped tendons is illustrated in the figure below.



Harped Tendon Drape

Harped tendons are commonly used in pre-tensioned, precast concrete members. In precast members, since the tendons are tensioned in the forms prior to concrete placement, precast manufacturers have ways to hold the pre-tensioned tendons in the proper position within the formwork until the concrete is placed and has reached a sufficient strength to release the tension. Harped tendons are also easily accommodated in post-tensioned construction since the tendons may be positioned within the formwork in the field in virtually any arrangement because they are not tensioned until after the concrete is placed and hardened. Harped tendons in post-tensioned construction are commonly used, for example, in a transfer beam that carries the concentrated load of a discontinuous column. The tendons in a transfer beam are sometimes stressed in stages to balance a certain portion of the column dead load as the construction progresses. Harped tendons are also commonly used in the repair or strengthening of existing beams where the post tensioning forces are applied externally.

The following figure illustrates the equivalent set of loads due to the harped tendons acting on the member shown above. The upward component of the tendon force P at the point where its direction changes from sloped to horizontal, B, is a function of the drape, a, and the distance from the end of the member, cL, and is equal to Pa/cL. Since the harped tendons are symmetrical in this case, the reactions in the free body diagram are equal. For unsymmetrical harped tendons, the upward component, B, and the reactions are found using statics. Thus, the equivalent loads acting on the beam consist of the axial force P, upward concentrated loads B where the tendon changes direction, and moments M at both ends due to the eccentricity of the tendon with respect to the neutral axis of the beam.

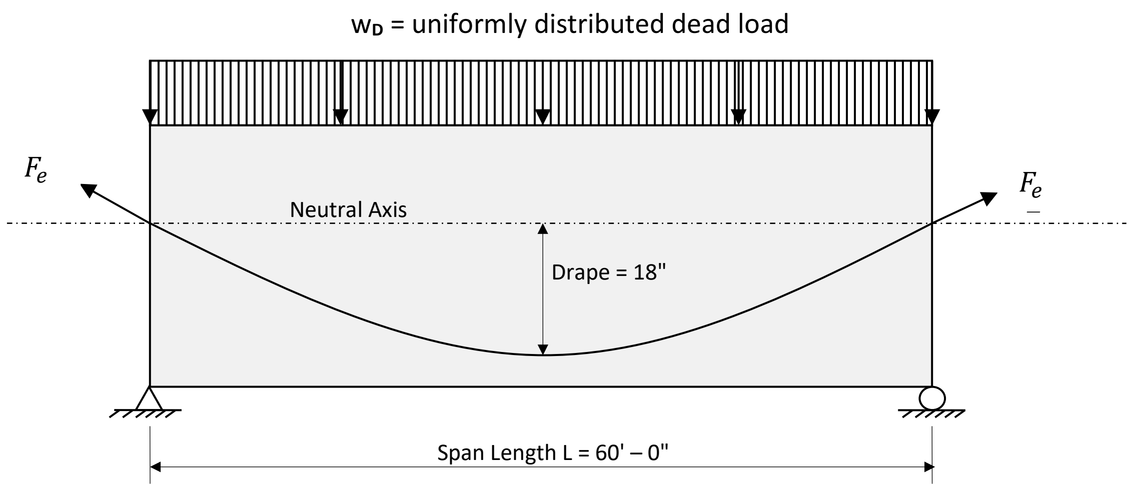


Equivalent Harped Tendon Loads Applied to Beam

Example

Find:

Choose an effective post-tensioning force, F_e, to balance 80% of the dead load for the following beam.



Given:

- o Beam Size 16" x 36"
- o w_D = 2.85 kips per foot (includes beam self-weight and tributary dead load)

Solution:

$$Fe = \frac{wL^2}{8a}$$

$$Fe = \frac{0.8(2.85)(60)^2}{8(1.5)}$$

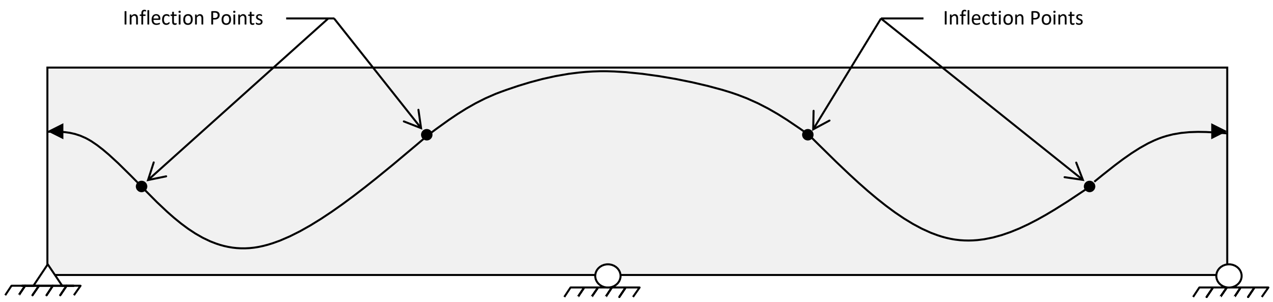
$$\underline{Fe = 684\ kips}$$

Thus, an *effective* post-tensioning force of 684 kips would be required to balance 80% of the dead load for this tendon profile. The effective force is the force remaining in the tendons after all pre-stress force losses. More on pre-stress losses later.

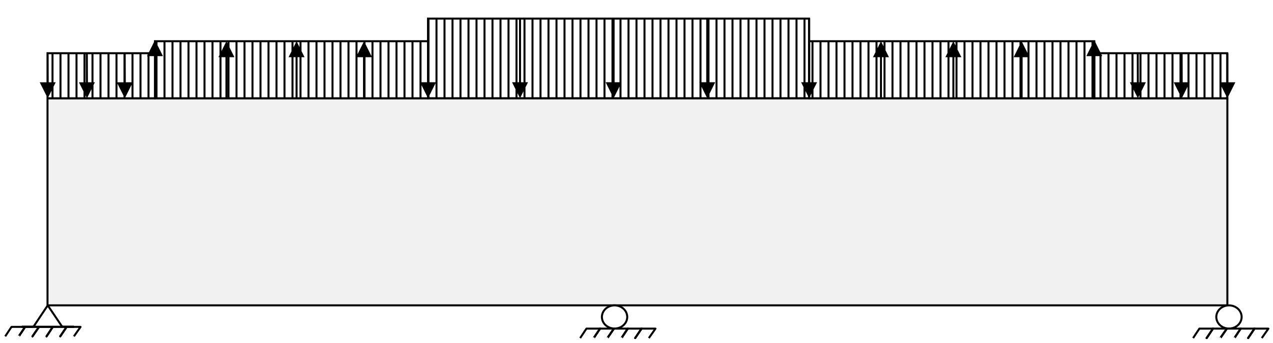
Load Balancing in Continuous Structures

Let's now turn our attention to the load balancing concept applied to continuous structures. As we will see later on, post-tensioning in continuous structures induces secondary, or so-called hyperstatic, forces in the members.

Consider the figure below. In real continuous structures, the tendon profile is usually a downward parabola at the supports and an upward parabola between supports such that the tendon drape is a smooth curve from end to end. The tendon curve changes at an inflection point. This tendon configuration actually places an equivalent downward load on the beam near the supports between inflection points while an upward equivalent load acts on the beam elsewhere, as shown in the figure below. This type of reverse loading can be taken into account in computer analysis, where a more rigorous approach to the structural analysis can be accommodated. However, for hand and more approximate calculations, tendon drapes are idealized as a single upward parabolic drape in each span. Thus, in this course, we will only consider a simplified single parabolic drape in each span as shown on the next page.

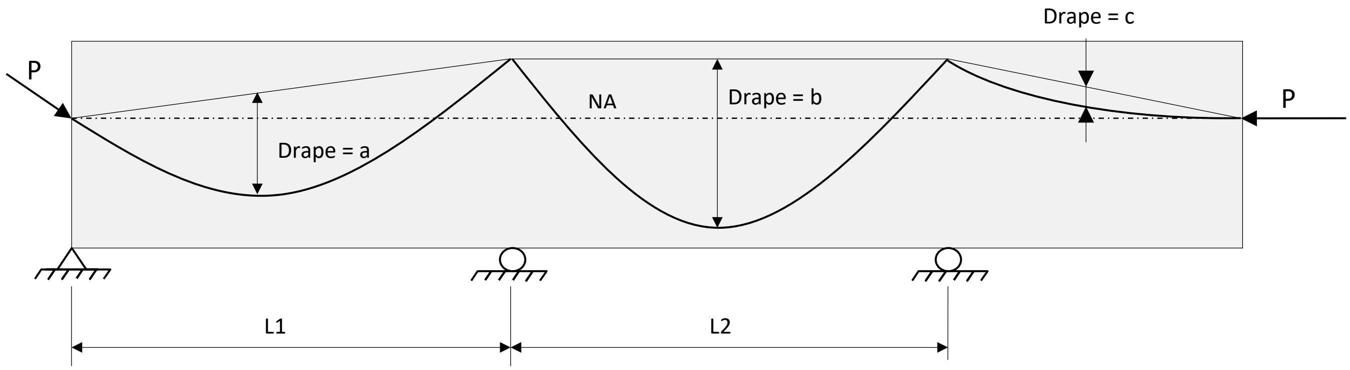


Usual Parabolic Tendon Drape



Equivalent Loads for a Reverse Parabolic Tendon Drape

Now let's consider the figure below, which shows a two span continuous beam with a cantilever on the right end. Each span has a different tendon drape as shown.

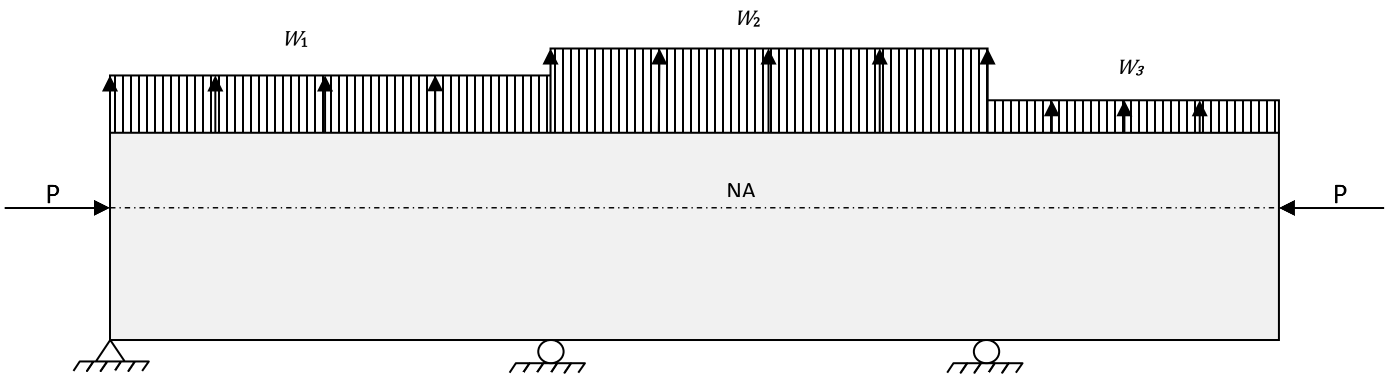


Idealized Continuous Parabolic Tendon Drape

As in the previous examples, the continuous beam shown above may be analyzed with an equivalent set of tendon loads acting on the member. Thus, the equivalent loads acting on the each beam span due to the pre-stressing force in the tendons consist of the axial force P and an upward uniform load. Since the tendon force, P, acts at the neutral axis at the ends of the beam in this example, there are no end moments induced due to the eccentricity of the tendon.

The diagram on the following page shows the equivalent set of tendon loads acting on the beam for the diagram above. Note that the drape "a" in span 1 is not equal to the drape "b" in span 2. Drape "c" in the right cantilever is also different. If we assume that the tendons are continuous throughout all spans of the beam, then the post-tensioning force, P, is also constant throughout all spans. Therefore, for a given post-tensioning force, P, we may balance a different amount of load in each span, depending on the drape in each span and the span length, according to the equation:

$$w = \frac{8Pa}{L^2}$$



Equivalent Tendon Loads Applied to Continuous Beam

However, it is not usually desirable or practical to balance different loads in each span. Nor is it practical to apply a different post-tensioning force in each span to balance the same load in each span, except in end spans or in spans adjacent to a construction joint, where tendons can be added. Therefore, the *drape* can be adjusted in each span to balance the same amount of load in each span. Thus, for a given post-tensioning force and balanced load, we can find the required drape:

$$a = \frac{wL^2}{8P}$$

<u>Example</u>

Given:

- o A post-tensioned, continuous, five-span concrete slab strip 1'-0" wide
- o Spans are 10'-0", 12'-0", 9'-0", 13'-0", and 15'-0"
- o Maximum available drape is 3.5" and 2.625" for interior spans and end spans, respectively

Find:

Choose the minimum effective post-tensioning force, F_e, to balance 60 psf and determine the drape in each span.

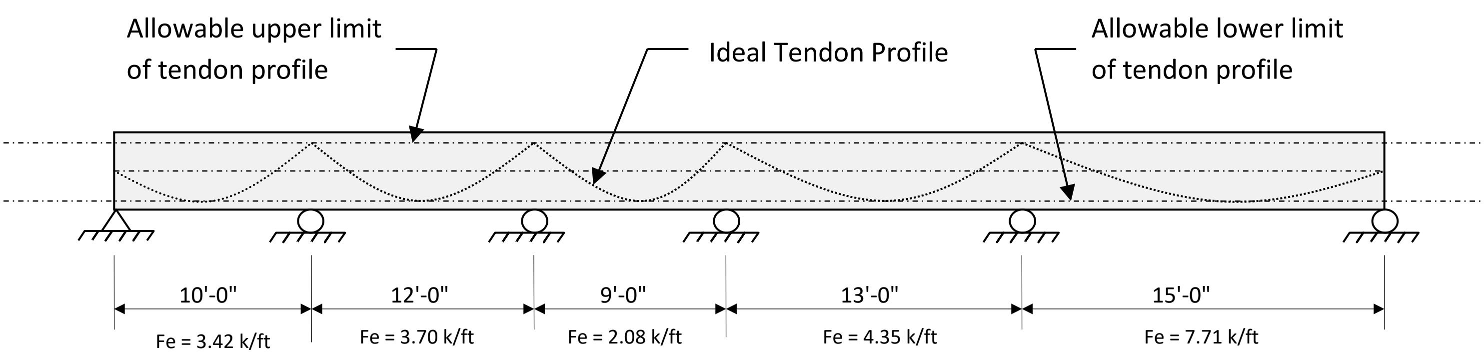
Solution:

Let's begin by examining the following equation which is used to compute the effective post-tensioning force:

$$F_e = \frac{wL^2}{8a}$$

We can see that the effective post-tensioning force, F_e, is a function of the span length and the available drape, for a given balanced load. By observing the above equation, we can see that the minimum effective post-tensioning force will be found in short spans with large available drapes. However, if the smallest possible effective post-tensioning force is selected (the left end span would only require 3.42 kips/foot) and used in all the spans, there would not be enough available drape to balance 60 psf in all the other spans.

Referring to the diagram below, the effective post-tensioning force has been calculated for each span using the maximum available drape.



Five-Span Slab Example

This would be the optimal use of pre-stressing since all spans would make use of the maximum amount of drape, and therefore the minimum amount of pre-stressing, to balance a given load. However, it is neither practical nor desirable to use a different amount of pre-stress in each span.

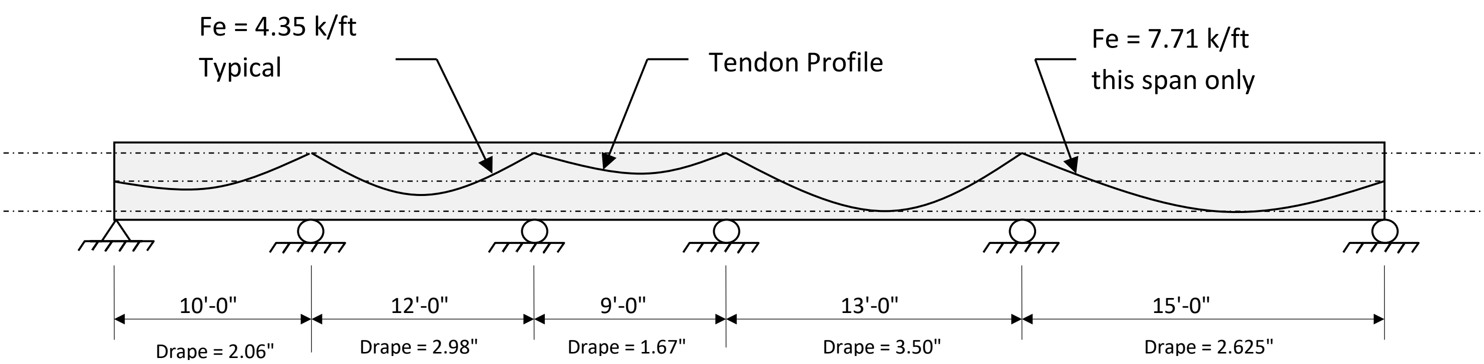
If we were to use the effective post-tensioning force of 7.71 kips per foot in all spans and adjusted the drapes to maintain the balanced load of 60 psf, we would have drapes of 1.68", 0.95", 1.97", and 2.625" for the first four spans left to right. Only the right end span maximizes the efficiency of the draped tendon to balance the load. Another way of stating this is that the full available drape in every span would not be utilized and it should be apparent that this would not result in the most efficient use of post-tensioning.

One technique that is commonly employed in continuous post-tensioned structures is to place additional tendons in the end spans to satisfy the controlling demand and use only enough tendons in the interior spans to satisfy the lower demand there. This is a more efficient use of post-tensioning. Even though this may seem to be a minor difference, if this is applied over a large multi-story structure, the total savings can be significant.

So, to make this example as efficient as possible, we will only use the 7.71 kips per foot in the right span and use 4.35 kips per foot in the other four spans. Thus, using the equation

$$a = \frac{wL^2}{8P}$$

We find the drapes as shown in the figure below.

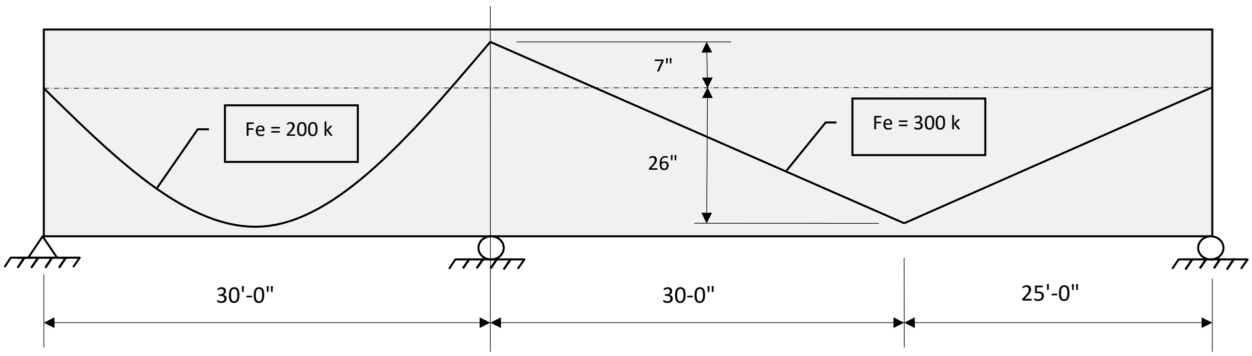


Five-Span Slab Example

Balanced Load Moment Diagrams

As we learned earlier, once the load to be balanced is selected, the effective pre-stressing force can be calculated for a given drape and span length. The member may then be structurally analyzed with this equivalent set of tendon loads applied. The results of this analysis may be combined with other load cases such as live load and superimposed dead load.

Let's now consider the two span beam shown below. The spans are unequal and each span requires a different effective post-tensioning force. Let's determine the balanced load moments.



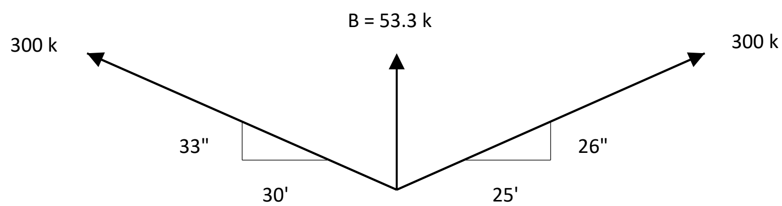
By inspection of the above diagram, we see that the drape for the left span is:

$$\text{Left Span Drape} = \frac{33+26}{2} = 29.5\ inches$$

And the balanced load is computed as:

$$w\ = \frac{8(200)(\frac{29.5}{12})}{(30)^2} = 4.37\ kips/ft$$

Next, we can find the concentrated balanced load in the right span due to the harped tendon by adding the vertical components on both sides of the harped point as follows:



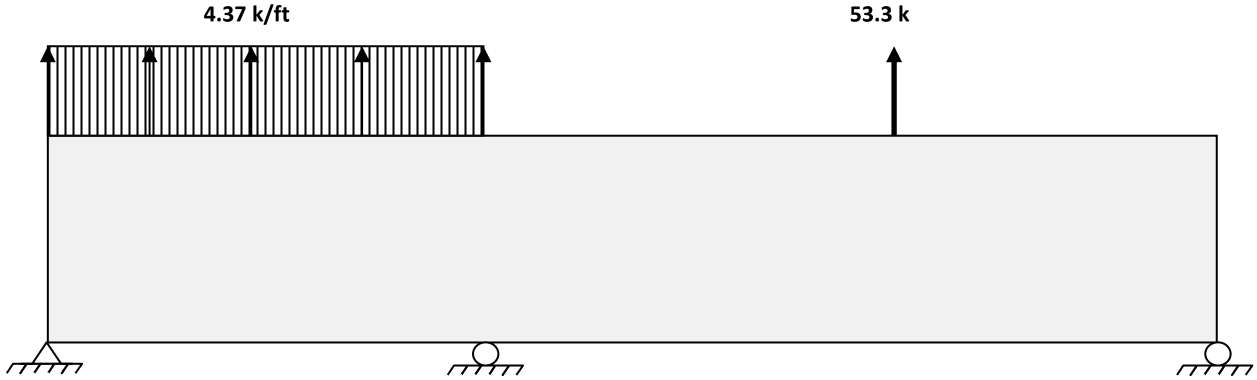
$$Vertical\ Component\ V = 300 \sin \emptyset$$

$$B = V_{left} + V_{right}$$

$$B = 300 \sin[\tan^{-1}(2.75/30)] + 300 \sin[\tan^{-1}(2.17/25)]$$

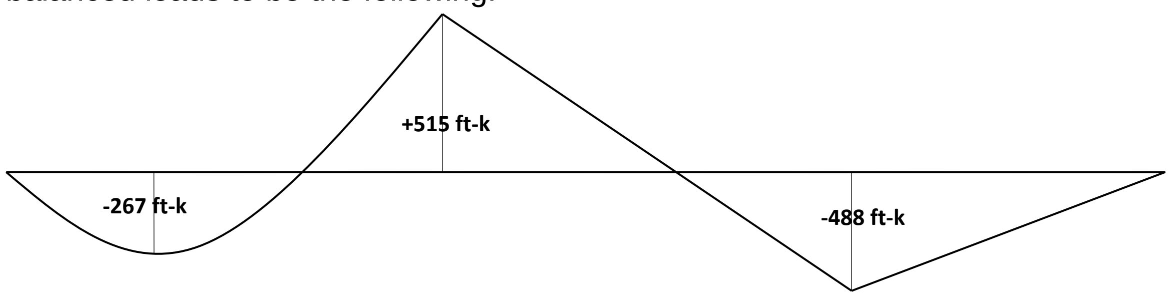
$$B = 27.4 + 25.9 = 53.3\ kips$$

Now we can construct the balanced loading diagram below. The left span has a uniformly distributed upward load of 4.37 kips/ft and the right span has a concentrated upward load of 53.3 kips.



Balanced Load Diagram Example

Using conventional elastic structural analysis, we find the moment diagram for the above balanced loads to be the following:



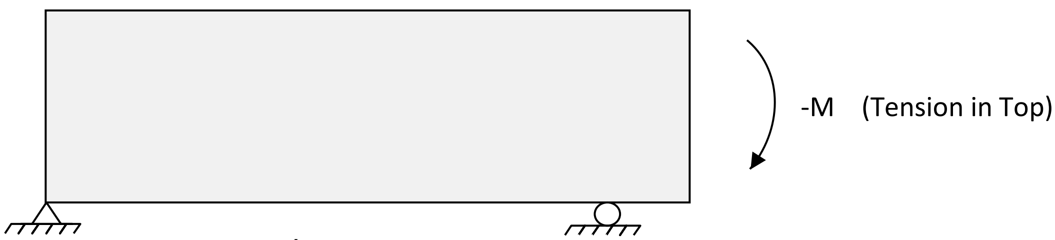
Balanced Moment Diagram

The above example illustrates the simplicity and straight-forwardness of the balanced load method. Once a load is selected to be balanced, and the tendon forces and drapes are chosen, it is a then a matter of elastic structural analysis, which is well suited to the computer, to find the moments due to the balanced load. The analysis for balanced loads is just another load case as far as the computer is concerned. In the balanced load method, the balanced load moments are used to determine the hyperstatic moments, which we will cover next. But first, we shall establish the sign conventions that we are using throughout this course.

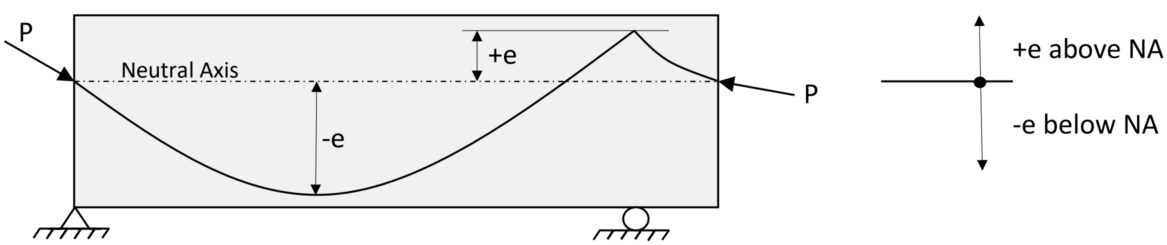
A Word About Sign Conventions

In this course, moments causing tension in the bottom fiber are considered positive. Moments causing tension in the top fiber are negative. Eccentricities below the neutral axis are negative and above the neutral axis they are positive. These sign conventions are illustrated below.

Positive Moment



Negative Moment

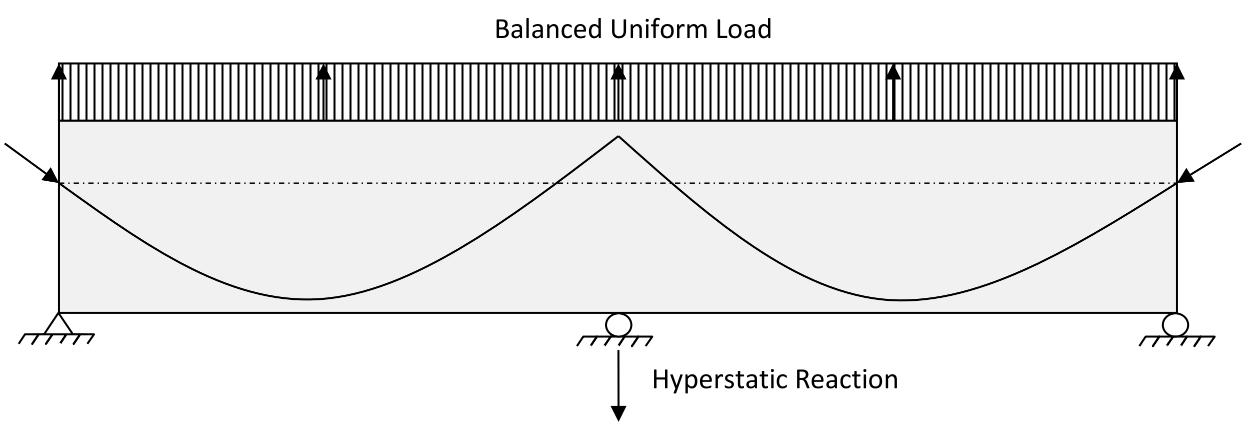


Eccentricity

Introduction to Hyperstatic (Secondary) Forces

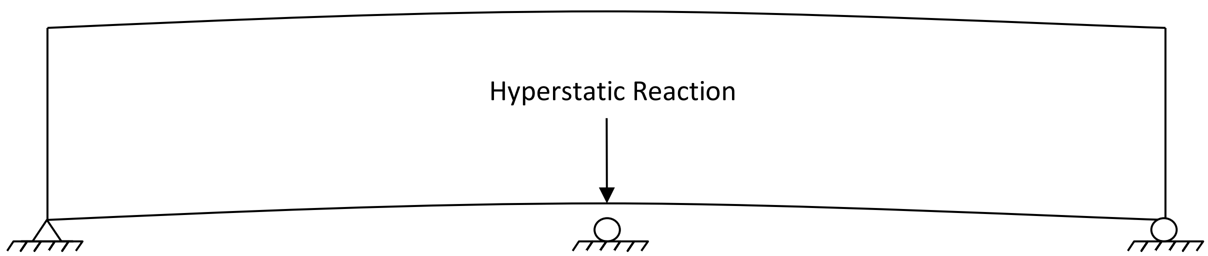
Hyperstatic, or secondary, forces are the forces generated in a statically indeterminate structure by the action of the post-tensioning. Generally, hyperstatic forces are generated due to support restraint. Hyperstatic forces are not generated in a statically determinate structure. It is important to isolate the hyperstatic forces as they are treated with a separate load factor when considering ultimate strength design.

Let's consider the two span post-tensioned beam in the following illustration. We know from previous examples that the tendon force will create an upward uniformly distributed load acting on the beam as shown in the figure. If the center support were not there, the beam would deflect upward due to the post-tensioning force. Since the center support is there, and it is assumed that it resists the upward deflection of the beam, a downward reaction is induced at the center support solely due to the post tensioning force.

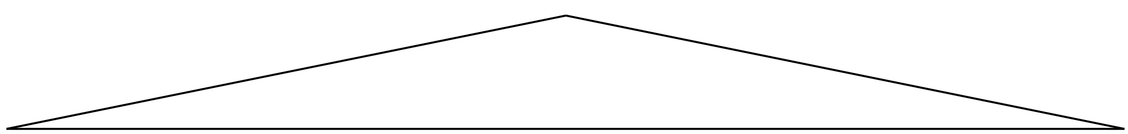


Post-Tensioning Induced Reaction

The next figure shows the deflected shape of the two-span beam due to the post-tensioning force (ignoring self-weight) as if the center support were not there. In order to theoretically bring the beam back down to the center support, a force equal to the center reaction would have to be applied to the beam. This induces a hyperstatic moment in the beam. The hyperstatic moment diagram is illustrated below for the two-span beam in this example. Note that the hyperstatic moment varies linearly from support to support. An example of a three-span hyperstatic moment diagram is also illustrated below.



Theoretical Deflected Shape due to Post-Tensioning



Hyperstatic Moment Diagram for a 2-Span Beam

Hyperstatic Moment Diagram for a 3-Span Beam

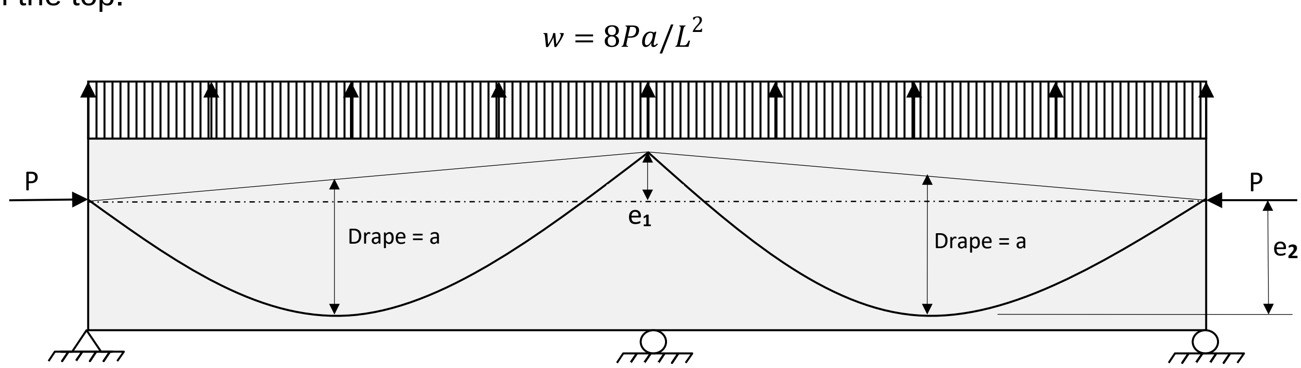
Note that the above example assumes that the beam is supported by frictionless pin supports and therefore no moments can be transferred into the supports by the beam. However, in real structures, the post-tensioned beam or slab is normally built integrally with the supports such that hypersatic *moments* are also induced in support columns or walls. Hyperstatic moments in support members are normally ignored in hand and approximate calculations, but can and should be accounted for in post-tensioning computer software. Thus, when designing supporting columns or walls in post-tensioned structures, it is important to take into account hyperstatic moments induced by post-tensioning forces. For purposes of this course, we will ignore hyperstatic moments that are induced in supports. We will assume our beams and slabs are supported by frictionless pin supports and deal only with the hyperstatic moments induced in the beam or slab members being considered.

The above illustrations serve to introduce the concept and source of hypersatic forces in continuous post-tensioned structures. The hyperstatic moment at a particular section of a member is defined as the <u>difference between the balanced load moment and the primary moment.</u> We refer to the primary moment as M_1 and this is the moment due to the eccentricity of the post-tensioning force with respect to the neutral axis of the member at any given section. In equation form,

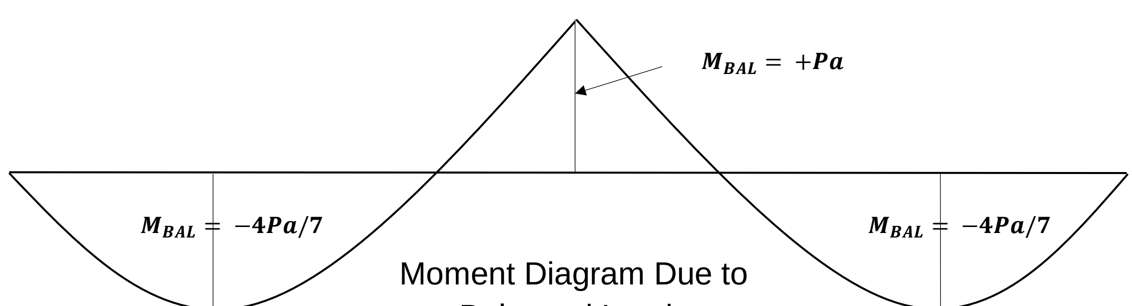
$$M_{HYP} = M_{BAL} - M_1$$

$$M_{HYP} = M_{BAL} - P \times e$$

Consider the two-span post-tensioned beam shown below. As we have seen previously, the beam can be analyzed with equivalent loads due to the tendon force. The draped tendons with force P may be replaced with an equivalent upward acting uniform load of $8Pa/L^2$. When the two-span beam is analyzed using this load, a moment diagram is developed as illustrated below. This is called the balanced moment diagram. Recall that the sign convention results in a negative moment when tension is in the top.

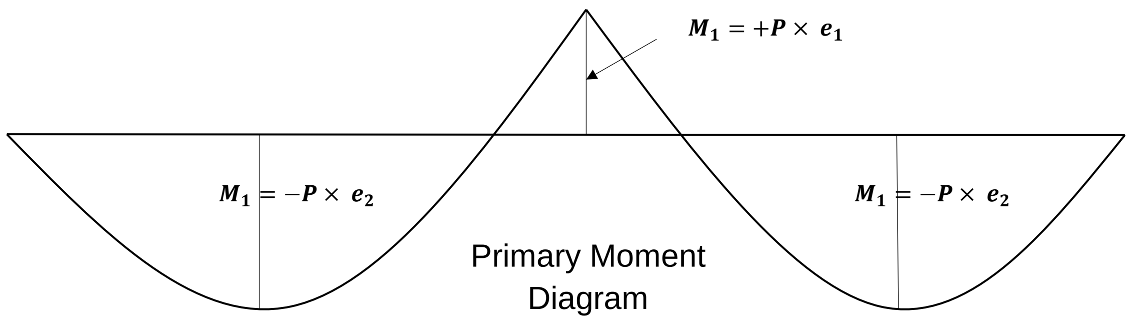


Equivalent Loading Due to Post-Tensioning



Moment Diagram Due to Balanced Loads

Using our sign conventions, we can construct a primary moment diagram for moments M_1. The primary moment is obtained from the product of the post tensioning force times its eccentricity with respect to the neutral axis of the beam at any given section. Thus, referring to the diagram above, the primary moment at the center support is P x e_1 and the primary moment at the mid-span of each span is P x e_2. According to our sign conventions, the primary moment at the center support is positive.



Now we can determine the hyperstatic moments using the equation

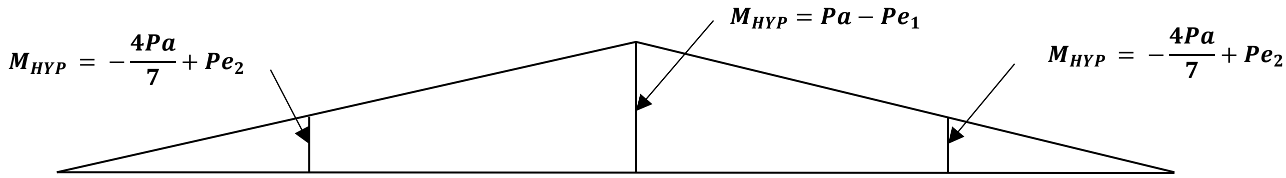
$$M_{HYP} = M_{BAL} - M_1$$

At the center support, we obtain

$$M_{HYP} = Pa - Pe_1$$

Near the mid-span of each span we have

$$M_{HYP} = -\frac{4Pa}{7} + Pe_2$$

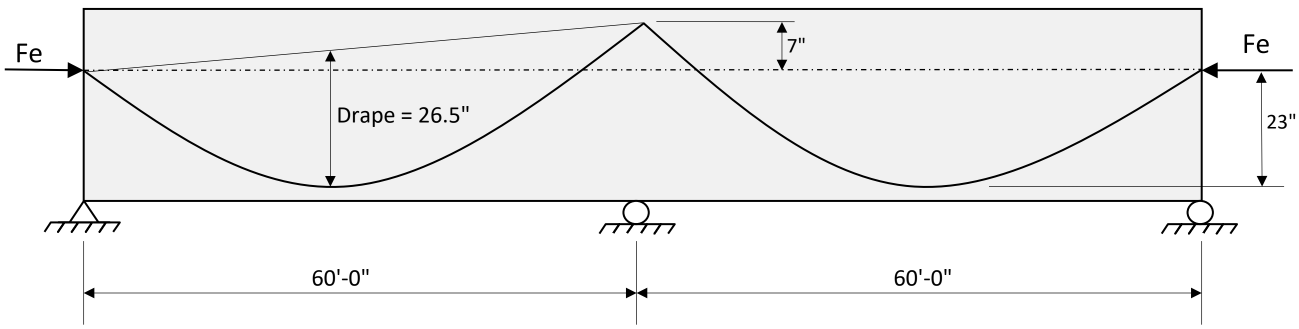


Hyperstatic Moment Diagram

Example

Given:

The two-span post-tensioned beam shown below.



Beam size 14 x 36
W_{Dead} = 1.84 kips/ft

Find:

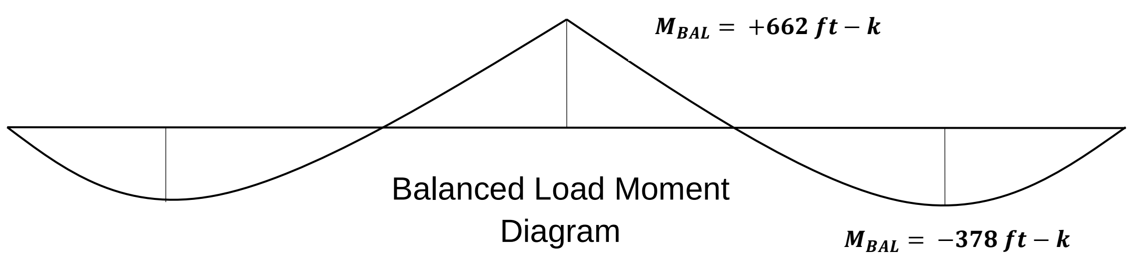
Determine the hyperstatic moments due to post-tensioning resulting from balancing 80% of the dead load.

Solution:

Determine Balanced Load Moments

$$Balanced\ w\ =\ 0.8\ \times 1.84\frac{kips}{ft} = 1.47\ kips/ft$$

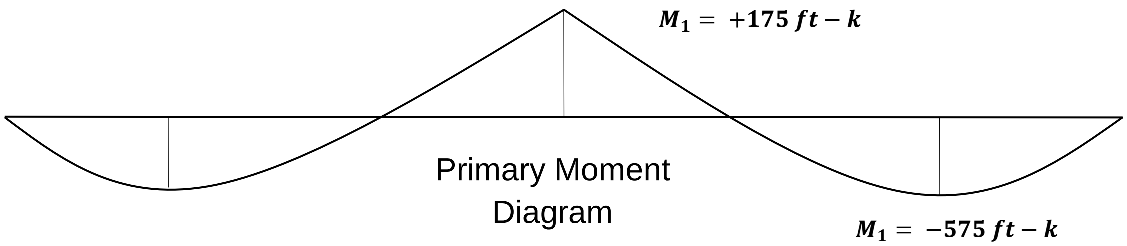
Using well known formulas for a beam with two equal spans subjected to a uniformly distributed load, we find the balanced moment diagram to be:



Next we can compute the required effective post-tensioning force:

$$Fe = \frac{wL^2}{8a} = \frac{(1.47)(60)^2}{8(26.5)/12} = 300\ kips$$

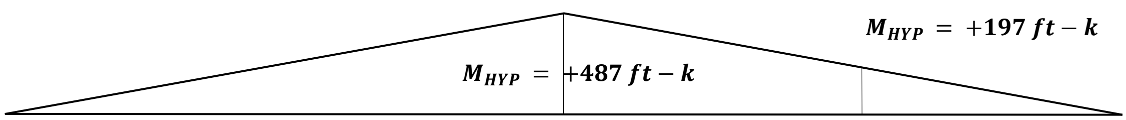
Referring to the beam diagram above, we see that the primary moments are 300 x 7" and 300 x 23" at the center support and at mid-span, respectively. Using our sign convention, we find the primary moment diagram to be:



Therefore, using the relationship of $M_{HYP} = M_{BAL} - M_1$, we find:

At Center Support: $M_{HYP} = 662 - 175 = +487\ ft-k$

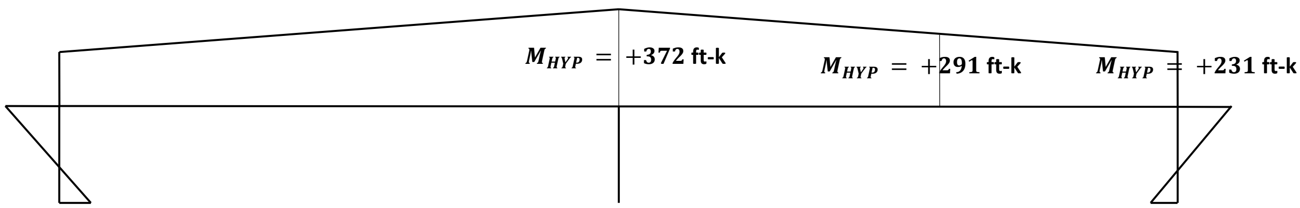
Near Mid-Span: $M_{HYP} = -378 - (-575) = +197\ ft-k$



Hyperstatic Moment Diagram

Discussion of Results

It is important to isolate the hyperstatic moments because there is a load factor of 1.0 applied to these moments in factored load combinations. Since we know that the hyperstatic moment diagram is linear, the location of the +197 ft-k hyperstatic moment is not at the center of the span – by similar triangles, it is actually 24.3 feet from the exterior support. This is usually considered to be close enough to the theoretical location of the maximum moment under uniform load to be able to algebraically add the factored moments at "mid-span" of the beam for design purposes. It is interesting to note that if we had included 18 inch square columns, which are fixed at the bottom, below the beam in our frame analysis, the hyperstatic moment diagram for the above example would look like this:



Hyperstatic Moment Diagram Including Columns

The above hyperstatic moment diagram is much closer to realistic conditions when a post-tensioned beam is built integrally with the columns below. Note that the hyperstatic moment in the beam at the center support is significantly less when the columns are included in the analysis, and that there is a substantial hyperstatic moment induced into the exterior columns. The center column has no hyperstatic moment, only because of the perfect geometrical symmetry, and because the tendon drapes and balancing loads in both spans are identical.

Pre-stress Losses

When an unbonded tendon is stressed, the final force in the tendon is less than the initial jacking force due to a number of factors collectively referred to as pre-stress losses. When a hydraulic jack stresses an unbonded tendon, it literally grabs the end of the tendon and stretches the tendon by five or six or more inches. As the tendon is stretched to its scheduled length, and before the jack releases the tendon, there is an initial tension in the tendon. However, after the tendon is released from the jack and the wedges are seated, the tendon loses some of its tension, both immediately and over time, due to a combination of factors, such as seating loss, friction, concrete strain, concrete shrinkage, concrete creep, and tendon relaxation.

Prior to the 1983 ACI 318, pre-stress losses were estimated using lump sum values. For example, the loss due to concrete strain, concrete creep, concrete shrinkage, and tendon relaxation in normal weight concrete was assumed to be 25,000 psi for post-tensioned concrete. This did not include friction and tendon seating losses. This was a generalized method and it was subsequently discovered that this method could not cover all situations adequately. Since the 1983 ACI 318, each type of pre-stress loss must be calculated separately. The effective pre-stress force, that is the force in the tendon after all losses, is given on the design documents and it is customary for the tendon supplier to calculate all pre-stress losses so that the number of tendons can be determined to satisfy the given effective pre-stress force. Even though most design offices do not normally calculate pre-stress losses, it is informative and relevant to understand how losses are calculated. Therefore, each type of pre-stress loss is discussed in more detail below.

- Seating Loss

When an unbonded tendon is tensioned, or stretched, to its full value, the jack releases the tendon and its force is then transferred to the anchorage hardware, and thereby into the concrete member. The anchorage hardware tends to deform slightly, which allows the tendon to relax slightly. The friction wedges deform slightly, allowing the tendon to slip slightly before the wires are firmly gripped. Minimizing the wedge seating loss is a function of the skill of the operator. The average slippage for wedge type anchors is approximately 0.1 inches. Seating losses are more significant on shorter tendons. There are various types of anchorage devices and methods, so the calculation of seating losses is dependent on the particular system used.

- Friction and Wobble

Loss of pre-stress force occurs in tendons due to friction that is present between the tendon and its surrounding sheathing material as it is tensioned, or stretched. Friction also occurs at the anchoring hardware where the tendon passes through. This is small, however, in comparison to the friction between the tendon and the sheathing (or duct) throughout its length. This friction can be thought of as two parts; the length effect and the curvature effect. The length effect is the amount of friction that would occur in a straight tendon - that is the amount of friction between the tendon and its surrounding material. In reality, a tendon cannot be perfectly straight and so there will be slight "wobbles" throughout its length. This so called wobble effect is rather small compared to the curvature effect. The amount of loss due to the wobble effect depends mainly on the coefficient of friction between the contact materials, the amount of care and accuracy used in physically laying out and securing the tendon against displacement, and the length of the tendon.

The loss in the pre-stressing tendons due to the curvature effect is a result of the friction between the tendon and its surrounding material as it passes through an intentional curve, such as drape, or a change in direction, such as a harped tendon. The amount of loss due to the curvature effect depends on the coefficient of friction between the contact materials, the length of the tendon, and the pressure exerted by the tendon on its surrounding material as it passes through a change in direction.

- Elastic Concrete Strain

When a concrete member is subjected to a compressive force due to pre-stressing tendons, it will shorten elastically. If this compressive force were removed, the member would return to its original length. Although ACI 318 does not specifically give procedures or requirements for calculating losses due to elastic strain, there are references available that provide some guidance. In general, the elastic strain shortening is a simplified computation involving the average net compressive stress in the concrete due to pre-stressing and the moduli of elasticity of the pre-stressing steel and the concrete at the time of stressing.

- Concrete Creep

A well known phenomenon of concrete in compression is that it creeps, or shortens, over time. The creep rate diminishes over time. Although ACI 318 does not specifically give procedures or requirements for calculating losses due to creep, there are references available that provide some guidance. In general, the creep shortening is a simplified computation involving the average net compressive stress in the concrete due to pre-stressing and the moduli of elasticity of the pre-stressing steel and the 28-day concrete strength. For post-tensioned members with unbonded tendons, for example, creep strain amounts to approximately 1.6 times the elastic strain.

- Concrete Shrinkage

The hardening of concrete involves a chemical reaction called hydration between water and cement. The amount of water used in a batch of concrete to make it workable far exceeds the amount of water necessary for the chemical reaction of hydration. Therefore, only a small portion of the water in a typical concrete mix is consumed in the chemical reaction and most of the water evaporates from the hardened concrete. When the excess mix water evaporates from a particular concrete member, it loses volume and therefore tends to shrink. Reinforcing steel and surrounding construction can minimize concrete shrinkage to some extent, but nonetheless shrinkage stresses are developed. Factors that influence concrete shrinkage include the volume to surface ratio of the member, the timing of the application of pre-stressing force after concrete curing, and the relative humidity surrounding the member.

- Tendon Relaxation

As a pre-stressed concrete member shortens, the tendons shorten by the same amount, thus relaxing some of their tension. The concrete member shortens due to the above three sources – elastic strain, creep, and shrinkage. Thus, the total tendon relaxation is a summation of these three concrete shortening sources. There may also be some relaxation in the pre-stressing steel over time, similar to concrete creep. This steel relaxation is a function of the type of steel used and on the ratio of actual steel stress to the specified steel stress.

As mentioned earlier, most design offices only show the required effective pre-stress force and the location of the center of gravity of the tendons on the construction documents. Remember that the effective pre-stress force is the force in the tendons after all losses have been accounted for. Calculating pre-stress losses for the design office can be very tedious and may not be exact, and therefore it is customary for the tendon supplier to calculate all pre-stress losses based on their experience and the specific tendon layout. Then, the number of tendons are determined to satisfy the given effective pre-stress force.

ACI 318-08 Requirements

We will now review some of the requirements contained in the 2008 edition of the Building Code Requirements for Structural Concrete ACI 318. We will be focusing on Chapter 18, Pre-stressed Concrete. The requirements in Chapter 18 have changed very little over the last several editions of the ACI 318.

ACI 318 places limits on the allowable extreme fiber tension stress at service loads according to the classification of a structure. Class U members are assumed to behave as uncracked sections and therefore gross section properties may be used in service load analysis and deflection calculations. Class C members are assumed to be cracked and therefore cracked section properties must be used in service load analysis, and deflection calculations must be based on an effective moment of inertia or on a bilinear moment-deflection relationship. Class T members are assumed to be in a transition state between cracked and uncracked, and the Code specifies that gross section properties may be used for analysis at service loads, but deflection calculations must be based on an effective moment of inertia or on a bilinear moment-deflection relationship. Pre-stressed two-way slabs must be designed as Class U and the extreme fiber tension stress must not exceed $6\sqrt{f_c'}$.

The allowable extreme fiber tension stresses in flexural members at service loads are as follows:

$$Class\ U\colon\ f_t \le 7.5\sqrt{f_c'}$$

$$Class\ T\colon\ 7.5\sqrt{f_c'} < f_t \le 12\sqrt{f_c'}$$

$$Class\ C\colon\ f_t > 12\sqrt{f_c'}$$

ACI 318 stipulates two cases of serviceability checks. The first is a check of the concrete tension and compression stresses immediately after transfer of pre-stress. The concrete stresses at this stage are caused by the pre-stress force after all short term losses, not including long term losses such as concrete creep and shrinkage, and due to the dead load of the member. These limits are placed on the design to ensure that no significant cracks occur at the very beginning of the life of the structure. The initial concrete compressive strength, f_{ci}', is used in this case. f_{ci}' is normally taken as 75% of the specified 28-day concrete compressive strength, but can be any specified minimum as long as it is greater than 3000 psi. The maximum permissible concrete stresses at force transfer are as follows:

Extreme fiber stress in compression at force transfer:

$$Ends\ of\ Simply\ Supported\ Members\colon\ f_{comp} \le 0.70\, f_{ci}'$$

$$All\ other\ Cases\colon\ f_{comp} \le 0.60\, f_{ci}'$$

Extreme fiber stress in tension at force transfer:

$$Ends\ of\ Simply\ Supported\ Members\colon\ f_{tens} \le 6\sqrt{f_{ci}'}$$

$$All\ other\ Cases\colon\ f_{tens} \le 3\sqrt{f_{ci}'}$$

If the above stresses are exceeded at force transfer, then additional bonded reinforcement shall be provided in the tensile zone to resist the total tensile force.

The second serviceability check is a check of the concrete tension and compression stresses at <u>sustained</u> service loads (sustained live load, dead load, superimposed dead load, and pre-stress) and a check at <u>total</u> service loads (live load, dead load, superimposed dead load, and pre-stress). These checks are to preclude excessive creep deflection and to keep stresses low enough to improve long term behavior. Note that the specified 28-day concrete compressive strength is used for these stress checks. The maximum permissible concrete stresses at the service load state are as follows:

Extreme fiber stress in compression: $f_{comp} \leq 0.45 f_c'$

Extreme fiber stress in tension: $f_{tens} \leq 0.60 f_c'$

If the above stresses are exceeded, then additional bonded reinforcement shall be provided in the tensile zone to resist the total tensile force. Also note that the above stress checks only address serviceability. Permissible stresses do not ensure adequate structural strength.

The tensile stress in pre-stressing steel shall not exceed the following:

Due to Jacking Force: $f_{tens} \leq 0.94 f_{py}$

But not greater than $0.80 f_{pu}$ or the maximum recommended by the anchor manufacturer.

Tendons Immediately After Transfer: $f_{tens} \leq 0.70 f_{pu}$

In the above, f_{pu} is the specified tensile strength of the pre-stressing steel and f_{py} is the specified yield strength of the pre-stressing steel. The most commonly used pre-stressing steel in the United States is Grade 270, low-relaxation, seven wire strand, defined by ASTM 416. Therefore, for this common pre-stressing steel, f_{pu} = 270 ksi and f_{py} = 0.90 f_{pu} or 243 ksi.

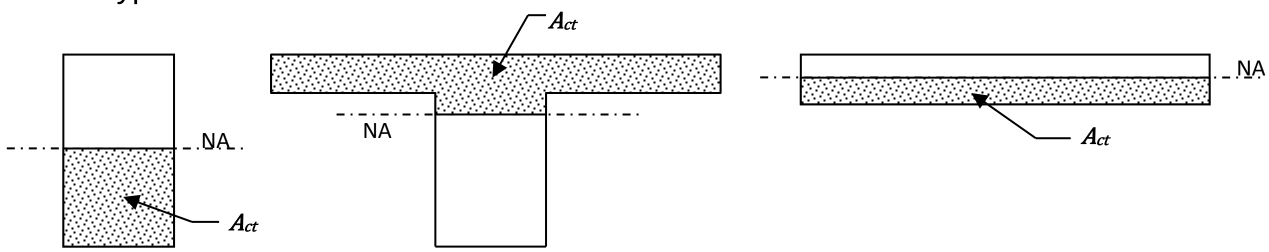
Permissible stresses for other type of pre-stressing steel, including deformed bars, vary slightly and can be found in the ACI 318 commentary. Since the scope of this course is limited to unbonded post-tensioned systems, and we will only be considering Grade 270 low-relaxation steel, the permissible stresses for other steels are not given here.

Minimum Bonded Reinforcing

All flexural members with unbonded tendons require some amount of bonded reinforcing. For beams and one-way slabs, this bonded reinforcing is required regardless of the services load stresses. For two-way slabs, the requirement for this bonded reinforcing depends on the services load stresses. This bonded reinforcing is intended to limit crack width and spacing in case the concrete tensile stress exceeds the concrete's tensile capacity at service loads. ACI requires that this bonded reinforcement be uniformly distributed and located as close as possible to the tension face. For beams and one-way slabs, the minimum area of bonded reinforcing A_s is:

$$A_s = 0.004A_{ct}$$

A_{ct} is defined as the area of that part of the cross section between the flexural tension face and the center of gravity of the cross-section. A_{ct} is graphically defined below for several typical cross sections.



Rectangular Beam TEE Beam (Negative Flexure) One-Way Slab

For two-way post-tensioned slabs with unbonded tendons, bonded reinforcing is not required in positive moment areas (in the bottom of the slab) if the extreme fiber tension at service loads, after all pre-stress losses, does not exceed the following:

$$f_t \leq 2\sqrt{f_c'}$$

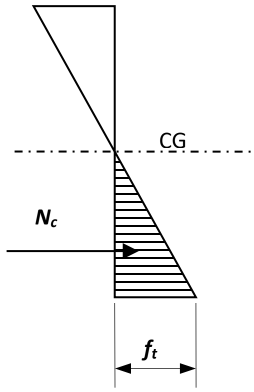
Recall that the maximum extreme tension fiber stress in a two-way slab is $6\sqrt{f_c'}$, so a minimum amount of bottom steel is required for tensile stresses in the range of:

$$2\sqrt{f_c'} < f_t \leq 6\sqrt{f_c'}$$

When it is required per the above equation for two-way post-tensioned slabs with unbonded tendons, the minimum area of bonded reinforcement in positive moment regions is:

$$A_s = \frac{N_c}{0.5\, f_y}$$

Where N_c is the tension force in the concrete due to unfactored (service) dead plus live load and is illustrated below.



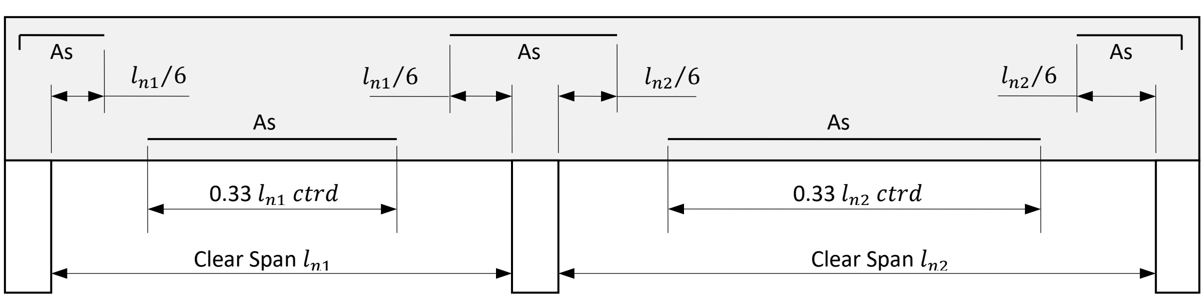
$$A_s = \frac{N_c}{0.5\, f_y} \qquad \text{When} \qquad 2\sqrt{f_c'} < f_t \le 6\sqrt{f_c'}$$

For two-way post-tensioned slabs with unbonded tendons, the minimum area of bonded reinforcement required in negative moment regions at column supports is

$$A_s = 0.00075 A_{cf}$$

Where A_{cf} is defined as the area of the larger gross cross-sectional area of the slab-beam strips in two orthogonal equivalent frames intersecting at the column. The dimensions and geometry are as defined in ACI 318 Chapter 13 – Two-Way Slab Systems and will not be reiterated here. The bonded reinforcing required in negative moment regions at columns shall be distributed between lines that are 1.5h outside both sides of the face of the column support, shall be a minimum of four bars, and shall be spaced no more than 12" on center.

ACI 318 also requires a minimum length of bonded reinforcing in positive and negative moment regions. It should be evident that the minimum lengths and areas of bonded reinforcement required by the ACI 318 may be exceeded by structural demand, as we will see later on. The minimum length of bonded reinforcing is shown below.

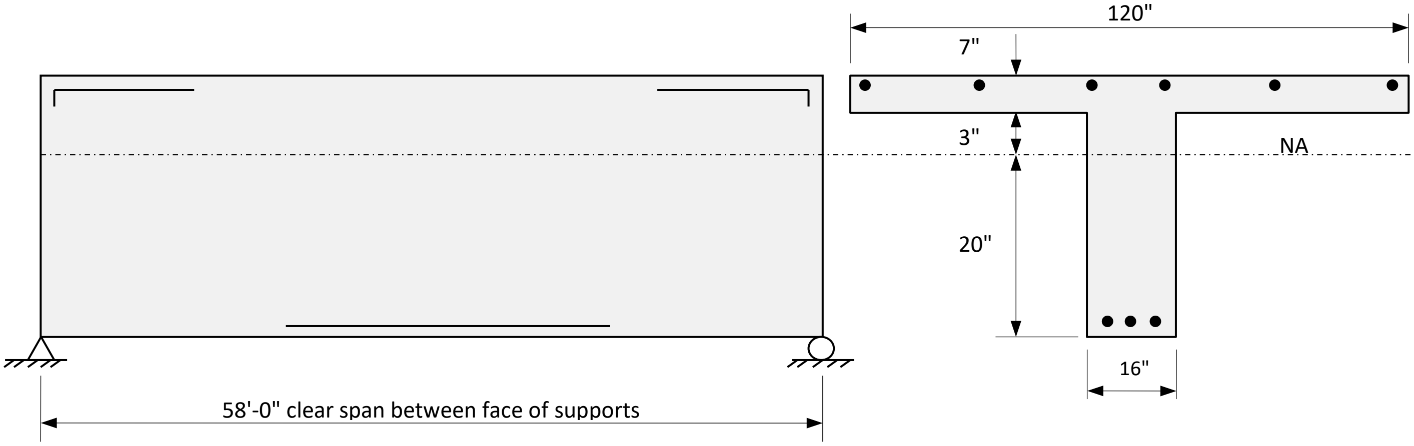


Minimum Lengths of Bonded Reinforcing

Example

Given:

The post-tensioned beam shown below.



Find:

Determine the minimum area and length of bonded reinforcing required.

Solution:

Let's begin by finding A_{ct}, the area of that part of the cross section between the flexural tension face and the center of gravity of the cross-section for the top and bottom of the beam. The neutral axis has already been determined to be 10 inches from the top of the beam.

Top A_{ct} = 3 x 16 + 7 x 120 = 888 sq. in.

Bottom A_{ct} = 16 x20 = 320 sq. in.

Now we can compute the minimum required area of bonded reinforcing using $A_s = 0.004A_{ct}$:

Top A_s = 0.004 x 888 = 3.55 sq. in. <u>Use 12#5 Top</u>

Bottom A_s = 0.004 x 320 = 1.28 sq. in. <u>Use 3#6 Bottom</u>

The ACI 318 states that this minimum bonded reinforcement shall be uniformly distributed as close as close as possible to the tension face. Therefore, the bottom steel would be spaced approximately 6 inches on center, allowing for concrete cover on the side of the stem and beam stirrups. This seems like an appropriate spacing, but in all likelihood, flexural demands will require a larger area of steel and possibly more than three bars.

The minimum bonded reinforcing in the top of the beam will be placed at both ends, where negative bending moments will occur due to support fixity. 12#5s were selected instead of 8#6s to avoid having fewer bars spaced farther apart. The 12#5s result in a spacing of approximately 10 inches on center, which seems to be better than almost 18 inches on center had 12#6s been selected. However, either spacing would probably meet the intent of the ACI 318.

The minimum length of the bottom bars is one-third the clear span, or 58/3 = 19.33 feet. The top bars need to extend into the clear span by at least one-sixth the clear span, or 58/6 = 9.67 feet.

Example

Given:

A continuous post-tensioned one-way slab is 5 1/2" thick. It is supported by 16" wide concrete beams that are at 22'-0" on center.

Find:

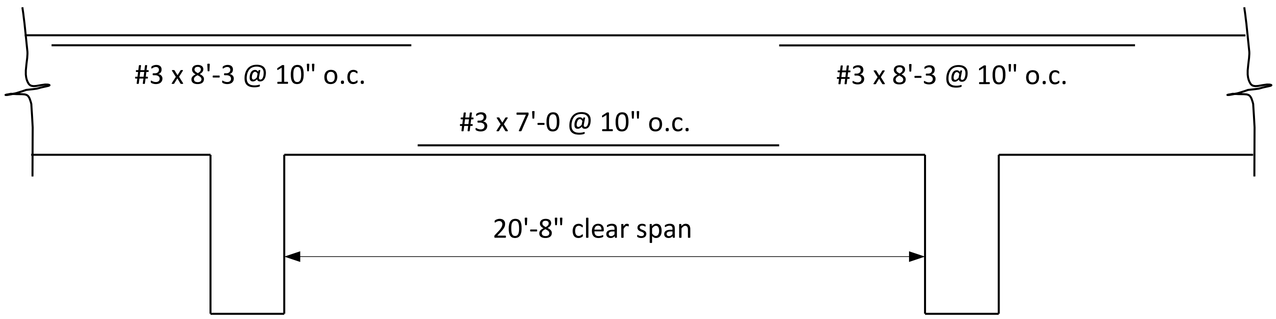
Determine the minimum area and length of bonded reinforcing required, without considering loading demands.

Solution:

Since the concrete slab has a symmetric cross section, the A_{ct} for the and bottom will be the same and equal to 12x5.5/2 = 33 sq. in. per foot. Using this, the minimum area of bonded reinforcing is:

A_s = 0.004 x 33 = 0.132 sq. in. per foot of slab Use #3@10" o.c.

The minimum length of the bottom bars is one-third the clear span, or (22-1.33)/3 = 6.89 feet and the top bars need to extend into the clear span by at least one-sixth the clear span, or (22-1.33)/6 = 3.44 feet. The results are illustrated below.



Minimum Bonded Reinforcing – One-Way Slab Example

Example

Given:

A two-way post-tensioned slab is 8" thick. It is supported by 16" x 16" columns that are at 20'-0" on center both ways. The 28-day specified concrete compressive strength is 5,000 psi and mild reinforcing with f_y = 60,000 psi will be used. The extreme fiber tension at service loads in positive moment areas (in the bottom of the slab), after all pre-stress losses, is 350 psi.

Find:

Determine the minimum area, length, and distribution of bonded reinforcing required, without considering loading demands.

Solution:

Because the bays are equal in both directions, we need only investigate one direction and apply the results to both. Since the tensile stress in the bottom of the concrete slab is greater than $2\sqrt{f_c'} = 140$ psi and less than $6\sqrt{f_c'} = 420$ psi, minimum bonded reinforcing is required in the bottom of the slab in the positive moment regions. Therefore, by referring to the figure on page 34, we have:

$$N_c = (350\,psi)(4\,in)\frac{12\,in}{2} = 8{,}400\,lbs/foot$$

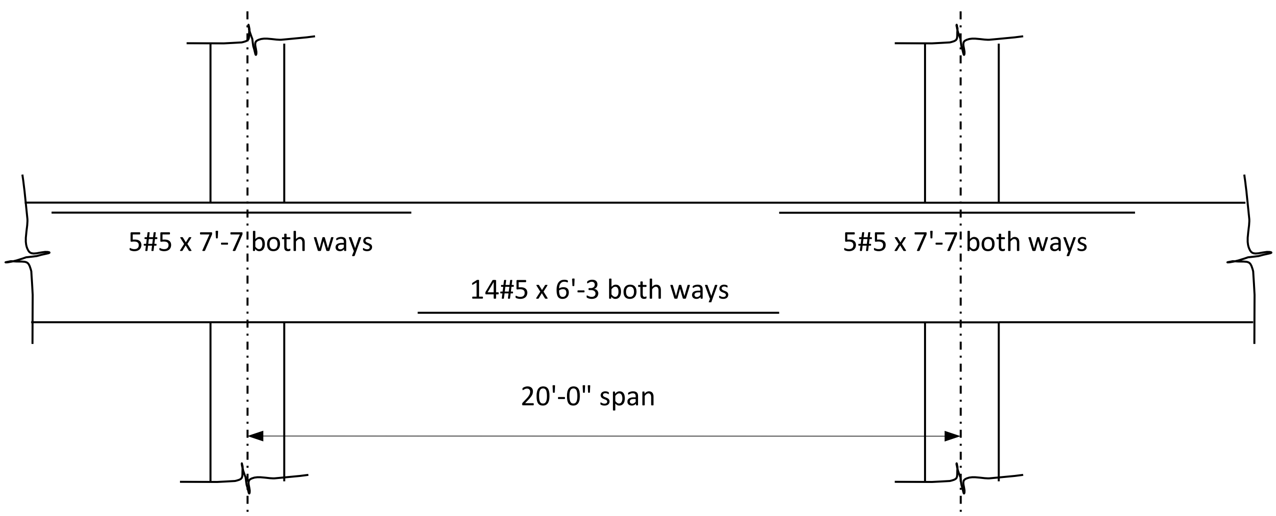
$$A_s = \frac{N_c}{0.5\,f_y} = \frac{8{,}400\,lbs/foot}{0.5 \times 60{,}000\,psi} = 0.28\,sq.in./foot$$

This bottom reinforcing should be placed in the middle strip. From chapter 13 of ACI 318, the middle strip width is defined as being bounded by two column strips, which is the smaller of the centerline spans of the slab. In this example, our column strip widths are 20/4 = 7'-6" and the middle strip width is 15'-0". Therefore, the total area of reinforcing is 15 x 0.28 = 4.20 sq. in. Recall that the minimum length of the bottom bonded reinforcing is one-third the clear span, or (20-1.33)/3 = 6.22 feet. So, we will use 14#5 bottom bars, 6'-3" long, equally spaced in a 15-foot wide middle strip, both ways.

For the minimum area of bonded reinforcement in negative moment regions at the column supports, A_{cf} in this example is the entire bay width times the slab thickness, or 20 x 12 x 8 = 1920 sq. in. For columns at the edge of the floor plate, only half of the larger bay is used. Therefore,

$$A_s = 0.00075A_{cf} = 0.00075(1920) = \ 1.44\, sq.in.$$

The width over which this area of bonded reinforcing is distributed is 1.5h on both sides of the column face, or 16 + (2x1.5x8) = 40 inches. The minimum length extended beyond the column face is one-sixth the clear span, or (20-1.33)/6 = 3.11 feet. So, we will use 5#5 top bars, 7'-7" long, equally spaced in a 40-inch wide strip centered on the column, both ways.



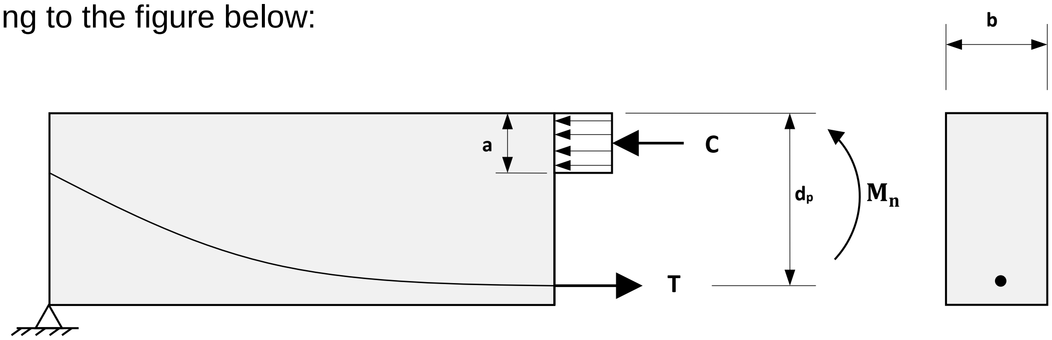
Minimum Bonded Reinforcing – Two-Way Slab Example

Ultimate Flexural Strength

Let us now begin our investigation into the ultimate flexural strength of a pre-stressed member. The design moment strength may be computed using the same methodology used for non pre-stressed members. That is, a force couple is generated between a simplified rectangular compression block and the equal and opposite tensile force generated in the reinforcing steel, which is in our case pre-stressing steel.

For purposes of this course, we will narrow our focus to post-tensioned members with unbonded tendons, and we will use the approximate values for the nominal stress in the pre-stressing steel, $\mathbf{f_{ps}}$, instead of using strain compatibility. For more accurate determinations of the nominal stress in the pre-stressing steel, and for flexural members with a high percentage of bonded reinforcement, and for flexural members with pre-stressing steel located in the compression zone, strain compatibility should be used.

Referring to the figure below:



The rectangular compression block has an area equal to a times b. Equating the compression resultant, C, to the tensile resultant, T, the nominal moment capacity can be written as:

$$M_n = T\left(d_p - \frac{a}{2}\right) = C\left(d_p - \frac{a}{2}\right)$$

$$\phi M_n = \phi A_{ps}\, f_{ps}\, (d_p - \frac{a}{2})$$

Where A_{ps} is the area of pre-stressed reinforcement, f_{ps} is the stress in the pre-stressed reinforcement at nominal moment strength, and ϕ is the strength reduction factor (0.90 for flexure). ACI 318 defines the approximate value for f_{ps} as the lesser of the following, depending on the span-to-depth ratio:

For $\frac{\textbf{Span}}{\textbf{Depth}} \leq 35$, the lesser of:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f_c'}{100\rho_p}$$

$$\leq f_{py} \ or \ f_{se} + 60{,}000$$

For $\frac{\textbf{Span}}{\textbf{Depth}} > 35$, the lesser of:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f_c'}{300\rho_p}$$

$$\leq f_{py} \ or \ f_{se} + 30{,}000$$

Where f_{se} is the effective stress in the pre-stressing steel after all losses. The depth of the compression block is defined as:

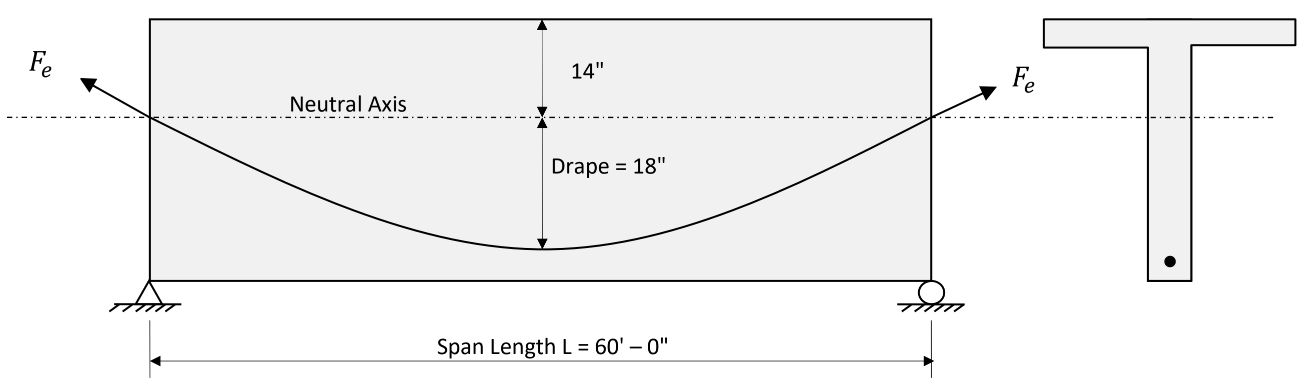
$$a = \frac{A_{ps} f_{ps}}{0.85 f_c' b}$$

<u>Example</u>

Given:

The simply supported post-tensioned beam from page 12 and shown below.

f_c' = 7000 psi
f_{py} = 270 ksi
F_e = 684 kips
16" x 36" beam with 7" x 100" flange
(26) ½" diameter tendons; A_{ps} = 26 x 0.153 = 3.98 sq. in.; ρ_p = 3.98/(16x32) = 0.00777



Find:

The nominal moment capacity, ϕM_n , neglecting any bonded reinforcing.

Solution:

The span-depth ratio is 60/3 = 20 which is less than 35. Therefore,

$$f_{ps} = f_{se} + 10{,}000 + \frac{f_c'}{100\rho_p} = \frac{684\ kips\ (1000)}{3.98\ sq.in.} + 10{,}000 + \frac{7000\ psi}{100(0.00777)}$$

$$f_{ps} = 190.9\ ksi \Longrightarrow \boxed{\text{Use } f_{ps} = 191 \text{ ksi}}$$

$$\leq f_{py} = 270\ ksi$$

$$\leq f_{se} + 60{,}000 = 231.9\ ksi$$

Next, determine depth of compression block, a:

$$a = \frac{A_{ps}\, f_{ps}}{0.85 f_c' b} = \frac{3.98\ sq.in.(191\ ksi)}{0.85\ (\ 7\ ksi)(100\ in)} = 1.28\ inches < 7\ inches - OK$$

Now we can compute the nominal flexural capacity at mid-span of this beam section:

$$\phi M_n = \phi A_{ps}\, f_{ps} \left(d_p - \frac{a}{2}\right)$$

$$\phi M_n = 0.90(3.98\ sq.in.)(191\ ksi)\left(32\ in. - \frac{1.28\ in.}{2}\right)\frac{1}{12}$$

$$\underline{\phi M_n = 1788\ ft - kips}$$

Let's now consider including the contribution of the bonded reinforcing steel to the nominal moment capacity. Assuming the bonded reinforcement is at the same depth as the pre-stressing steel in the beam (a slightly conservative assumption since the center of gravity of the mild reinforcing is often closer to the face of the beam, i.e. deeper, than the center of gravity of a bundle of tendons), the tensile component of the moment couple becomes the sum of both pre-stressing steel and the bonded reinforcement and can be written as follows:

$$\phi M_n = \phi(A_{ps}\, f_{ps} + A_s\, f_y)(d_p - \frac{a}{2})$$

The depth of the compression block then becomes:

$$a = \frac{A_{ps}\, f_{ps} + A_s\, f_y}{0.85 f_c' b}$$

Example

Given:

The simply supported post-tensioned beam from the previous example. In addition to the post-tensioning tendons, the beam has 3#10 bars in the bottom with a yield strength of 60 ksi.

Find:

The nominal moment capacity, ϕM_n , including the bonded reinforcing.

Solution:

From the previous example, the effective stress in the pre-stressing steel at nominal moment strength is unaffected by the bonded reinforcing and so it is the same:

$$f_{ps} = 191\ ksi$$

Determine depth of compression block, a:

$$a = \frac{A_{ps}\, f_{ps} + A_s\, f_y}{0.85 f_c' b}$$

$$a = \frac{3.98\ sq.in.(191\ ksi) + 3(1.27\ sq.in)(60 ksi)}{0.85(7\ ksi)(100\ in.)} = 1.66\ in. < 7\ inches - \ OK$$

And the nominal flexural capacity of this beam section is:

$$\phi M_n = \phi(A_{ps}\, f_{ps} + A_s\, f_y)(d_p - \frac{a}{2})$$

$$\phi M_n = 0.90[(3.98)(191) + (3)(1.27)(60)](32 - \frac{1.66}{2})\frac{1}{12}$$

$$\underline{\phi M_n = 2311\ ft\ kips}$$

Ultimate One-Way Shear Strength

The treatment of one-way, or so called "beam shear," will not be rigorous in this course. ACI 318 offers a simplified approach to computing the shear capacity of a pre-stressed member and this is what will be used here. In general, just as in a non pre-stressed member, the following basic equation must be satisfied:

$$\emptyset V_n \geq V_u$$

Where Ø is the strength reduction factor (typically 0.75 for shear), V_n is the nominal shear capacity, and V_u is the ultimate, or factored, applied shear demand. The nominal shear capacity is the sum of the nominal shear strength provided by the concrete, V_c, and the nominal shear strength provided by the steel, V_s, and is written as follows:

$$V_n = V_c + V_s$$

For pre-stressed members, the nominal shear strength provided by the concrete is:

$$V_c = (0.6\lambda\sqrt{f_c'} + 700\frac{V_u d_p}{M_u})b_w d$$

But V_c need not be less than $2\lambda\sqrt{f_c'}b_w d$ and shall not be greater than $5\lambda\sqrt{f_c'}b_w d$ and $V_u d_p/M_u$ shall not be greater than 1.0. λ is a modification factor for lightweight concrete.

A minimum amount of shear reinforcement is required anytime the ultimate applied shear demand, V_u, is greater than $0.5\emptyset V_c$. The minimum amount of shear reinforcing is the largest of:

$$Min\ A_v = 0.75\sqrt{f_c'}\frac{b_w s}{f_{yt}}$$

$$Min\ A_v = 50\frac{b_w s}{f_{yt}}$$

$$Min\ A_v = \frac{A_{ps} f_{pu} s}{80 f_{yt} d}\sqrt{\frac{d}{b_w}}$$

Note that the only place the pre-stressing force comes into play is the third equation above. Otherwise, the minimum amount of shear reinforcement for pre-stressed members is identical to that for non pre-stressed members. The design of concrete members for torsional forces is outside the scope of Part One of this course. Likewise, the investigation of the shear strength of two-way slabs is not covered here in Part One but is covered in Part Two.

Where shear reinforcement is required by structural demand, that is when $V_u \geq \emptyset V_c$, and it is perpendicular to the axis of the member, the nominal shear strength provided by the steel is:

$$V_s = \frac{A_v f_{yt} d}{s}$$

Therefore, the nominal shear capacity of a pre-stressed concrete member may be written as:

$$V_n = V_c + V_s \geq \frac{V_u}{\emptyset}$$

$$V_c + \frac{A_v f_{yt} d}{s} \geq \frac{V_u}{\emptyset}$$

$$\emptyset V_c + \frac{\emptyset A_v f_{yt} d}{s} \geq V_u$$

Solving for the required area of shear reinforcing A_v gives:

$$A_v \geq \frac{V_u - \emptyset V_c}{\emptyset f_{yt} d} \times s$$

The maximum spacing of shear reinforcement in pre-stressed members shall not be greater than 0.75h nor 24 inches.

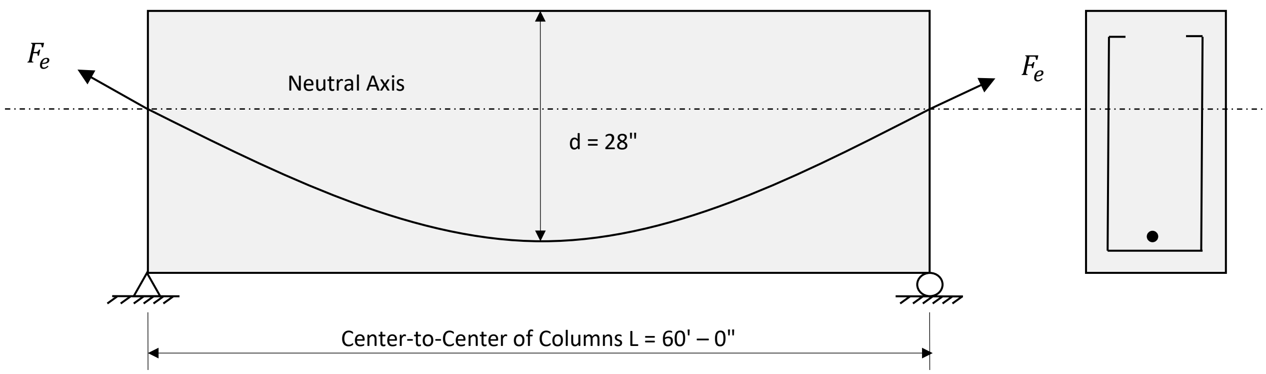
<u>Example</u>

Given:

The simply supported post-tensioned beam shown below and:

- f_c' = 5000 psi
- f_{yt} = 60 ksi
- f_{pu} = 270 ksi

- o 16" x 36" beam
- o 16-½" diameter tendons; A_{ps} = 16 x 0.153 = 2.45 sq. in.
- o Beam dead load = 3.0 klf, unfactored; beam live load = 1.5 klf, unfactored
- o 24" x 24" columns



Find:

Design the shear reinforcing at a distance "d" from the face of the support.

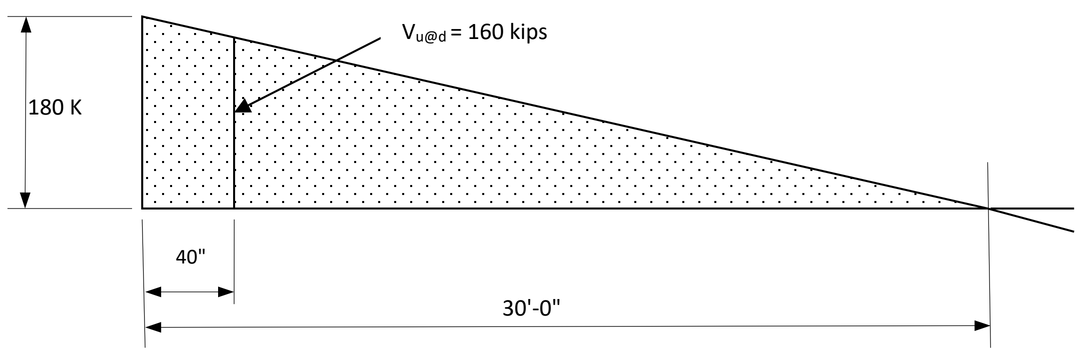
Solution:

The factored uniformly distributed load is:

$$w_u = 1.2 \times 3.0 + 1.6 \times 1.5 = 6.0\ kips/foot$$

$$R_u = 6.0 \frac{kips}{foot} X\ 30\ feet = 180\ kips$$

This yields a shear diagram as follows:



At a distance d = 28 inches from the face of support, or 40 inches from the centerline of the support, the factored shear and moment are:

$$V_{u@d} = 180\ kips - 6.0\ k/ft\left(\frac{12+28}{12}\right) = 160\ kips$$

$$M_{u@d} = 3.33\ ft\left(\frac{180+160}{2}\right) = 567\ ft - kips$$

$$\frac{V_u d_p}{M_u} = \frac{160(2.33)}{567} = 0.66 < 1.0\ \Rightarrow OK$$

$$\emptyset V_c = \emptyset\left(0.6\lambda\sqrt{f_c'} + 700\frac{V_u d_p}{M_u}\right) b_w d = \ 0.75\left(0.6\sqrt{5000} + 700 \times 0.66\right)(16 \times 28)/1000$$

$$\emptyset V_c = \ 169\ kips$$

Remember that:

$$\emptyset 2\lambda\sqrt{f_c'} b_w d < \emptyset V_c < \emptyset 5\lambda\sqrt{f_c'} b_w d$$

$$0.75 \times 2 \times 1.0\sqrt{5000}(16 \times 28) < \emptyset V_c < 0.75 \times 5 \times 1.0\sqrt{5000}(16 \times 28)$$

$$47.5 < 169 \nless 118.8$$

Therefore we must use $\emptyset V_c = \ 118.8\ kips$. This is the maximum nominal shear strength that can be provided by the concrete alone.

Let's now compute the amount of shear reinforcing required by structural demand at a distance "d" from the face of the support:

$$A_v \geq \frac{V_u - \emptyset V_c}{\emptyset f_{yt} d} \times s$$

$$A_v \geq \frac{160 - 118.8}{0.75 \times 60 \times 28} \times 12 = 0.39\frac{in^2}{foot} \Rightarrow \mathbf{\#5@18\ Double\ Leg}$$

As a check, let's find the minimum amount of shear reinforcing required and will be the largest of the following three results:

$$Min\ A_v = 0.75\sqrt{f_c'}\frac{b_w s}{f_{yt}} = 0.75\sqrt{5000}\frac{16 \times 12}{60{,}000} = 0.17\frac{in^2}{foot} \ \Rightarrow \textbf{Largest}$$

$$Min\ A_v = 50\frac{b_w s}{f_{yt}} = 50\frac{16 \times 12}{60{,}000} = 0.16\ in^2/foot$$

$$Min\ A_v = \frac{A_{ps} f_{pu} s}{80 f_{yt} d}\sqrt{\frac{d}{b_w}} = \frac{2.45 \times 270 \times 12}{80 \times 60 \times 28}\sqrt{\frac{28}{16}} = 0.08\ in^2/foot$$

None of these are greater than the reinforcing required by structural demand and so the minimum shear reinforcing does not control.

Therefore, #5 @ 18 " o.c. double leg stirrups would work, giving us 0.41 in^2 per foot. Most designers would space the stirrups in convenient groups of incrementally larger spacing away from the support until they are no longer required by structural demand or minimum reinforcing requirements. However, it is common practice to use stirrups throughout the entire span to have something to which to tie the tendon bundle support bars.

Conclusion

Part One of this three-part course covers many of the fundamentals of post-tensioned concrete design for building structures using unbonded tendons. With a good understanding of the material in Part One of this course, you should know something about the historical background of post-tensioned concrete and the difference between post-tensioned members and pre-tensioned members. You should also understand the load balancing concept, hyperstatic moments, pre-stress losses, and the basic requirements of ACI 318 (Building Code Requirements for Structural Concrete). We also covered nominal flexure and shear capacities of post-tensioned members, including a few examples. Specifically, you should now be able to:

- Compute effective pre-stress force F_e for a given drape and balanced load
- Understand allowable stresses according to ACI 318-08
- Understand pre-stress losses
- Compute the balanced and hyperstatic moments for a continuous structure
- Determine the minimum amount of flexural and shear reinforcing required
- Calculate the nominal moment capacity ϕM_n and nominal shear capacity ϕV_n of a cross section

To be comfortable performing a preliminary design by hand or to be able to quickly check a computer generated design or an existing design by hand, Parts Two and Three of this course should also be completed. In Part Two, we will use the material learned in Part One to work design examples of two common structural systems used in buildings and parking structures, namely a one-way continuous slab and a continuous beam. Then in Part Three, the more advanced topic of two-way slabs is covered, including the Equivalent Frame Method, punching shear, and moment transfer at columns, with several examples.

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