A SunCam online continuing education course

Structural Concepts

for Non-Structural Engineers

by

Warren H. Miller, P.E.

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CREDENTIALS

The author, a PE, has 50 years of experience designing structures and mechanisms. He holds 13 US Patents ranging from sawmills to spacecraft.

STRUCTURE

What do we mean by this term "Structure"? Simply, it is any kind of framework that holds components in relationship to other components. A house has a structure. An airplane is a structure. A car is a structure. A swing set is a structure. Obviously, a structure must be strong enough not to break under expected loads and stiff enough to hold the components where we need them to be. "Structure" is in contrast to "Mechanism" which is an assembly of parts that move with respect to one another. While each of the parts of a mechanism is a structure of its own, the primary function of a mechanism is to perform a task.

Let's look at what drives the design of some common structures: Take a house. The floor of a house is obviously a structure. If it is a floor over an opening, like a first floor over a basement, the primary requirement is that it be stiff. This is mainly an esthetic thing, for no one wants to feel the floor bouncing under their feet. So building codes require that a floor, when loaded with a certain large load, should not deflect more than a certain small amount. This is a stiffness-driven design. In general, you could sit an elephant in the center of your floor, and it would not break. As more and more elephants sat on the floor, it would eventually break, but before it did, it would have sagged alarmingly.

Contrast this with the roof design. No one cares what a roof feels like to walk on, but they really don't want it to blow away. So the design of a roof depends on strong trusses to resist the wind loads (or snow loads) on the roof deck, held to the rest of the structure with strong attachments. This is a strength-driven design.

So what drives the design of a particular structure? Does it have to be really stiff, like the frame of an automobile, so that the body doesn't rattle, and strong enough to keep the engine attached? Should it be very strong, like the wing of an airplane, and flexible enough to isolate the passengers from the shocks of air turbulence? There is a very real difference between strong structures and stiff structures.

Before we get into the business of analyzing structures, we need to define some terms. Everyone is familiar with the terms "Force", "Moment," and "Torque". Fortunately, force means just the same to a structural engineer as it does to anyone else: A load applied to push or pull on the structure. Moment means something completely different to a structural engineer. To most folks, moment means a short period of time. "I'll be with you in a moment." But to a structural engineer, moment means a force applied at a distance from the point being investigated, and at some angle to that point. A force of 1 pound applied at right angles to a beam will create a moment of 1 pound-foot at a point on the beam 1 foot from the point of application of the pound. It will create a moment of 2 pound-feet at a point 2 feet from the point of application of the pound, and a moment of 0 pounds-feet at the point of application of the 1-pound force. The fact that the moment caused by the application of a force increases as the distance from the force increases gives us a clue as to where we got the term "moment".

In the nineteenth century, moment not only meant a short period of time, but it also meant "importance". "We are undertaking a work of great moment." A residue of that still exists in our phrase "A momentous occasion." Pick up a dining-room chair and hold it up near your chest. For most people and most chairs, the weight will be trivial. Now extend the chair to arms length in front of you, and the weight of that chair will have become "important". Hence the term "moment".

Finally, most folks associate the term "torque" with twist. That is basically true, but in a technical sense, torque is caused by two equal forces in opposite directions, separated by a distance. Two 1-pound forces separated by a foot cause a torque of 1 pound-foot to be applied to the structure. It is like a moment (note the units of measurement) but is constant wherever it is applied.

There are three kinds of stress to be found in a structure after a load has been applied: Tensile Stress, Compressive Stress, and Shear Stress. Tensile and compressive stresses typically run along the length of the beam while shear stress runs across the height of the beam. Tensile stresses result in the molecules of the material being pulled apart from one another. Shear stresses result in the molecules trying to slide past one another. Compressive stresses result in the molecules being pushed together, which they don't mind, but eventually they try to "squirt out sideways" from the load, which converts them into shear stresses.

In general, we try to compare the stresses that we calculate in the structure to the stresses that we have found that the material is capable of resisting. These stresses are found in tables, and are listed as Ultimate Tensile Stress, which is the stress at which the material breaks, and Yield Stress, which is the stress at which the material just starts to become permanently deformed. For most metals, compressive stress is assumed to be the same as tensile stress, but is sometimes listed in the tables also.

Ultimate and yield shear stresses are also listed in the tables, and have the same meanings as tensile stress.

THE DIFFERENCE BETWEEN STIFFNESS AND STRENGTH

A rope is flexible; an eggshell is stiff. Two properties essentially determine the **stiffness** of a structure: Shape and modulus. (Modulus is the inherent stiffness property of the material itself. We shall talk more about modulus later). From a distance, the shape of a rope is a long thin rod, a shape which is naturally flexible. From close up, we see that the rope is made up of many tiny strands, which make it even more flexible. It is an inherently flexible shape.

An eggshell is a three-dimensional arch. It is a shape which supports itself from all directions, and hence, is very stiff.

The rope can be made from many different materials, from cotton to nylon to steel. The stiffer materials make the rope stiffer, but it is still very flexible. The eggshell, on the other hand, is made primarily from calcium, which is almost limestone. Limestone is stiffer than nylon, but less stiff than steel. An eggshell made of either nylon or steel would still be very stiff, because of its shape.

BUT a rope is strong and an eggshell is weak. Again, two properties determine the **strength** of a structure: Shape and tensile strength. (Tensile strength is the inherent strength property of the material itself. We shall talk more about tensile strength later). When a rope is in use, it is pulled into a straight line. Every fiber in the rope is under tension, nothing is being bent. The strength of the rope is only a function of the tensile strength of each fiber.

The eggshell is made of a quite weak material (like chalk or limestone), and even though its shape makes it self-supporting, it takes little force on any point on the shell to crush it. But while the shell has a little strength to resist crushing (such as when under the weight of the mother hen), when pushed against from the inside, now it is no longer self-supporting and it is easy for a chick to break out of it.

It is important early on, when looking at a structure, to decide what it does, or when designing a structure, to decide whether the major requirement for the structure is to be stiff or to be strong.

STRETCHING OF BEAMS

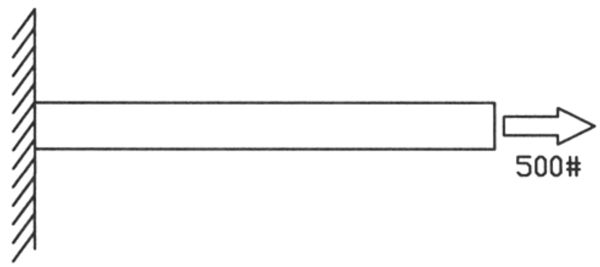

FIGURE 1

Let's look at the beam in Figure 1. It is a simple aluminum bar 10" long and 1" by 1" in cross section. It is firmly anchored into the wall on the left. (The symbolism of "a wall with hairs" is universally used by structural engineers to indicate that the end of a beam is absolutely rigidly anchored. It cannot move or twist in any direction). We call that beam "cantilevered". A load, P, of 500 pounds is applied to the end of the beam, pulling it to the right. We immediately want to know: How strong is this beam? Can that little piece of aluminum support a load of 500 pounds without breaking? And then we might want to know: How much will this beam stretch under that load? The formula for figuring out the stress in the beam is very simple:

$$S = \frac{P}{A}$$

where S equals stress, P equals the load, and A equals the cross-sectional area. (We will be using a few formulas in this lesson, and I will not try to derive them. They are all available in a book called *Formulas for Stress and Strain*, by Raymond J. Roark. This book is the "Bible" for structural engineers. It is published by McGraw-Hill and all the formulas in it have been incorporated into the application "MathCAD". In the future, I will refer to it as "Roark").

So, the stress in the beam is 500 pounds divided by 1 square inch of area, or 500 pounds per square inch (generally abbreviated psi). What does that mean for us? Since aluminum has a tensile strength (strength in tension) of anywhere between 20,000 psi and 80,000 psi, the bar certainly will not break. Why this wide range of tensile stresses? Because the strength (tensile stress) of aluminum is a function of what alloy it is and what processing it has had. If you want to know how strong a material is, you must look it up. Knowing that a material is "aluminum" or "steel" or "plastic" is of very little use. You must know what kind of aluminum it is, and you can't tell just by looking.

But perhaps we need to have the end of the beam stay rigidly in one place when the load is applied to it. How do we find out how much the beam will stretch under that load? The formula is a little more complicated:

$$d = \frac{PL}{AE}$$

where d equals deflection, L equals the length of the beam, and E equals the "modulus" of the material.

"Modulus" or more properly "Young's Modulus", is the inherent stiffness of the material. You look it up in tables. Importantly, unlike tensile strength, it is essentially independent of alloy or processing. If you are holding a piece of aluminum in your hand, it has a modulus of 10 million psi. Note the psi, the units of stress. The real units of modulus are: inches of stretch per inch of length of the beam per psi of applied stress. That simplifies from in/in/psi to psi.

If you are holding a piece of steel in your hand, it has a modulus of 30 million psi, whether it is a coat hanger or a ball bearing.

So, plugging numbers into the formula, d = 500 # x 10" / 1 in^2 x 10 x 10^6 psi, or d = .0005". Our beam will only stretch one half of a thousandth of an inch under a 500 pound load. Pretty stiff.

LATERAL DEFLECTION OF BEAMS

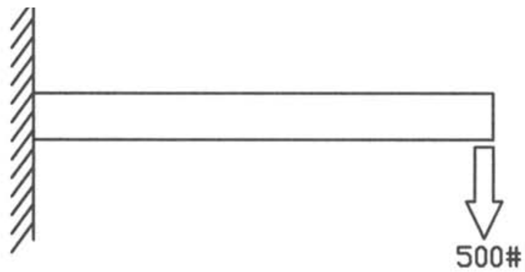

FIGURE 2

Let's look again at our same beam. (Figure 2) Note that this time, we are applying the load downward instead of lengthwise, which means that we are bending the beam instead of stretching it. We ask the same questions: Will the beam break, and how much will it bend? I now need to introduce a new concept, "Moment of Inertia". It is defined as the sum of all of the little pieces of area that make up the cross section of the beam, times the square of their distances from the "Neutral Axis". That is another new term. When you pull down on the end of a beam, for example, you cause the material forming the top of the beam to stretch and the material at the bottom of the beam to compress. We talk about the material in the beam as if it were made up of fibers, so we say that the closer the fibers are to the top of the beam, the more that they stretch, and the closer the fibers are to the bottom of the beam, the more that they compress. Obviously, somewhere in the middle of the cross-section of the beam are fibers that have no load on them at all. They lie along the neutral axis. Technically, the neutral axis is at the "centroid" of the area, but as a practical matter, if you cut the shape of the cross-section of the beam out of paper and balance the paper on a knife-edge, the line of contact of the knife-edge with the paper is the neutral axis.

So, if we are going to find out how strong the beam is, we must first find out what the moment of inertia of the cross-section of the beam is. Looking up the formula for the moment of inertia of a square in Roark, we find that

$$I = \frac{bh^3}{12}$$

where b is the width and h is the height.
Our I = .0833 in^4. Remember, I is area (in^2) times distance (inches) squared, or in^4.

The formula for stress in a beam that is being bent is

$$S = \frac{Mc}{I} \quad .$$

This formula is called the "Flexure Formula" and it is as basic to structural analysis as E = IR is to electrical analysis. It is not new. It was originally (almost) derived by Galileo about 1630, and corrected into its present form by Navier in 1820. M is the moment applied to the beam at the point at which we are calculating the stress, and c is the distance of the farthest fiber from the neutral axis.

Plugging in the numbers, S = 500# x 10" x .5"/.0833 in^4, or S =30,000 psi. That's significant. We need to use one of the stronger aluminum alloys.

But let's go back for a minute. Where is that stress occurring? If the beam were to break, where would it break? We all know just by looking that the beam will break right at the wall, but what does the formula say? $S = Mc/I$, and M is P times the distance from the point at which P is applied. If we look at the section right where P is applied, then the distance is zero and the moment is zero. The farther we get from where P is applied, the longer the distance gets, and the higher M gets. So the farthest we can get from P is at the wall, and that is where the highest stress is, so our intuition is correct. It will break at the wall.

As before, we need the end of the beam to remain rigid under the load, so we need to calculate the deflection. The formula for the deflection of a cantilevered beam from Roark is

$$d = \frac{PL^3}{3EI} \; .$$

Let's look at that formula for a minute. P is linear, so if our 500# load went to 1000#, the deflection would double. E is the property of the material, and I is the property of the cross-sectional area, so they don't change. But L is cubed, so if L went from 10" to 20", the deflection would increase by a factor of 8! This is a big deal.

Plugging in,

$$d \ = \frac{PL^3}{3EI} \text{ , or } d \ = \frac{500x10^3}{3x10x10^6x0.0833} \quad \text{or} \quad d = 0.200".$$

This is 400 times as much as when we pulled on the beam. This is typical. **A beam will generally be two to three <u>orders of magnitude</u> stiffer under an axial load than under a bending load**.

OTHER APPLICATIONS OF BEAMS

Of course, there are many other ways of using beams other than simply sticking one end into the ground and pulling on the other end. As a matter of fact, most beams are used in the mode of being laid horizontally, with the ends supported and the span of the beam carrying a load. For example, the floor joists of your house, where the beams span from wall to wall, carrying the weight of the floor and its occupants along its length. A beam used this way is termed "simply supported" For example, we have the beam in Figure 3, which is supported on each end and carries a load "P" in the middle.

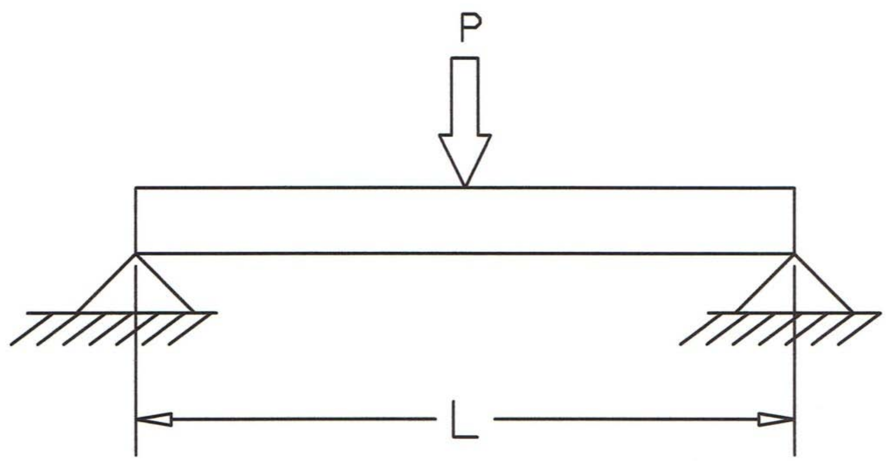

FIGURE 3

The formula for the deflection of this beam in the middle, from Roark, is:

$$d = \frac{PL^3}{48EI}\ .$$

Note that it looks just like the formula for the cantilevered beam, except that the constant in the denominator has changed from 3 to 48. Which tells us instantly that for the same beam and load, the deflection is only 1/16th as much. All the other relationships that we learned from the cantilevered beam still hold. If you double the load, you double the deflection. If you double the length, the deflection increases eight times.

Now look at Figure 4, a simply supported beam with a load uniformly distributed across it (Think floor joist).

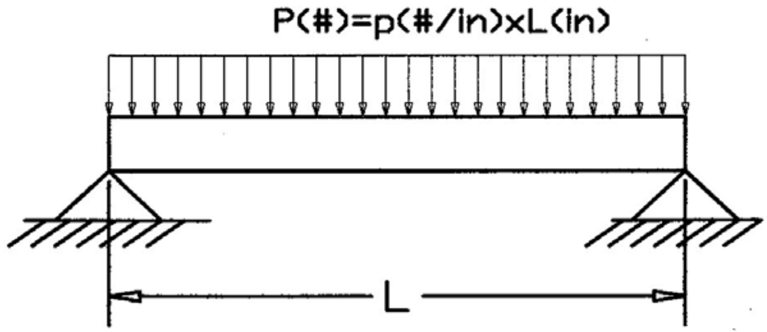

FIGURE 4

From Roark, we have:

$$d = \frac{5PL^3}{384EI}.$$

All the same properties, different constant. My edition of Roark contains 38 variations of beam configuration and load case. Look it up.

A brief digression here: Some of you are thinking "Oh good! Now I can design my vacation cottage." In Engineering, there is science and there is law. Buildings are designed to lie within the constraints of a law known as a "Building Code". The Building Code decrees that the floor will be designed to deflect less than a certain amount under the loading condition decreed by the Code. It then will refer you to a "Span Book" to determine what the cross section of the beam shall be, made out of what species of wood, to span the distance called for in the plan.

The previous two chapters, "Stretching of Beams" and "Lateral Deflection of Beams" were put together to illustrate one fundamental point: Lateral deflection is huge compared to axial deflection. Where this shows up is in the detailed design of structures. Trusses (see a later chapter) are made up of members which all come together at a point. If the attaching bolt is at that point, there will be no bending, and hence no lateral deflection in those truss members. But "members" are real pieces of structure. Suppose you want your structure to be comprised of many members made from 4" pipe. You put one pipe such that the centerline goes directly to the attaching bolt, but then when you try to put the second pipe to the same bolt, the pipes interfere. Once you figure that one out, how do you get to that bolt which is buried under all that pipe? The way many people solve these problems is with offsets. For example, bring the pipes as close together as you can, weld them to a flat flange, and put a bolt through the flange on either side of the pipe to hold the structure down.

It is very easy to look at small offsets and think "It's only a couple of inches. That won't matter." But if the parameters of this structure are anything like the beams in the previous chapters, one inch of distance out along that bending flange adds as much bending to the structure as 400 inches more length of primary members. Minimizing offsets to increase stiffness is important.

THE SIGNIFICANCE OF SHAPE

Earlier, we found that the bending stress in our beam was 30,000 psi, which is just about at the limit for the most common aluminum alloy, 6061-T6. Pretty scary. If someone doesn't hang that 500 pounds on the end of that beam very carefully, it may break off. So what can we do to get some safety factor? Obviously, we could use a bigger beam, but the Boss doesn't want to have to buy more aluminum, so we are stuck with only one square inch of cross-sectional area. But earlier we were talking about how the stresses are largest in the upper and lower fibers of the beam, and how the fibers in the center have no stress at all. So if some of the fibers aren't carrying any load, what are they there for? Good question.

Remember that the discussion was leading to the concept of Moment of Inertia, and that by the formula $S = Mc/I$, if you increase the moment of inertia, then you decrease the stress. So let's take our one square inch of area and divide it into two 3/8th square inch pieces and one 1/4 square inch piece. Let's then assemble them as shown in Figure 5 and calculate the moment of inertia of the new section.

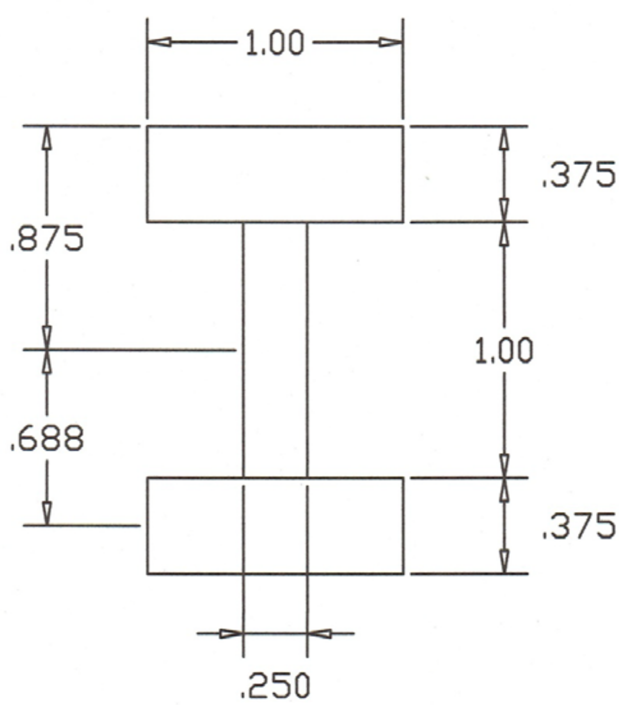

FIGURE 5

Remember that moment of inertia is defined as the sum of all of the little elements of area times the squares of their distances from the neutral axis. A tip: Some things are important and some things are unimportant. Life's like that. In this case, big pieces of area far from the neutral axis are important. Little pieces of area near the neutral axis are unimportant.

Just allowing for the two big pieces, I = .375in^2 x (.688in)^2 x 2 (both pieces) = .355in^4. That's already over four times larger than the moment of inertia of the square section. The formula for the little vertical piece (from Roark) is

$$I = \frac{bh^3}{12}$$

Plugging in, I = .25 x 1^3/12 = .021 in^4. The total is .376 in^4, or four and a half times our original moment of inertia. In truth, the real moment of inertia is just a bit larger, because we ignored the moments of inertia of the top and bottom pieces about their own neutral axes, but so what? If we use a moment of inertia a bit less than actual, then we will calculate a stress a bit larger than actual. So the difference between our calculated stress and the actual stress is a bit of safety factor. In general, the moments of inertia of most cross sections are available in tables and are accurate. Using simplified methods like the one above will get you comfortably into the ballpark.

Remembering that c, the distance from the neutral axis to the outer fiber, just got larger, from 1/2 inch to 7/8 inch, we re-calculate the stress caused by the 500 pound load:

$S = Mc/I$, or S = 500 lb x 10 in x .875 in / .376 in^4 = 11,636 psi,

which is much better than 30,000 psi, and the Boss doesn't have to buy any more aluminum. If you look at Figure 3, you can see that we have turned the square bar into an I-beam. Not surprisingly, the I-beam is the most efficient (most strength for the least weight) shape for resisting bending loads <u>in one plane</u>. Many people think that the tube is the most efficient section, but that is only for when the loads come from all (or any) directions.

Checking the new beam for deflection, we have

$$d = \frac{PL^3}{3EI} \rightarrow d = 500\# \times 10^3 / 3 \times 10^7 \times .376, \text{ or } .044 \text{ inch} \quad .$$

This is 4.51 times less than before, which figures, because the new I is 4.51 times larger than before.

TORSION

Now let's see what happens to our 1 inch square bar when we put a twist on it (Figure 6).

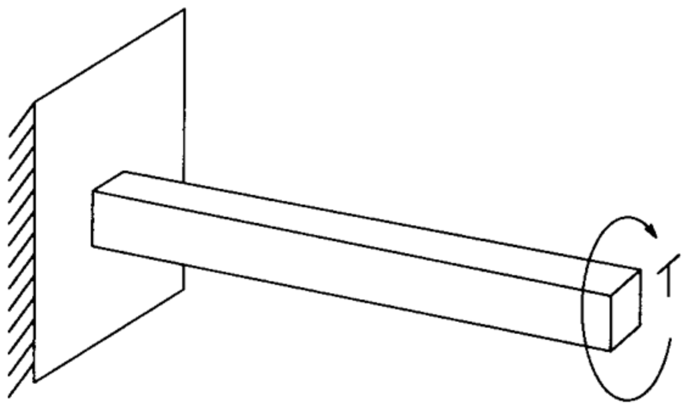

FIGURE 6

We apply a torque T to the free end of the bar of 1000 inch-pounds. You might be saying "Wait a minute! If we are going to twist it, shouldn't it be a round bar?" While the most common shape for most shafts under torsion, like axles, is round, any shape can be twisted, but with varying results.

When we load a bar in torsion, we cause a shearing stress in the material. The molecules making up the bar are trying to slide across themselves, rather than being pulled apart from one another as in tensile stress. Shearing stress, like tensile stress, is a property of both the alloy of the material and its processing. Like tensile stress you have to look it up, generally in the same table as you find tensile stress. The formula for shearing stress in a bar loaded in torsion is much like the Flexure Formula:

$Ss = Tz/K$ (Compare with $S = Mc/I$).

This time, Ss equals Shearing Stress, T equals Torque, z equals the distance of the farthest fiber from the centroid of the section, and K is a function of shape like I, but harder to define. For circular (or tubular) sections, K is just 2 x I, but for most other sections, the formulas are given in Roark and must be looked up. For a round shaft, $Ss = Tr/J$, which is simple and easy to calculate. (J = K = 2 I) But for our square shaft,

K = .1406 x 1 in^4 = .1406, and Ss = 1000 in-lbs x .50 in x √2/ .1406, or 5746 psi.

Since the yield shear stress for 6061-T6 aluminum is 20,000 psi (from Roark) we are safe, but remember that we were applying a load of 1000 inch-pounds to our bar, and many of us are used to thinking in terms of foot-pounds, as in the torque of an engine. Since all of the other quantities in the formula are in inches (or inches^4) then the torque has to be expressed in inch-pounds also.

If we need to know how much our bar twists under the applied load, the formula is:

$$\theta = \frac{TL}{KG} \ .$$

Compare this to the formula for simple stretch,

$$d = \frac{PL}{AE}$$

θ is the angle of twist <u>in radians</u>. T, L, and K are the same as before, but G is the Shear Modulus of the material, which you look up in Roark. (In general, G is about one-third E). Note that like the formula for axial stretching, the L is linear. There is no L^3, so in general, things are pretty stiff in torsion.

For our beam, $\theta = TL/KG \ \rightarrow \ \theta$ = 1000 in-lbs x 10 in / .1406 x 3.75 x 10^6 = .0019 radian or .11 degrees. Again, that's pretty stiff.

The thing that is important when designing to resist torsion is the K factor. That is very much like the Moment of Inertia that we learned was so important in controlling deflection in bending. For example, take a round shaft. It has a certain K. Like I, it is the fibers near the outside that have the most effect, so you can bore out the center of the round shaft and still have most of the original stiffness left in the remaining tube.

Fat sections, like the circle and the square have a lot of their material a long way from the center of the section, so have high values of K and are stiff. Thin sections, like a piece of sheet metal, have little torsional stiffness. Interestingly, the shape that a piece of sheet metal is bent into has little effect on its stiffness. Therefore, if you have a piece of sheet metal bent into a hat-section, or some other section that is not listed in the tables, calculate the unbent length, and use the formula for a rectangle. The error will be small.

BUT, if the ends of the bent-up section are joined together, we have a closed section, which is very stiff. Conversely, if we take the nice, stiff tube that we made earlier by boring the center out of a circular shaft and slit it lengthwise as if we were going to open it out flat, it would have about the same stiffness as a flat sheet of the same thickness and length. This would reduce the torsional stiffness by orders of magnitude.

As an experiment, take a shoebox without a lid. Twist it, and it will twist readily. Then put the lid on it and hold the lid snugly against the box. Try to twist it, and you will find that it has become very stiff. You have made it into a closed section. For another experiment: roll up a sheet of paper into a couple of layers. Twist it in your hands, and it will twist readily. Note that the exposed edge of the paper is slipping edgewise on the roll beneath it. Then tape that edge down to the roll. Note that the stiffness has increased dramatically.

The message here is that for a light, torsionally stiff section, choose a fat, closed section like a tube, with a thin wall.

BUCKLING

Buckling. The word conjures up images of crumpled automobiles and twisted bridges lying in rivers. While the metal in question has certainly buckled, in reality, buckling is a more subtle and dangerous phenomenon.

The study of buckling and the ways of calculating it literally fill volumes, and I will only touch on it lightly here. Buckling happens when a long, slender beam is loaded in compression. Theoretically, the beam is perfectly straight and the loads are applied in the exact centers of the ends, and the loads are exactly in line with one another. But all those "perfectly"s and "exactly"s aren't real life. In real life, the fact that the loads are not exactly aligned with the centerline of the beam means that there is a small moment due to the resistance of the beam being on one line and the forces being on another, and there being a small distance between the two lines. Forces in opposite directions separated by a distance form a couple which causes the beam to start to bend. As the beam bends, the distance between the load forces and the beam resisting forces increases, so the moment increases and the beam bends more. Very soon, the beam is no longer strong enough to support the moment, and it collapses. This can happen at a simple compression stress (P/A) much lower than the ultimate compressive stress number found in the tables.

Think of a tall flag pole next to a building. If you pile enough lead on top of the flag pole, it will suddenly start to lean over. If you are lucky, it will lean toward the building, and the lead will rest against the building. The flag pole has buckled. If you remove the lead, the flag pole will stand back up, none the worse for wear. If you are not lucky, the flag pole will have leaned away from the building, leaning farther and farther, until the metal of the pole fails (in a phenomenon called "crippling") and the lead lies on the lawn and the pole must be replaced.

For a simple experiment, take a wooden yardstick and set one end on the floor. With the yardstick vertical, push down on the upper end with your hand. At a load of a very few pounds, the yardstick will suddenly bow sideways, relieving the load in your hand. If you then relax, the yardstick will straighten itself up and be no worse for wear. It has buckled.

Suppose you have been asked to design columns to support a mezzanine in a new shopping center, and for aesthetic reasons, the columns must be as slender as possible. You divide the load on the column by the area of the column (remember $S = \frac{P}{A}$) and the answer comes out nice and safe. When the column buckles and the mezzanine comes crashing down, you are astounded. What can have happened?

To continue on toward a mathematical solution to what load it takes to buckle a beam, and what a beam that might buckle looks like, we have to introduce a new term, "Radius of Gyration". Remember that the moment of inertia was defined as the sum of all of the little pieces of area times the squares of their distances from the neutral axis. If you divide the Moment of Inertia by the Area of the section, you will have the sum of all of the squares of the distances, or r^2. The square root of this is r, the Radius of Gyration. You might think of it as the average of the squares of all of the distances from the neutral axis, or a "characteristic distance". For any section of any area A, the larger the r, the larger the Moment of Inertia, and the stiffer the section.

As a rule of thumb, if the length of the beam, divided by its Radius of Gyration (L/r) is greater than 20, the beam will probably buckle before its compressive stress reaches the Yield Compressive Stress of the material.

The famous equation that defines what the maximum load is that a beam can sustain without buckling is known as "Euler's Formula":

$$Pcr = \frac{C\pi^2 E}{\left(\frac{L}{r}\right)^2} .$$

Pcr is the critical load at which the beam begins to buckle. C is the coefficient of constraint, which takes into account whether the ends of the beam are pinned (C = 1) or fixed (C = 4), etc. The other factors are as before.

So, what if L/r is less than 20? What if one end of the beam is fixed and one end is pinned? That's why there are whole books written on the subject, but in general, it is well-treated in Roark.

ACCURACY

Before we tackle more complex structures, we need to understand the concept of vectors, which are a graphical way of representing quantities that have both a magnitude and a direction. It is frequently convenient to work with quantities such as "60 MPH northeast" or "500 pounds along the beam" by representing them as arrows with lengths proportional to their magnitude, and directions as described by the quantity, such as "northeast" or "along the beam". But you might say "Graphics. Isn't that a little sloppy? Isn't a computer more accurate?"

True. Graphical methods are accurate to around one percent, while computers are accurate to however many decimal places you want. But how much accuracy do you need? Suppose you were getting low on gas and stopped and asked a surveyor "Where is the nearest gas station?" He might answer "7922 feet, 7 inches north-northwest on US 1". That might be a very accurate answer, but wouldn't "One and a half miles up US 1" be more useful? Your speedometer is calibrated in miles. It can't handle feet and inches.

It is the same with vectors. The quantities that we work with in real life are seldom known to an accuracy of one percent. For example, is that 60 MPH really accurate to 0.6 MPH? Is that 500 pound load really within 5 pounds? Was it applied in such a way that it didn't jar the structure at all? When we solve for the stress in the beam, we then compare it to the yield strength in the tables for the material used. The number in the table is the minimum stress that the material can have and still meet its specification. It can easily be ten percent stronger than that. While the modulus in the table is pretty accurate (to within about one percent) the moment of inertia of the actual section can be significantly different from the theoretical dimensions shown in the handbooks. So a solution that is accurate to one percent is generally more accurate than the data that you are working with, and is entirely acceptable.

Finally, the real benefit of a graphical solution is that it gives you an immediate feel for the magnitudes of what are involved, so you can see what is important and what is not. Also, if your finger slips on the computer, and you enter a speed for your automobile of 600 MPH instead of 60 MPH, the computer doesn't care, and the answer will be nonsense. But if you are representing speed by a scale of 10 MPH per inch, then an arrow 60 inches long would immediately show as ridiculous.

VECTORS

A vector is a graphical way of representing quantities that have both a magnitude and a direction. Vectors are useful if you either want to add two or more different quantities together, or of you want to see what the effect of a single quantity is when broken down into equivalent components in different directions. For example, if you have driven 5 miles east, and then 2 miles northeast, how far are you from your starting point, and in what direction? Graphically, the answer is quick and obvious. With trigonometry, the answer may be right (and very accurate) or you might have made a sign error. For another example, you have propped a door closed with a rake stood at a 30-degree angle to the door, and there is a 100-pound wind load on the door. What is the load in the rake?

Let's look at two vectors, **A** & **B**. (Figure 7A).

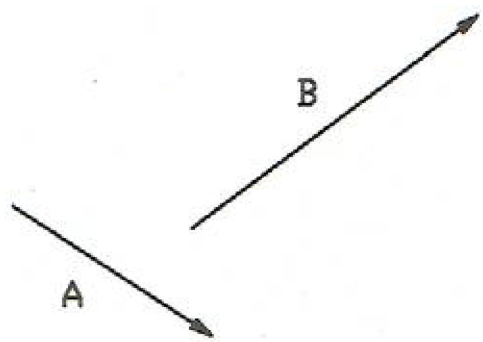

FIGURE 7A

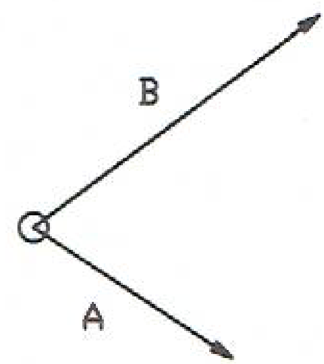

FIGURE 7B

A represents some quantity with a magnitude that is represented by the length of the arrow. For example, the arrow is 2 inches long, representing a force of 20 pounds. It is pointed at some angle down and to the right, because that is part of the description of the quantity. **B** represents another quantity. It is 3 inches long, representing 30 pounds, and acts up and to the right. Suppose both forces are pulling on a stake in the ground (Figure 5B). What single force would pull on the stake with the same force as the two original forces, and in what direction would it act?

There are two ways to solve this problem, and the solution will tell us a lot about using vectors. (Figure 8).

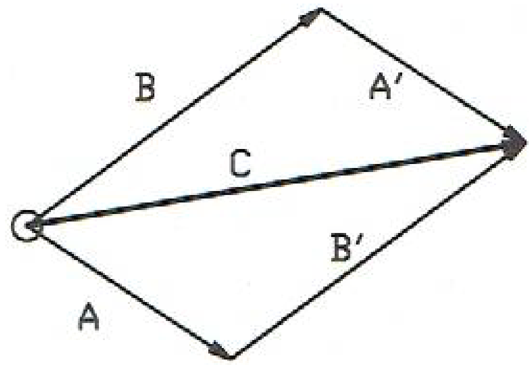

FIGURE 8

Vectors are added by attaching the tail of one vector to the head of another. So, for this problem, we create a duplicate to vector **B** (call it **B'**) and attach it to the head of vector **A**. The sum of the two is the vector from the stake to the head of **B'**, which we will call **C**. Alternately, we could create a duplicate of vector **A** (call it **A'**) and attach it to the head of vector **B**. Note that it comes exactly to the head of vector **B'**, so the sum of **A'** and **B** (**C**) is the same as the sum of **A** and **B**'. This is the same as for simple addition: 1 + 2 = 2 + 1. It is also the same for many vectors: A + B + C + D = C + A + B + D. (Or any other order). The parallelogram **A B'A'B** is called the "parallelogram of forces" and is the standard solution for many forces acting on the same point. Of course, you don't have to draw **B'** exactly the same length as **B**. A simple line parallel to **B**, intersecting a simple line parallel to **A** gives the same parallelogram.

Vectors don't have to act on the same point. In the example at the beginning of this section, you can find out how far you are from your starting point by drawing a vector 5-inches long eastward, representing the 5 miles that you drove east, and add to that arrow another that is 2-inches long pointing northeast. The distance from the tail of the first arrow to the head of the second arrow is how far you are from the start, and the direction can be measured from the horizontal (first arrow) with a protractor.

A frequent use of the parallelogram of forces is to take a resultant force such as vector **C**, and break it down into two component forces that lie in directions that are useful to us. Frequently, those directions will be horizontal and vertical (X & Y), but as we have seen, they do not have to be at right angles to one another.

COMPLEX STRUCTURES

So far, we have only talked about individual beams. Now, we need to assemble these beams into structures. Let's take our original beam, the one-inch square beam that was 10 inches long, and bolt it diagonally to two other beams forming a 3-4-5 triangle (Figure 9).

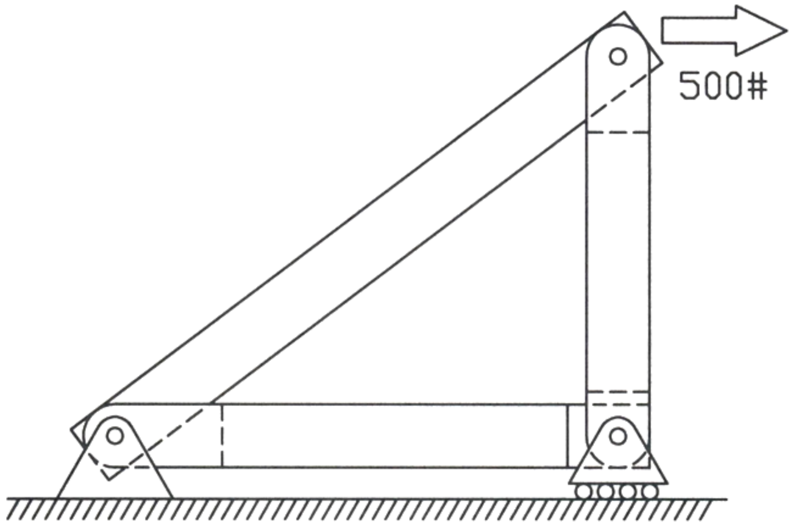

FIGURE 9

We still have the 500 pound force pulling on the end of the beam. So, to figure out what is going on in this structure, we first need to know what forces are in each member. For our purposes, we assume that the bolts have been assembled loosely, so that they act as pins at each joint. Let's look at this structure for a minute. It is naturally rigid. No matter how we push or pull at any corner of the triangle, it will only deflect the tiny amounts similar to what we calculated for the simple cantilevered beam that we first loaded axially. Partly, this is because there is no way to apply a moment to any of the members of the triangle, because we can only apply loads at the corners of the triangle. (Certainly, we can add a feature to the triangle to allow moments to be applied to it, as by welding a bracket to one of the members, but then it is no longer a simple triangle, and the stiffness will be impacted.) Because of its inherent stiffness, even when the joints are only pinned together, the triangle is the basic element of all structural design. **The bolts that join a triangle of structural members can be either tight or**

loose to make the structure rigid. Note that all trusses are composed of arrays of triangles.

Let's look at how we have attached our frame to the world. The left corner is pinned to a rigid surface and the right lower corner is free to slide or roll on a rigid surface. When we pull on the end of the diagonal member, that pull is resisted by the pin which is anchored to the rigid surface.

Before we start with a bunch of calculations to find out how much force is in each member of the structure, let's look the structure over and see what we can find out just by looking. First, if the vertical member wasn't there, that 500 pound load would immediately pull the diagonal member into a horizontal position. So it is pretty obvious that the vertical member is pushing up pretty hard to keep the diagonal member in position. What about the horizontal member? One end is pinned to the same bracket that attaches the diagonal member to the rigid surface, and the other end is pinned to the same "roller skate" that supports the vertical member. So if there was any load in the horizontal member, the "roller skate" would just roll sideways a bit and relieve the load. Therefore, the load in the horizontal member is zero, without having to do any calculations at all. The horizontal member is just there to keep the "roller skate" where it belongs, under the vertical member. In a perfect world, we wouldn't even need the horizontal member, but in the real world, it needs to be there just to be sure that someone doesn't kick the "roller skate" or something. But still in the real world, the horizontal member can be made very small and light (and cheap).

But how much load will be induced into the diagonal member by the 500 pound pull? Is it just 500 pounds? This is where vectors are useful. Referring to Figure 10, we have some unknown force running along the centerline of the diagonal member. We can resolve that force into two forces, choosing one to be horizontal to resist our applied force, and one to be vertical to find the load in the vertical member.

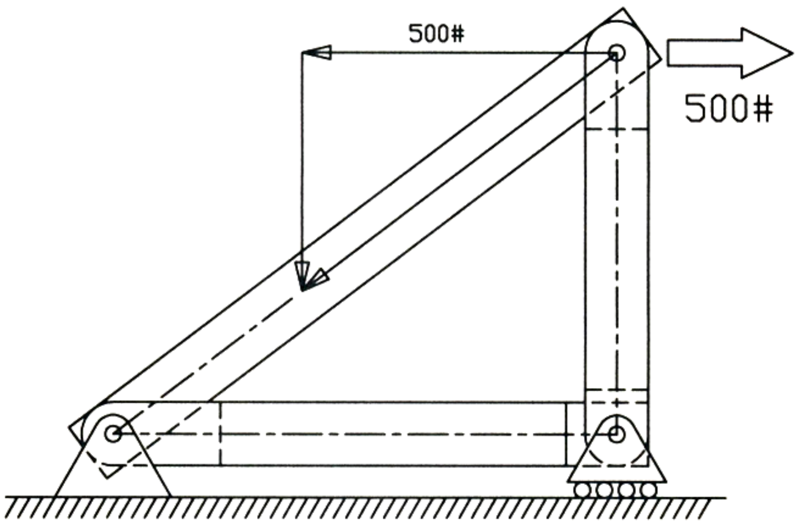

FIGURE 10

In our case, we are not as interested in the force applied as we are in the forces that are generated in our structure to resist that load. Let's decide to represent the force resisting the applied load with an arrow 5 inches long (a scale of 100 pounds per inch) pointed horizontally to the left.

We have a force of unknown magnitude acting along the axis of the diagonal beam, and another force of unknown magnitude acting downward along the axis of the vertical beam. From the arrowhead of the horizontal 5-inch long force, we draw a line down vertically until it intersects the axis of the diagonal beam. Then we draw an arrowhead pointing down on the vertical line where it intersects the axis line, and draw an arrowhead pointing down and left on the diagonal beam axis where it intersects the vertical line. (Figure 11).

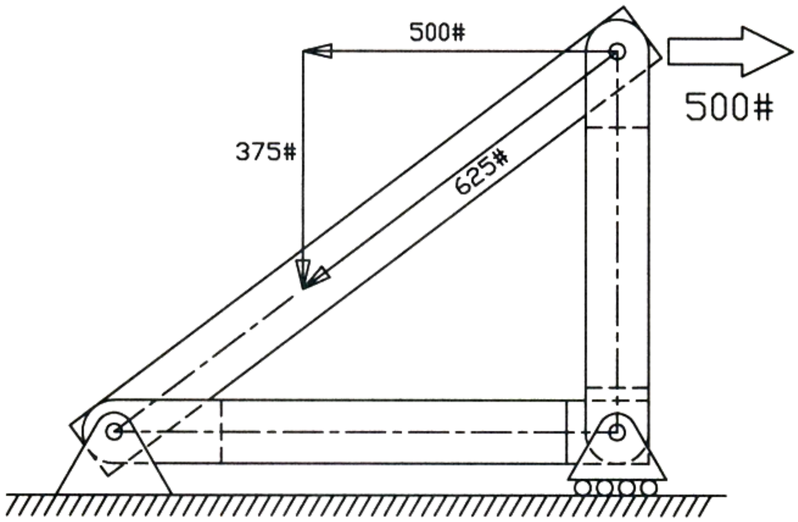

FIGURE 11

We now have three vectors: One horizontal representing the force resisting the applied load, one down representing the load in the vertical beam, and one diagonal, representing the load in the diagonal beam. Measuring the lengths of the vectors and applying our scale factor, we find that the vertically down load is 375# and the load in the diagonal beam is 625#. If we wanted to, we could calculate the deflections that these loads cause in the individual beams, represent them as vectors, and add them. But there is one potentially confusing thing about the graphical calculation that we just did and diagrammed in Figure 11. We ended up with a 625-pound force acting downward and to the left on the pin, and a 500-pound force acting horizontally leftward on the pin, but what is this 375-pound force that seems to act in the middle of the beam? It is not really there. It is just shown that way to make the graphical calculation easier. What we really had is a force that we now know is 625 pounds that was caused by the 500-pound load applied to the structure. That 625-pound force can be replaced by the sum of a 500-pound force acting to the left on the pin, and a 375-pound force acting down on the pin. We have created a "parallelogram of forces" as we saw earlier. Compare Figure 12A and Figure 12B. They are exactly equivalent. Chose the one that makes your calculation easier.

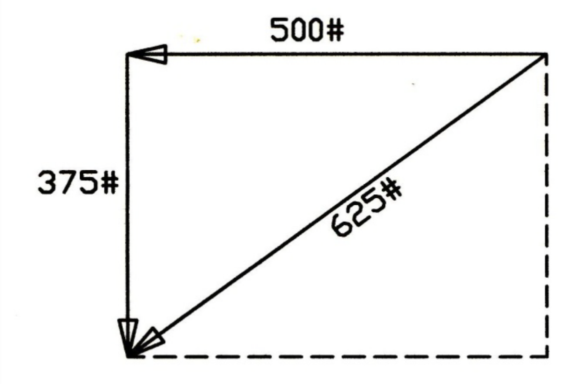

FIGURE 12A

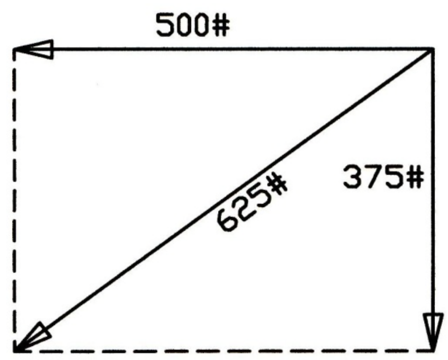

FIGURE 12B

THE FUTURE

Two things will profoundly affect the future of structural design: Finite Element Modeling (FEM) and carbon fiber. Yes, I know that both of them are now in existence, and are generally thought of as being "High Tech". Both of them will, I believe, trickle down from the aerospace industry to the local shops.

FEM is classic software. Over the years, it will become more powerful, cheaper, and easier to use. It will soon be possible for a person in an average weld shop to draw a simple model of a proposed structure on a laptop, apply a load to it, and have the model show what the stresses and deflections are. You can then optimize your choices of shapes and materials to get the most cost-effective structure possible. Remember that the basic reason for making first, a drawing, and second, a model, is so that you can change it until it is what you really want.

We all know that satellites and race cars are currently being made of carbon fiber. In industry, we have a joke that you can solve any structural problem with carbon fiber, except cost. If your structure is too weak, add carbon fiber. If your structure is not stiff enough, add carbon fiber. If your structure is too heavy, replace some of the steel with carbon fiber. In a properly laminated form, carbon fiber is as strong as high-tensile steel, as stiff as steel, and only about 1/4 the weight of steel. However, carbon fiber does have two problems: 1) As a laminate, it generally can't stand temperatures much over 150F although there are special resins and processing that can raise that temperature significantly (at a lot of extra cost), and 2) carbon fiber is brittle. Unlike most metals, it will not bend or dent. It will generally fracture.

The only problem with carbon fiber currently is cost. There are two factors to that cost: the cost of the raw material and the cost of laminating it. Cost is currently coming down rapidly. Once, good quality carbon fiber cost upwards of $100 per pound. It is currently at $15 per pound, and headed for $5 per pound, which is not far from the cost of steel. Also, remember that you get four times as much carbon fiber for your pound as you do steel. Finally, carbon fiber is made from, as you might suspect, carbon. Carbon will never become scarce and costly. Carbon fiber is laminated in much the same way as fiberglass. To get the best properties, extra care must be taken, but you can still get pretty good properties from the fiber by using standard fiberglass laminating techniques, with the exception that the resin used must be epoxy instead of polyester. Thousands and thousands of small boats are made very cheaply by mostly unskilled labor. Now that the cost of carbon fiber is getting cost-competitive, we will likely see a lot more of it.

