A SunCam online continuing education course

What Every Engineer Should Know About Structures

Part A – Statics Fundamentals

by

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This is the first course of a series in the area of study of engineering mechanics called Statics. The series will provide the tools for solving most of the common statics problems.

What Every Engineer Should Know About Structures Part A – Statics Fundamentals is an applied statics course focusing on presenting simplified methods of solving statics problems. The emphasis is on an intuitive rather than theoretical approach. The methods presented are simple solutions to what can sometimes appear to be complicated problems. Anyone with a working knowledge of high school trig and algebra can complete this course.

Part One – WHAT IS STATICS is an introduction to the field of mechanics called statics.

Part Two – LOADS and FORCES introduces the concept of loads and forces on members.

Part Three – RESULTANTS and COMPONENTS introduces methods to resolve a single force into components, or components into a single force.

Part Four - MOMENTS and RESULTANTS introduces moments and applies the concept to determining resultants.

Part Five – EQUILIBRIUM develops the methods for applying Sir Isaac Newton's laws of motion to the solution of statics problems.

The second course in the series, **What Every Engineer should Know About Structures Part B – Statics Applications**, focuses on applying the basic principles presented in this first course to real-life statics problems.

Basic Trigonometry, Significant Figures, and Rounding – A Quick Review is a zero credit course intended for those who might find themselves a bit rusty and would like a quick refresher. The information in the course is useful for application to the solution of structural problems especially in the fields of statics and strength of materials.

This course is free and can be downloaded at:

http://www.suncam.com/authors/123Glon/TRIG.pdf

The trigonometry review includes demonstrating - through the use of several example problems – the use of the basic trigonometric functions including: the sine, cosine and tangent and their inverse; the Pythagorean Theorem; the Sum of the Angles; the Law of Sines; and the Law of Cosines. The significant figures and rounding review includes a discussion of the precision and validity of an answer, along with rules and guidelines for using the appropriate number of significant figures, and for rounding answers appropriately.

Part One – WHAT IS STATICS

Statics is part of the division of physics known as mechanics. The word "static" means stationary. Statics is the study of bodies that are stationary; at rest; not moving. The study of bodies that are in motion – or moving – is known as dynamics. We won't be considering anything that is in motion.

When beginning the study of mechanics, the first area of study is typically called Statics and includes the forces acting **on** a structural member – that structural member could be a bench used by football players on the sideline, a column in a high-rise building, a retaining wall in a nicely landscaped lawn, or a rack of storage shelves used in your garage or in a warehouse. This is the first course of a series in the area of study of engineering mechanics called Statics.

The second area of study, after Statics, is typically called **Strength of Materials** and includes the calculation of, and the distribution of the forces acting **within** a structural member – i.e., the stresses in a member. The principles learned in Statics are necessary to learn the principles in Strength of Materials.

- **Statics** is concerned with determining the forces acting **on a body**.
- **Strength of Materials** is concerned with determining the effect of those forces on the internal structure of the body – i.e., **within a body.**

These two areas of study, Statics and Strength of Materials, form the basis of the field of structural engineering – which includes structural analysis and structural design.

Statics was understood in a basic form way back in 200 B.C. Archimedes (287 – 212 B. C.) figured out levers and the mechanical advantage of using a long lever to "pry" on a short lever. This concept was used to move big stones for pyramids and things like that – i.e., to lift real heavy loads. Today we can thank Archimedes for the claw hammer used to pull nails, and for the piece of playground equipment called the see-saw, or the teeter-totter, which can be adjusted to allow a heavy person on one end to enjoy the ride with a very light person on the other end. We can also thank him for the ability to pop the top off of our favorite bottled beverage with a bottle opener – or, as it was called when I was a young lad, a "church key".

A long time later, Sir Isaac Newton (1642 – 1727) thought about the law of gravity and some other stuff and proposed his three Laws of Motion. That's when the field of mechanics really took off. Sir Isaac Newton is considered the founder of the field of mechanics.

The underlying principles of Statics are contained in his Three Laws of Motion. Newton's Three Law's of Motion are stated here:

- **First Law -** If the forces acting on a body are in equilibrium, a body at rest will stay at rest and a body in motion will stay in motion.
- **Second Law -** If the forces acting on a body are not in equilibrium, the body will have an acceleration proportional to and in the direction of the unbalanced force.
- **Third Law -** To every action there is an equal and opposite reaction.

In statics, we are **not** concerned with structures and elements that are in motion. Therefore, Newton's First Law can be restated as: **A body at rest will stay at rest if the forces acting on it are in equilibrium.**

His Second Law of Motion points out that if the forces on an object are not balanced - not in equilibrium - the object will accelerate. Since we are not dealing with motion when we study Statics, the **Second Law does not apply**.

Therefore, we are only interested in a portion of Newton's First Law and his Third Law of motion. These two laws as stated here are the fundamental basis for the study of Statics. You should commit these two laws to memory.

- His **First Law** states that a body at rest will stay at rest if the forces acting on it are in equilibrium.
- His **Third Law** states that for every action there is an equal and opposite reaction.

And here we are today. In this course - **What Every Engineer Should Know About Structures Part A – Statics Fundamentals**. We will now learn how to apply Newton's first and third laws of motion to bodies at rest. Wheee....

Part Two – LOADS AND FORCES

LOADS

Definition A load can be defined as a weight that is imposed on a structural member. The weight of the member itself is also a load. A load is a scalar quantity which means that it is completely specified by a number on an appropriate scale – such as pound, Newton, etc.

Units The units for load in the American system are pounds or kip. Kip stands for kilopound which is 1000 pounds. A ton, which is two kips, is also a unit for load; but it is seldom used in mechanics.

In the SI system, the units for loads are Newtons or kilonewtons. One Newton is equal to approximately 0.225 pounds. A kilogram is converted to a Newton by multiplying by the gravitational constant of 9.81 (1 kg = 9.81 N). An easy way to remember the magnitude of a Newton is to first recognize that an apple weighs approximately one Newton, then think of an apple falling on Sir Isaac Newton's head. A kilonewton is 1000 Newtons (225 pounds) – about the weight of an average size football player.

Types of Loads A loads is classified according to the area over which it is applied. A **concentrated load** is applied at a point, and a **distributed load** is applied along a length or over an area.

Loads are also classified according to the duration of the load and the nature of the load. A permanent load is called a **dead load**. A dead load is always present – it never leaves. The weight of a structure is a dead load. A temporary load is called a **live load**. Live loads come and go. Live loads are also identified by more descriptive terms such as wind load, snow load, floor load etc.

Concentrated loads are not actually applied at a point, but rather are applied over a small area. For example, a 12" x 12" column setting on a 6' x 6' square footing is considered to be a concentrated load even though a 12" x 12" area is far from a point.

Loads that are distributed along a length are very common. The weight of a steel beam is an obvious example – its own weight is a dead load – it never changes. The magnitude of the distributed load can be determined from the dimensions of the section and the density of the material. The usual units for this type of load are:

American Units	SI Units
pounds / foot (#/ft)	Newtons / meter (N/m)
kips / foot (k/ft)	kilonewtons / meter (kN/m)

A load that is distributed over an area is also called pressure. The weight of the structure itself is a dead load. Some common examples of live loads are the weight of snow on a roof; the weight of people on the floor of a building; the pressure of the wind on the side of building; and the lateral pressure of wet concrete on wall forms. The usual units for this type of load are:

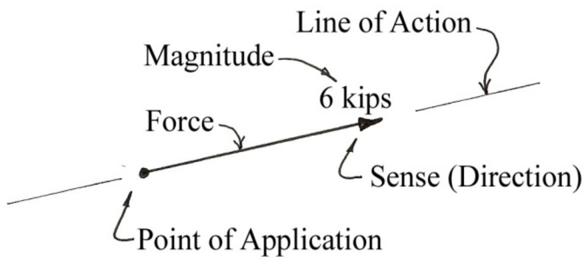
American Units	SI Units
pounds / foot ^{2} (#/sf or psf)	kilopascals (kPa)
	(1 kPa = 1 kN/m^2)

The solution to statics problems – procedure, format, assumptions, etc. – are the same whether the units are SI or American. In this course, we will only work in American Units.

FORCES

Characteristics of a Force A force is defined as an action that tends to change the state of the body on which it acts. A force is a vector quantity which means that it has three characteristics: (1) magnitude, (2) a line of action, and (3) a sense of direction. A fourth characteristic, the point of application can also be indicated – although, in statics, the point of application is often obvious and therefore, not shown.

Forces are shown on a diagram by an arrow with a magnitude and a direction, as shown below. Arrows that appear to be horizontal or vertical are assumed to be horizontal or vertical without specifying an angle. It is frequently desirable to draw the length of the arrow proportional to force. This can assist in visualizing the magnitude of the answer. However, this is not appropriate for all types of problems.



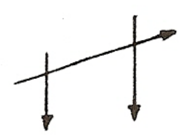
Graphic Representation of a Force

Coplanar and Noncoplanar Forces If all of the forces in a problem are in the same plane, they are said to be **coplanar**. If not, they are said to be **noncoplanar**. Stated another way, coplanar problems are 2-dimensional and noncoplanar problems are 3-dimensional. The majority of real-life statics problems are 2-dimensional or can easily be represented as 2-dimensional.

Concurrent, Parallel, and Nonconcurrent Forces If all of the lines of action of the forces in a problem pass through a single point, the forces are said to be **concurrent**. If all of the lines of action of the forces in a problem are parallel, the forces are said to be **parallel**. If the lines of actions of the forces in a problem include both parallel and non-parallel lines, the forces are said to be **nonconcurrent**. The force systems are illustrated below – all 2-dimensional, all coplanar.

Concurrent Forces

Parallel Forces



Nonconcurrent Forces

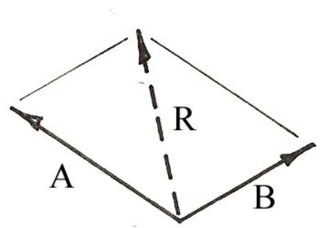
Coplanar Forces

External and Internal Forces Forces that act on a structure are **External forces**. The weight of a vehicle on a bridge is an example of an external force. The weight of the bridge itself is also considered as an external force because the bridge structure has to support itself in addition to the other applied loads (like the vehicles). **Internal forces** are forces that act within a structural member. The tension within the cable of a suspension bridge is an example of an internal force.

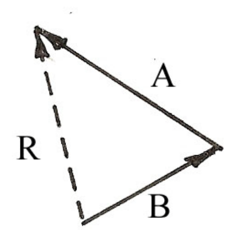
Part Three – RESULTANTS and COMPONENTS

Resultants

The resultant of a group of forces is a single force that has the same effect on the body on which it acts as the group of forces. The resultant is equal to the vector sum of the forces: **R = A + B.** The resultant can replace the forces without affecting the body on which it acts. The resultant is shown as a dashed line so that it will not be misinterpreted as an additional force.



Two Forces and Their Resultant



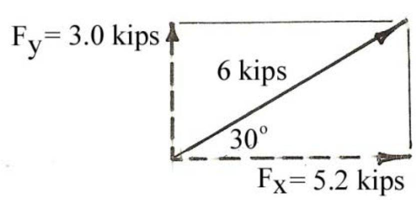
Vector Sum: **A + B = R**

Resultants

Components

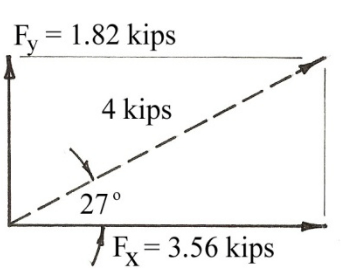
A component of a force is the projection of that force on another axis. Any force may be resolved into <u>two</u> components that will produce the same effect as the force they replace. If the force is resolved into two components that are perpendicular to each other, the components are called rectangular components. A force can be resolved into nonrectangular components, but this is a rarely used procedure. (Expressed in vector algebra terms, a force is equal to the sum of its components. Thus if $\mathbf{F_x}$ and $\mathbf{F_y}$ are the components of force $\mathbf{F}$, then $\mathbf{F_x} + \mathbf{F_y} = \mathbf{F}$.)

To avoid confusion, either the original force or the components are shown as dashed lines so that they will not be misinterpreted as additional forces.



Components

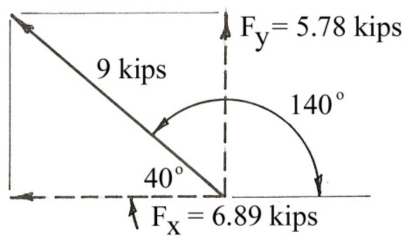
Example: What are the x component and the y component of the 4 kip force? Check the answer using the Pythagorean Theorem.



$$Sin\ 27 = \frac{F_y}{4}$$
$$F_y = (4)\sin 27 = \underline{1.82\ \text{kips}}$$
$$F_x = (4)\cos 27 = \underline{3.56\ \text{kips}}$$

$$a^2 + b^2 = c^2$$
$$1.82^2 + 3.56^2 \stackrel{?}{=} 4^2$$
$$3.3 + 12.7 = 16$$
check ✓

Example: What are the x component and the y component of the 9 kip force?

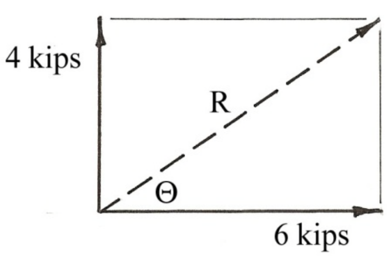


$$F_y = (9)\sin 40 = \underline{5.78\ \text{kips}}$$
$$F_x = (9)\cos 40 = \underline{6.89\ \text{kips}}$$

RESULTANT OF CONCURRENT FORCES

Resultant of Two Orthogonal Forces The resultant of two forces at 90° (**orthogonal** forces) is the diagonal of the rectangle defined by the forces. Both the magnitude and the direction of the resultant can be determined by right triangle relationships – the Pythagorean Theorem plus the tangent relationship.

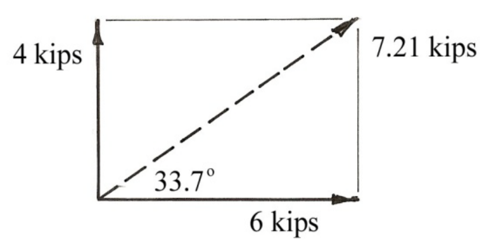
Example: Find the resultant of two orthogonal forces and the angle the resultant makes with the horizontal.



Problem

$$R = \sqrt{4^2 + 6^2} = \underline{7.21\ \text{kips}}$$
$$\Theta = tan^{-1}\left(\frac{4}{6}\right) = \underline{33.7°}$$

Solution

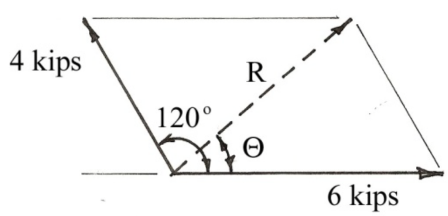


Answer

Resultant of Two Oblique Forces The resultant of two forces at an angle other than 90° (**oblique** forces) can be determined by either of two procedures.

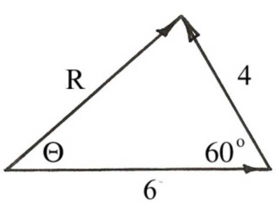
The **first** procedure, which we will not typically use in this course, is to complete the parallelogram defined by the two forces. The resultant is the diagonal of the parallelogram. This procedure works well with graphic solutions, and therefore is presented here for illustration purposes only. However the arithmetic can get complex when solving with trig functions as shown in the example below. The magnitude and direction of the resultant are found by solving a triangle with two sides and the included angle as the known quantities.

Example: Find the resultant of two oblique forces using the parallelogram method – not the preferred method.



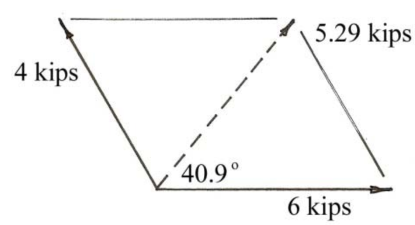
Solution

First use the Law of Cosines to determine R. Then use the Law of Sines to determine theta.



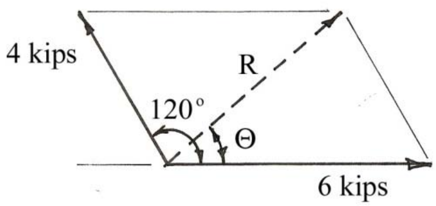
$R^2 = 4^2 + 6^2 - 2\,(4)\,(6)\cos 60°$
$R^2 = 28.0$
$R = \underline{5.29\text{ kips}}$
$\sin \Theta = \frac{4}{5.29} \sin 60° = .655$

$\Theta = \sin^{-1}(.655) = \underline{40.9°}$



The **second** procedure, which we will use in this course, requires as a first step, that the two forces be resolved into their horizontal and vertical components. Then, the horizontal components are added to find a single resultant horizontal force, and the vertical components are added to find a single resultant vertical force. The problem is then one of finding the resultant of two orthogonal forces.

Example: Find the resultant of two oblique forces using the component method.



Solution

In this solution the "tally" method of solving problems is introduced. The tally method is a way of eliminating complex mathematical equations; it is a simple way to keep track of all the forces, components, and directions without worrying about plus and minus signs in long equations; and, it also has the advantage of being able to only deal with one component of an equation at a time, thus avoiding confusion about whether or not something has already been included.

The first step is to resolve each of the two original forces into their individual horizontal component. Then add the horizontal components to find a single horizontal component. In this case F_x = 4 kips→. This single horizontal component is the horizontal component of the Resultant. The next step is to resolve each of the two original forces into their individual vertical components and add the vertical components to find a single vertical component (F_y = 3.46 kips↑). This single vertical component is the vertical component of the Resultant.

$\sum F_x$

←	→
4 cos 60	6
2	6
↳	-2
	4 kips →

$\sum F_y$

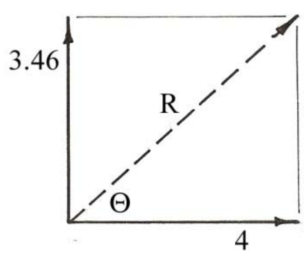
↑	↓
4 sin 60	0
3.46↑	0
-0	↲
3.46 kips ↑	

Tally Method Explained One of the biggest concerns in solving any structural engineering problem is that of making a mistake. A simple mistake, often mental, can frequently be the hardest to find – for example, misplacing a decimal, or using a plus sign instead of a minus – or a large mistake like forgetting to include a component of an oblique force will lead to incorrect answers. At worst, a mistake could be disastrous. At best, a mistake can cause others to doubt your competence. The tally method is a way of eliminating many careless mistakes.

Make it a point to use the tally method on every single applicable problem. Try to do it the same way every time. To use in practice, try this method. Look at the two calculations above while reading the following steps. Beginning at the top say to yourself "sum of the forces horizontal" while writing $\sum F_x$; then say "some go to the left and some go to the right" while drawing two arrows below, one to the left and one to the right; then draw a horizontal line under the arrows and a vertical line to split the space into two columns. Then any force that has a left horizontal component goes under the left arrow and any force that has a right horizontal component goes under the right arrow. Now draw a totals line. Add up the entries under the left arrow. Add up the entries under the right arrow. Subtract the smaller from the larger and that is the component of the resultant

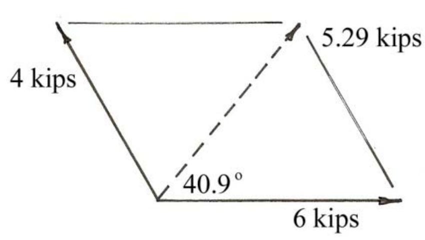
The results of the tally above are the horizontal component of the resultant and the vertical component of the resultant.

Now find the resultant of the two orthogonal forces.



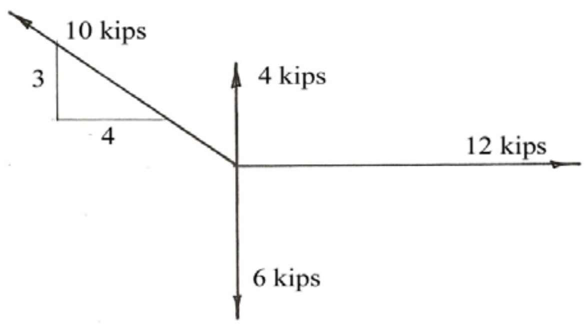
$R = \sqrt{4^2 + 3.46^2} = \underline{5.29 \text{ kips}}$
$\Theta = \tan^{-1}(3.46 / 4) = \underline{40.9°}$

Answer



Resultant of Multiple Concurrent Forces Finding the resultant of multiple concurrent forces can best be done by using the second procedure – the tally method. First, replace each of the forces with their respective horizontal and vertical components. Second, add all the horizontal components to find a single horizontal force, and add all the vertical components to find a single vertical force. Then, find the resultant of the horizontal and vertical forces.

Example: Find the resultant of multiple concurrent forces.



The slope or direction of a force is often stated by specifying the horizontal and vertical relationship – in this case 3 and 4. The hypotenuse of this triangle is 5. It is a 3-4-5 triangle. Another common one is the 5-12-13 triangle. Notice that 10k is twice 5. Therefore, the vertical component is twice 3, or 6 kips; and the horizontal component is twice 4, or 8 kips.

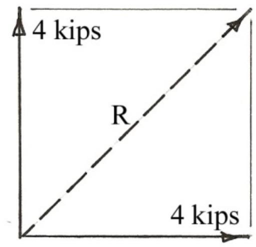
Solution

Using the tally method, first resolve each of the four original forces into their individual horizontal component. Then add the horizontal components to find a single horizontal component (F_x = 4 kips→). This single horizontal component is the horizontal component of the Resultant.

Next, resolve each of the four original forces into their individual vertical component. Then add the vertical components to find a single vertical component (F_y = 4 kips↑). This single vertical component is the vertical component of the Resultant.

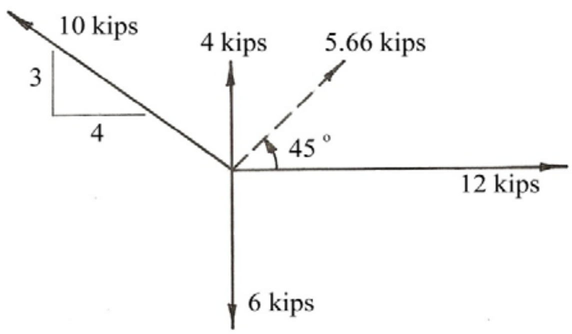
$\sum F_x$		$\sum F_y$	
←	→	↑	↓
10(.8)	12	4	6
		10(.6)	
8	12	10	6
↳	-8	-6	↲
	4 kips →	4 kips ↑	

Now find the resultant of two orthogonal forces.



$R = \sqrt{4^2 + 4^2} = \underline{5.66 \text{ kips}}$
$\theta = \tan^{-1}(4 / 4) = \underline{45°}$

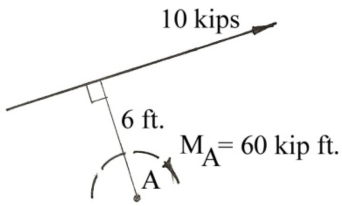
Answer



Part Four –MOMENTS and RESULTANTS

MOMENTS

Moment A moment is an action that tends to rotate the body on which it acts. The moment at a point is produced by a force acting at a distance from the point. The moment is equal to the force times the perpendicular distance from the point to the line of action of the force. A moment is a vector quantity. It has magnitude, direction, and a point of application. The direction of the moment, either clockwise or counterclockwise, is indicated with a curved arrow. The original forces (or force) are solid lines; the resultant moment is dashed.



$M = (F) \text{ x } (d)$
$M_A = (10 \text{ kips}) (6 \text{ ft.})$
$M_A = 60 \text{ kip-ft}$

Moment of a Force about Point A

Units The units for moment are force times distance.

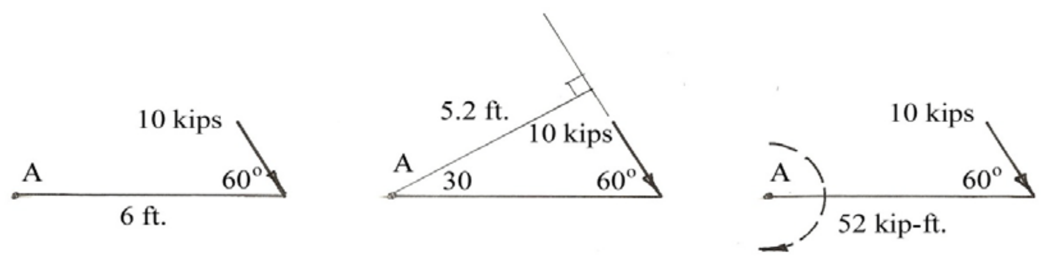
American Units	SI Units
pound-feet (lb-ft)	Newton-meter (N-m)
kip-feet (k-ft)	kilonewton-meter (kN-m)

Moment of an Oblique Force If the perpendicular distance from the reference point to the line of action of the force is a known quantity, the moment is simply the force times the distance.

If the force that produces a moment is drawn at an oblique angle (any angle other than 90°) to the coordinate axis, either of two procedures can be used to compute the moment.

The **first** procedure is to construct a perpendicular from the reference point to the line of action of the force. The moment is then computed as the force times the perpendicular distance. Again, this procedure works well with graphic solutions, however the arithmetic can get messy when solving with trig functions. The following example is for demonstration purposes only. We will not use this method.

Example: Find the moment about point A by computing the perpendicular distance to Point A.



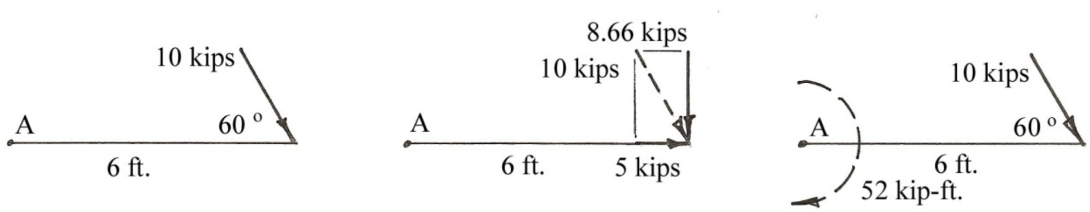
$M_A = 10\ (5.2) = 52$ kip-ft

Problem Solution Answer

The **second** procedure – the procedure we will use from now on in this course – requires that the force be replaced by its horizontal and vertical components. The moment is then computed as the horizontal force times the vertical distance to its line of action plus the vertical force times the horizontal distance to its line of action.

Example: Find the moment about Point A by using horizontal and vertical components.

Hint: sometimes it's helpful to show the original force as a dashed line and the horizontal and vertical components as solid lines during the solution phase of a problem. This helps keep clear in your mind which forces you are working with. It will be one less thing you have to think about if you are always making calculations with solid lined forces.



$M_A = 8.66\ (6) + 5\ (0) = 52$ kip-ft.

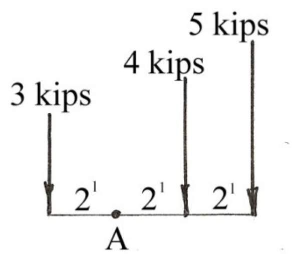
Problem Solution Answer

Notice that the line of action of the horizontal component of the force (5 kips→) passes though the Point A and therefore there is no vertical distance. The moment due to the horizontal component of the force then is $M_{A\text{-Horiz}}$ = 5 kips x zero feet = zero.

Moment of Several Forces If several forces are acting on a body, the moment is the sum of the moments produced by each of the forces. When computing this sum, the clockwise or counterclockwise direction of the moments must be taken into account. This can be done using two methods. The first method is accomplished by assuming a sign convention of positive for clockwise, and negative for counterclockwise. Again, this is an algebraic formula requiring full concentration to "get it right". We will not use this method.

The second method is done using the tally method. Remember, when summing moments about a point, use clockwise and counterclockwise arrows to indicate the direction of the moment.

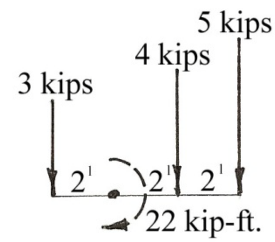
<u>Example:</u> Find the moment about Point A of several forces using the tally method.



Problem

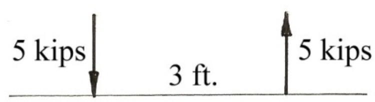
↶	↷
3(2)	4(2)
	5(4)
6	28
↳	-6
	22↷

Solution



Answer

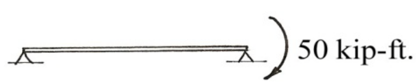
Couple A couple is a pair of parallel forces that have the same magnitude, but are in the opposite direction. The moment produced by a couple is equal to the force times the distance between them. The moment generated by the couple is the same regardless of the point of reference. The couple is a useful concept. There are occasional problems in statics that can be solved more easily by recognizing the special nature of a couple.



M = (5 kips) (3 ft.) = 15 kip-ft.

Couple

Concentrated Moment Occasionally a moment will occur as a load on a beam. In this case, it is shown as being concentrated at a point. A moment of this type is usually located at the end of the beam and is commonly produced by internal or connection forces.



Concentrated Moment

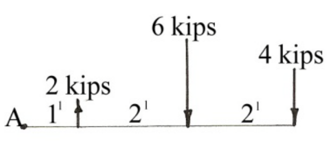
Torque A torque is an action that tends to rotate the body on which it acts. This is the same definition as for a moment. **Torque and moment are equivalent.** Torque is the appropriate term to use when describing the twist delivered by a shaft or motor. Moment is the appropriate term to use when describing the bending of a beam.

RESULTANT OF PARALLEL FORCES

Resultant of Concentrated Parallel Forces The resultant of a group of forces must produce the same effect as the original forces. If the forces are parallel, then the line of action of the resultant will be parallel to the other lines of action, and the magnitude of the resultant will be equal to the algebraic sum of the forces. What remains to be determined is the location of the line of action of the resultant. The line of action must be located so that the resultant will produce the same moment about any given point as the original forces.

Example: Find the resultant of a group of parallel forces.

The solution is a three step process: first, determine the resultant force by summing forces vertically; second, determine the resultant moment from the original forces; and, third, determine where the resultant force must act – by dividing the moment by the resultant force.



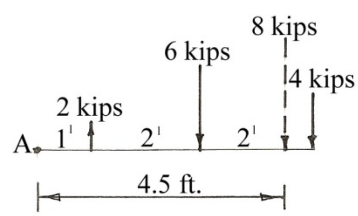
$\sum F_y$

↑	↓
2	6
	4
2	10
↳	-2
	8↓

$\sum M$ @ A

↶	↷
2 (1)	6 (3)
	4 (5)
2	38
↳	-2
	36↷

$$x = {}^{36}/_{8} = 4.5'$$



Problem Solution Answer

RESULTANT OF DISTRIBUTED FORCES

Distributed Forces Distributed forces are usually referred to as distributed loads. They are, in essence, a continuous set of parallel forces. Real life loads such as snow loads, hydrostatic loads, earth pressures, etc., can be approximated by distributed loads

Distributed loads are shown on structures as a diagram. The length of the diagram is the length over which the load acts – it is usually plotted on either the vertical or horizontal axis. The magnitude of the distributed force is plotted on the other axis – at 90° to the length. The length units for the diagram are length (ft.) and magnitude units for the diagram are force/length (#/ft. or kips/ft.).

Resultant of Distributed Forces The total magnitude of a distributed force is equal to the area of the load diagram. The units for this area are force.

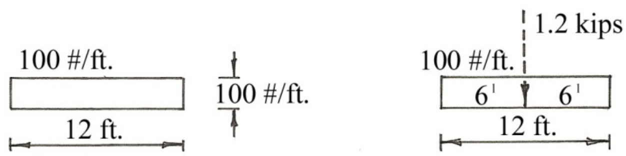
(force / length) x (length) = force
(kips / ft.) x (ft.) = kips

The line of action of the resultant is parallel to the lines of action of the distributed force. And the resultant passes through the centroid of the load diagram area. The resultant is shown as a dashed line so it is not mistaken for an additional force.

Some loads are uniformly distributed loads and are plotted as a rectangle. Others are uniformly varying distributed loads and are plotted as triangles or trapezoids.

Example: Find the resultant of a uniformly distributed load.

A uniformly distributed load is a constant along its length. The magnitude of the unit load does not change from one end of the load to the other. The resultant of a uniformly distributed load is the area under the load diagram and is simply the length times the height – (ft.) x (#/ft.) = #, (or kips). The location of the resultant of a uniformly distributed load is at the centroid of the load diagram – at the center of the length.

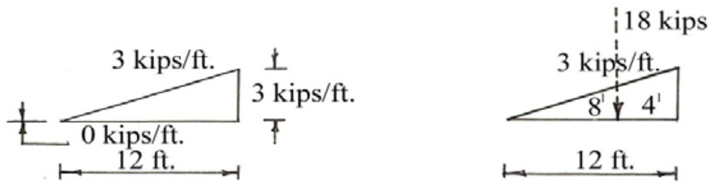


R = (12 ft.) (100 #/ft.) = 1,200 # = 1.2 kips
x = (1/2) (12 ft.) = 6' from each end

When showing the resultant on the load diagram, show it as a dashed line. Show the location of the resultant by placing the distance from each end of the load within the load diagram. The original values of the uniformly distributed load are also shown on the diagram – the length and its magnitude.

Example: Find the resultant of a triangular distributed load.

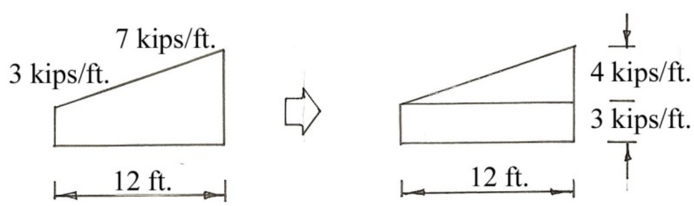
A triangular load is a uniformly varying distributed load. Its value at one end is zero. The magnitude of the unit load varies from one end of the load to the other. The resultant of a triangular load is the area under the load diagram – (1/2) b h. The resultant is located at the third point.



R = (1/2) b h
R = (1/2) (12 ft.) (3 kips/ft.) = 18 kips
x_1 = (2/3) (12 ft.) = 8'
x_2 = (1/3) (12 ft.) = 4'

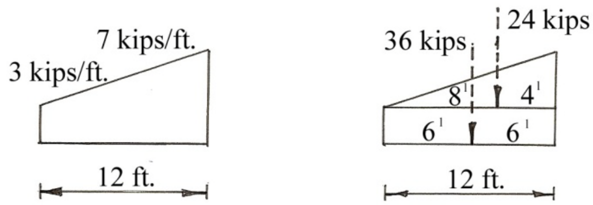
Example: Find the resultant of a trapezoidal distributed load.

A trapezoidal distributed load can be thought of as a uniformly varying load consisting of two separate distributed loads – a uniformly distributed load and a uniformly varying (triangular) distributed load. This type of load has a magnitude at each end of the load, and they are different. To find the resultant of a trapezoidal load, break the load into two components – a uniform load and a triangular load – and find the resultant of each. Show both resultants on the diagram and note their locations.

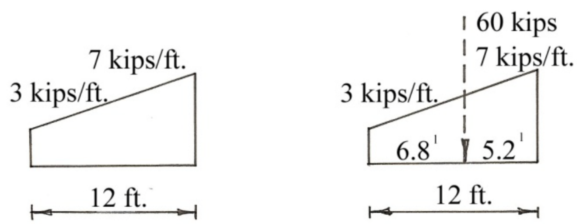


<u>Uniformly distributed load –</u>
R = (12 ft.) (3 kips/ft.) = 36 kips
x_1 = (1/2) (12 ft.) = 6'

<u>Triangular load –</u>
R = (1/2) (12 ft.) (4 kips/ft.) = 24 kips
x_1 = (2/3) (12 ft.) = 8'
x_2 = (1/3) (12 ft.) = 4'



There is an alternate solution to the trapezoidal load. Instead of replacing the trapezoidal load with two resultants, replace the entire trapezoidal shape with only one resultant. The math to do



that is slightly more difficult, and requires a bit more thinking than breaking the load into two components. Using two components is the simplest way to do it and is the preferred way - there is less chance to make an error using the real simple math for two resultants. Shown below is the "answer" for replacing the trapezoidal load with only one resultant.

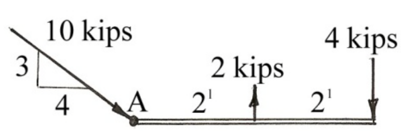
RESULTANT OF NONCONCURRENT FORCES

If the forces in a problem are neither concurrent nor parallel, and it is desirable to determine a single resultant, the resultant can be determined by the following procedure.

1. Replace each diagonal force with its horizontal and vertical components.
2. Find the resultant of the vertical forces.
3. Find the resultant of the horizontal forces.
4. Find the resultant of the resultants of steps 2 and 3.

Example: Find the resultant of the nonconcurrent forces.

Problem

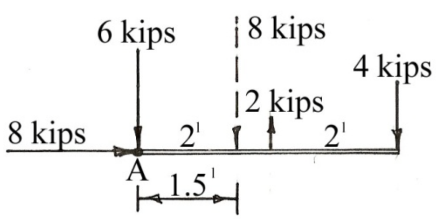


Solution:

Step 1. Replace all diagonal forces with their horizontal and vertical components, and

Step 2. Find the resultant of the vertical forces (magnitude and where it acts)

<u>NOTE:</u> The vertical and horizontal components of the diagonal force are solid lines because they represent the given original force. Only the resultant of the vertical forces is shown as a dashed line.



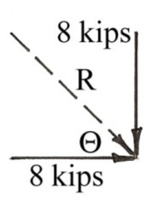
$\sum F_y$ ↑	$\sum F_y$ ↓	$\sum M$ @ A ↶	$\sum M$ @ A ↷
2	6	2 (2)	4 (4)
	4		
2	10	4	16
↳	-2	↳	-4
	8 kips↓		12↷

$$x = {}^{12}/_{8} = 1.5'$$

Step 3. Find the resultant of the horizontal forces

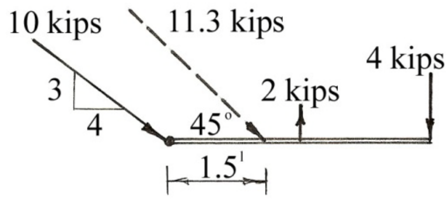
<u>NOTE:</u> There is only one horizontal force (8 kips→). Therefore the resultant of the horizontal forces is that force (8 kips→). And its line of action is through point "A".

Step 4. Find the resultant of the resultants



$$R = \sqrt{8^2 + 8^2} = 11.3 \text{ kips}$$
$$\Theta = \tan^{-1}({}^{8}/_{8}) = 45°$$

Answer:



Part Five – EQUILIBRIUM

EQUILIBRIUM BACKGROUND

Newton's Laws At the beginning of this course, we discussed Sir Isaac Newton, the founder of the field of mechanics, and his Three Laws of Motion. And, for our use in Statics, we reduced them to the following two rules:

- His **First Law** states that a body at rest will stay at rest if the forces acting on it are in equilibrium.
- His **Third Law** states that for every action there is an equal and opposite reaction.

Again, these two laws as stated here are the fundamental basis for the study of statics. You should commit these two laws to memory.

Equal and Opposite Forces Newton's third law is important because it states that forces come in pairs. For example, if you are standing, your weight is a downward force and the floor is pushing upward with an equal and opposite force. In our study of forces, we normally show only one of the forces, but never forget that the other force exists.

EQUILIBRIUM EQUATIONS

Summation of Forces and Moments Equals Zero If a body is in equilibrium, then all of the forces and all of the moments must be in balance. This condition is stated by the following six equations:

Forces in Balance	Moments in Balance
$\Sigma F_x = 0$	$\Sigma M_{xy} = 0$
$\Sigma F_y = 0$	$\Sigma M_{xz} = 0$
$\Sigma F_z = 0$	$\Sigma M_{yz} = 0$

Equilibrium Equations for Two Dimensional Problems If a problem is two dimensional (coplanar), then the equilibrium equations that have a z-component are meaningless. Eliminating all the equations with a z-component leaves three equations – two force equations and one moment equation.

Forces in Balance	Moments in Balance
$\Sigma F_x = 0$ or $\Sigma F_h = 0$	$\Sigma M_{xy} = 0$ or $\Sigma M = 0$
$\Sigma F_y = 0$ or $\Sigma F_v = 0$	

Notice that the equations above are shown with two styles of identification. One style uses the x-y notation and the other style uses "h" and "v" notations which stand for horizontal and vertical. Both styles are acceptable.

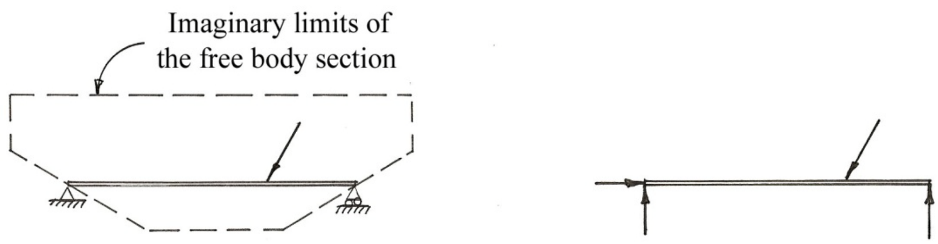
FREE-BODY DIAGRAMS

Definition A free-body diagram is a simplified sketch of the physical conditions that describe a problem. In a free-body diagram, the body, or section of the body, under consideration is cut free from all other bodies. The forces that the adjacent bodies exert on the free-body are shown on the diagram – supports replaced with force arrows, connecting members replaced with forces, etc. – along with all other information needed to solve the problem.

The drawing of a proper free-body diagram is an essential first step in the solution of a statics problem.

External Free-Body Diagram An external free-body diagram cuts the member to be studied free from its supports. The member remains intact, but the supports are removed. In place of the supports, force arrows are shown that have the same magnitude and direction as the force of the reactions that support the member.

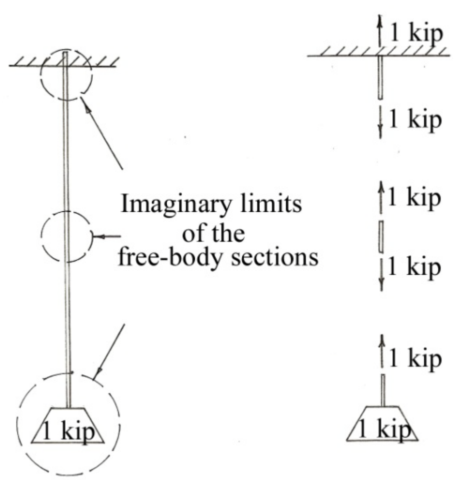
Imagine a box around the beam below that "cuts loose" the supports from the member. Everything that is outside the box must be replaced with an appropriate force or forces.



Load Diagram Free-Body Diagram

External Free-Body Diagram

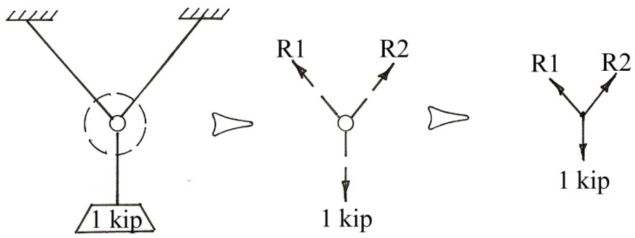
Internal Free-Body Diagram An internal free-body diagram is a free-body diagram in which the boundaries of the free-body cut through the member. At the cut surface, force arrows are shown that have the same magnitude and direction as the internal forces in the member.



Load Diagram Free-Body Diagram

Internal Free-Body Diagram

Example: Draw the free body diagram for the suspended weight shown on the left.



Internal Free-Body Diagram

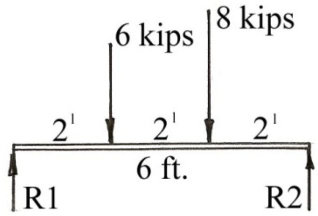
Notice that the intermediate drawing shows the center connection device and a portion of the three "ropes" with three force arrows that are not "attached" to the ropes. While that is an acceptable way of drawing a free-body diagram, the drawing on the right is the preferred way of drawing it. The drawing on the right takes less effort to draw, is simpler, and still clearly shows the intended simplification of the actual, real life, situation of a hanging one-kip weight.

USING THE EQUILIBRIUM EQUATIONS

Summation of Forces The concept that the forces in both the horizontal or vertical directions should add to zero is a simple one – "left equals right" and "up equals down". What is needed is a workable procedure for converting the information from a fee-body diagram into an equation.

One method is to adopt a sign convention where up is plus and down is minus, and then write the algebraic equation. This can get cumbersome and confusing if you're not paying close attention, especially when summing horizontal forces. While trying to get the direction and value of the forces and distances correct, you're also trying to remember the sign convention - is left positive? Or is it negative? Mistakes can easily happen while looking back and forth from the free-body diagram to the equation being written. We won't be using this method.

The other method, the tally method, is very easy to use. It is the same method we have been using - simply tally the "up" forces, tally the "down" forces. Only, now set them equal to each other to arrive at the equation. And, for horizontal forces, simply tally the "left" forces, tally the "right" forces, and set them equal to each other to arrive at the equation. There is nothing to "remember" while summing the vertical forces except to account for all up forces under the "up" arrow and all the down forces under the "down" arrow. Same for the horizontal forces except there are "left" and "right" arrows. Using up and down arrows and left and right arrows helps eliminate mistakes – it simplifies the mental process of solving statics problems. We will continue to use the tally method in this course.



$$\sum F_y = 0$$
$$R1 + R2 - 6 - 8 = 0$$

$\sum F_y = 0$

↑	↓
R1	6
R2	8

R1 + R2 = 14

Equation

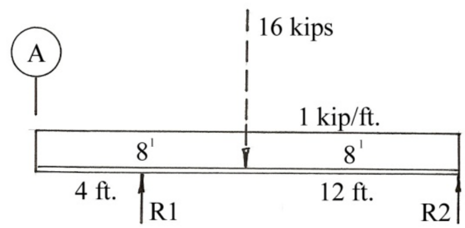
Tally

Summation of Forces

Summation of Moments When using the tally method, the comments about the summation of forces also apply to the summation of moments, with two additions. First, it is important to **note the point about which the moment is being taken**. Notice the heading for the moment tally – the sum of the moments at A equals zero ($\sum M$ @ A = 0).

And second, you should develop the habit of showing both the distance and the force that were used to compute the moment instead of just their product. This makes it much easier to check your work.

$\sum M$ @ $A = 0$
$4R1 + 16R2 - 16(8) = 0$



$\sum M$ @ $A = 0$

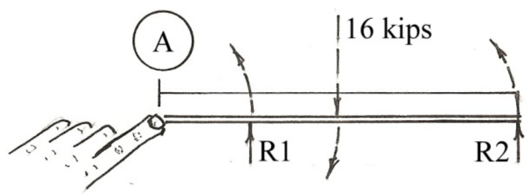
↶	↷
4R1	16(8)
16R2	
4R1 + 16R2 =	128

Equation

Tally

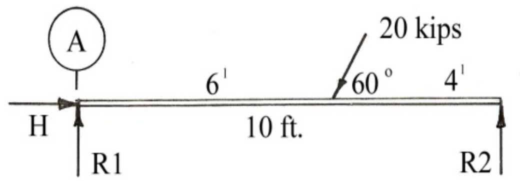
Summation of Moments

<u>Hint:</u> Placing your left index finger on the point about which moments are to be taken is a simple, but very effective, way of keeping track of the point AND which direction the moment rotates about the point for each force. After many years of calculating even the simplest of moment problems, I still place my left index finger (because I'm right handed) on the point about which the moments are being taken while tallying the moments about the point.



Simultaneous Equations In statics, because nothing is moving through space, all of the equilibrium equations must be satisfied all of the time. This means that for the most complex problems there are potentially six unknowns and six equations that must be solved simultaneously. Fortunately the number of unknowns and equations is reduced to three if the problem is two-dimensional. And, in many problems there are only two unknowns and therefore only two equations. The solution to virtually every problem in statics is found by solving a set of equilibrium equations.

Shown below is the solution for a relatively simple two-dimensional problem. All the equations of equilibrium are satisfied. Notice that the equations are nothing more than the results of the tally method – forces up equal forces down, forces right equal forces left, and the sum of the moments clockwise equal the sum of the moments counter-clockwise.



H
R1
R2

Problem — Three Unknowns

$\sum Fx = 0$

←	→
20 cos 60	H
20 cos 60 =	H

$\sum Fy = 0$

↑	↓
R1 R2	20 sin 60
R1 + R2 =	20 sin 60

$\sum M @ A = 0$

↶	↷
R2 (10)	(20 sin 60) (6)
10R2 =	(20 sin 60) (6)

Equilibrium Equations:

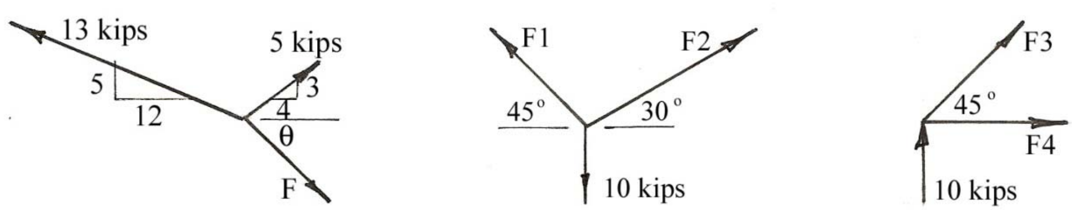
$\sum F_x = 0$	H = 20 cos 60
$\sum F_y = 0$	R1 + R2 = 20 sin 60
$\sum M_A = 0$	10R2 = (20 sin 60) (6)

Notice in the problem above, that the line of action of both the horizontal component of the 20 kip force and the force R1 both pass through the point A, the point about which the moment is being taken. Therefore their contribution to the moment about point A is zero.

EQUILIBRIUM OF CONCURRENT FORCES

Free-Body Diagrams The free-body diagrams shown below are typical concurrent, coplanar force problems that can be solved by using the equations of equilibrium. Since these are two-dimensional problems, there are potentially only three equilibrium equations. However, **no moment is created in a concurrent force system** because the lines of action of all forces pass through a single point. Since there are no moments in the problem, there is no moment equation. This leaves only two equations: $\Sigma F_x = 0$ and $\Sigma F_y = 0$. Because there are only two equations, the problem can have only two unknowns.

There are two types of concurrent force problems: one, where the magnitude and direction of a single force are unknown (Problem A), and two, where there are two unknown forces (Problem B and Problem C).



Problem A Problem B Problem C

Concurrent Force Problems

In Problem A, the direction and magnitude of the two known forces (13 kip and 5 kip) are given. The line of action of the unknown force F and its direction ϴ as shown are just educated guesses at this point. When the problem is solved below, we'll see that the "guess" was correct. The unknown force F does indeed act downward and ϴ is less than 90°.

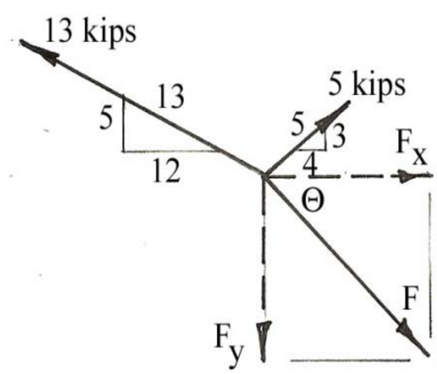
In Problem B the 10 kip force acts downward and the line of action of F1 and F2 are given at 45° and 30° respectively.. By looking at the problem and noticing that the only given magnitude is 10 kips downward, it is fairly easy to "guess" that the two unknown forces (F1 and F2) are up as shown. When the problem is solved below, we'll see that the "guess" was correct. The unknown forces F1 and F2 do indeed act upward.

In Problem C the 10 kip force acts upward and the line of action of F3 is given as 45° and F4 acts horizontally. It is assumed ("guess") that the forces F3 and F4 will act to the right in the drawing. When the problem is solved below, we'll see that the "guess" was incorrect. The unknown force F3 acts in the opposite direction shown.

Magnitude and Direction of a Force is Unknown This type of problem is shown in Problem A above – where the magnitude of a single force and its direction is unknown. The procedure for solving this type of problem is to first determine the magnitude of the horizontal component of the unknown force and the magnitude of the vertical component of the unknown force. In the case of Problem A, the unknown force and its direction are initially unknown. The drawing represents a "best guess" at what direction it will ultimately be located. If the "guess" was wrong, i.e., the force ends up to be in the opposite direction, the answer would contain a negative sign.

These horizontal and vertical components of the unknown force are simply the sum of the vertical components and the sum of the horizontal components of the known forces in the problem – no matter how many known forces there are in the problem. Once these two components are computed, it is an easy second step to find the magnitude and direction of the unknown force.

__**Example:**__ Find the magnitude and direction of the unknown force in Problem A above.



First determine the magnitude of the horizontal component and the magnitude of the vertical component of the unknown force. <u>Note:</u> because the forces are in equilibrium, the unknown force is equal in magnitude and <u>opposite in direction</u> to the resultant of the other forces.

$\sum F_x$

←	→
12	4
	F_x
12 =	$4+F_x$

$F_x = 8\rightarrow$

$\sum F_y$

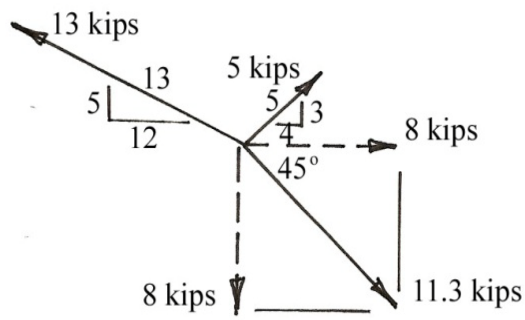
↑	↓
5	F_y
3	
8 =	F_y

$F_y = 8\downarrow$

Now find the magnitude and direction of the unknown force.

$$F = \sqrt{8^2 + 8^2} = \underline{11.3 \text{ kips}}$$
$$\theta = \tan^{-1}(8 / 8) = \underline{45°}$$

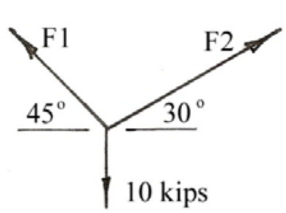
Answer



Two Unknown Forces If the two unknowns of a concurrent force system are the magnitude of two forces, you must write two equations, each equation summing the forces in a different direction. Usually the directions are horizontal and vertical (but they can be any direction). If each of the unknowns appears in each equation, the equations must be solved simultaneously. However, in some problems the equations can be written so that they are independent and can be solved independently.

Example: Find the magnitude of each of the unknown forces in Problem B above.

Note: Because both unknown forces each have a component in the x- and the y-direction, both unknown forces will show up in each of the two equations. Therefore, the solution to this problem requires the solving of two simultaneous equations.



First sum the forces horizontally to generate an equilibrium equation; then sum the forces vertically to generate another equilibrium equation. Both F1 and F2 appear in both equations.

$\sum F_x = 0$

←	→
.707 F1	.866 F2
.707 F1 =	.866 F2

.707 F1 - .866 F2 = 0

$\sum F_y = 0$

↑	↓
.707 F1 .500 F2	10
.707 F1 + .500 F2 =	10

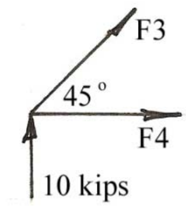
Now subtract the sum of the forces in the x-direction equation from the sum of the forces in the y-direction equation to solve for F2:

.707 F1 + .500 F2 = 10
<u>(-).707 F1 (+) .866 F2 = 0</u>
1.366 F2 = 10
<u>F2 = 7.32 kips</u>

Then substitute the found value for F2 into the sum of the forces in the x-direction to solve for F1:

.707 F1 - .866 F2 = 0
.707 F1 - .866 (7.32) = 0
.707 F1 = 6.34
<u>F1 = 8.97 kips</u>

Example: Find the magnitude of each of the unknown forces in Problem C above.



Since the final directions of forces F3 and F4 may not be obvious, it is convenient to "guess" their direction as tension as shown above before beginning the solution for their values. If the solution for either, or both, produces a negative value, it simply means that the assumed direction was not the correct direction.

One of the forces (F4) is acting along a major axis, in this case the x-axis; it only has a horizontal component. It does not have a vertical component. Therefore this problem can be solved without using simultaneous equations. Simply choose to sum the forces vertically first. Doing so produces an equation with only one unknown, F3 as shown below.

$$\sum F_y = 0$$

↑	↓
10	0
.707 F3	
10 + .707 F3 =	0

.707 F3 = -10

<u>F3 = - 14.1 kips</u>

The answer is negative 14.1 kips which simply means that the assumed direction of F3 was incorrectly "guessed" at the beginning of the problem. F3 is actually a compressive force instead of the assumed tensile force.

Now, sum the forces horizontally to determine F4 as follows:

$$\sum F_x = 0$$

←	→
.707 F3	F4
.707 F3 =	F4

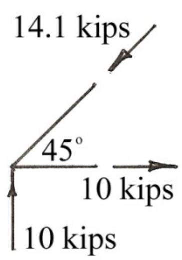
Notice in the tally above that F3 was put in the correct column – acting to the left – its correct direction. Because the correct direction of F3 was used, the result for F4 will have its correct sign – plus for the correct assumed direction, minus for the incorrect assumed direction.

Substituting the value of F3 yields:

.707 (14.1) = F4
F4 = 10 kips

> NOTE: F4 is positive which simply means that the correct direction of the force was assumed at the beginning of the problem. Because the tally method is used to generate the equilibrium equations, there is no reason to clutter your mind with positive and negative signs while creating the equation (except, of course, when "guessing" the direction of the force at the beginning of a problem – but that is not creating an equation. It is setting up the problem getting ready to create the equilibrium equations). The tally method uses arrows for direction and sets left equal to right and up equal to down. Any positive or negative signs in the equilibrium equations will take care of themselves during the arithmetic process of adding the two directions and setting them equal to each other.

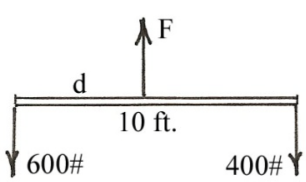
The "answer" to Problem C is shown below.



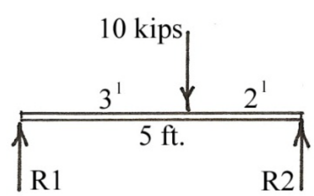
EQUILIBRIUM OF PARALLEL FORCES

Free-Body Diagrams The free-body diagrams shown below are typical parallel, coplanar (all forces in the same plane) force problems that can be solved by using the equations of equilibrium. Since these are two-dimensional problems, there are potentially three equilibrium equations. However, all of the forces are parallel which means that there are no forces in one direction and the summation of forces in that direction is meaningless. That leaves two equations: $\Sigma F_y = 0$ and $\Sigma M = 0$. With two equations there can only be two unknowns.

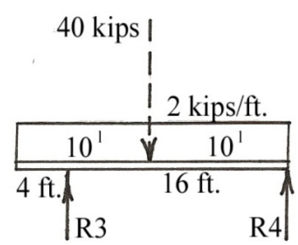
There are two types of parallel force problems: one, the magnitude and location of a single force are unknown (Problem A), and two, where there are two unknown forces (Problem B and Problem C).



Problem A



Problem B

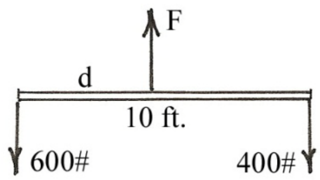


Problem C

Parallel Force Problems

Magnitude and Location of a Force is Unknown The line of action of the unknown force must be parallel to the other forces. Its magnitude is found by summing forces in the same direction – in this case the vertical direction. The force must be located at a point such that the summation of moments at any point (or, at all points) is zero. The unknown force is equal in magnitude and opposite in direction to the resultant of the other forces, and the unknown force has the same line of action as the resultant of the other forces.

Example: Find the magnitude and location of the unknown force in Problem A above:

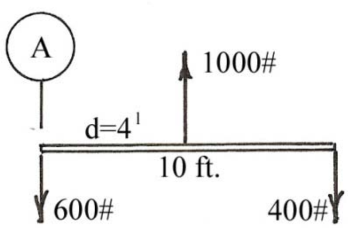


First determine the magnitude of the unknown force by summing forces vertically using the tally method.

$\sum F_y$

↑	↓
F	600
	400
F =	1,000 #

Next, determine the location of the 1,000# force by summing moments around any point. For ease, we will choose Point A as the point about which to sum the moments. By choosing Point A, we can eliminate one calculation from the solution – the moment of the 600# force – and we can solve directly for the distance



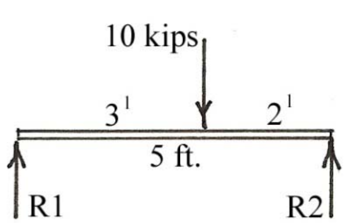
$\sum M @ A = 0$

↶	↷
1000 d	400 (10)
1000 d =	4000

<u>d = 4 ft</u>

Two Unknown Forces To solve a parallel force problem, you can sum moments about each of the unknown forces to solve for the other unknown force. Then sum forces in the direction of the parallel forces to check your work.

Example: Find the magnitude of the two unknown forces in Problem B above.



Sum the moments about R1 and solve for R2. Then sum the moments about R2 to find R1.

$\sum M$ @ R1 = 0

↶	↷
R2 (5)	10 (3)
5 R2 =	30

<u>R2 = 6 kips</u>

$\sum M$ @ R2 = 0

↶	↷
10 (2)	R1 (5)
20 =	5 R1

<u>R1 = 4 kips</u>

Check the work by summing forces vertically

$\sum F_y$

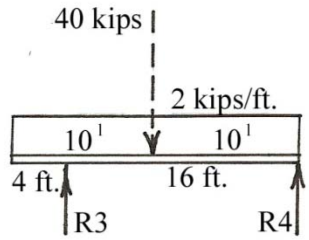
↑	↓
4	10
6	
10 =	10

Check ✓

Two Unknown Forces To solve a parallel force problem, you can also solve two simultaneous equations. To get the two equations, first write one equation that sets the sum of the forces equal to zero, and second, write another equation that sets the sum of the moments about any point equal to zero. If each unknown appears in each equation, they must be solved simultaneously. However, most problems can be set up in way that does not require solving simultaneous equations.

<u>Example:</u> Find the magnitude of the two unknown forces in Problem C above by solving simultaneous equations.

> <u>NOTE:</u> This problem could be solved without the use of simultaneous equations by following the procedure used in the previous example problem – i.e., sum the moments about each unknown force to find the other unknown force. Simultaneous equations are being used here to show that there is often more than one way to solve a problem.

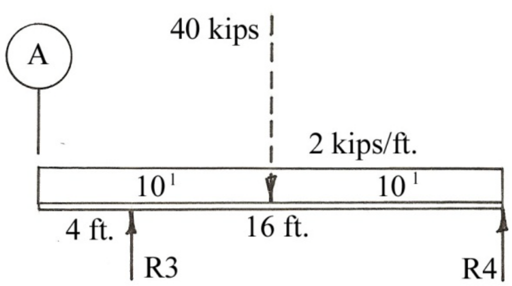


First determine one equilibrium equation by summing forces vertically using the tally method.

$$\sum F_y = 0$$

↑	↓
R3	40
R4	
R3 + R4 =	40

Next, determine the other equilibrium equation by summing moments about Point A. Notice that Point A is purposely NOT at one of the reactions. In this example we want two equations, each with both unknown forces. <u>Note:</u> Don't forget to place your left index finger on Point A when you are completing the tally method for moments.



$$\sum M @ A = 0$$

↶	↷
R3 (4)	40 (10)
R4 (20)	
4 R3 + 20 R4 =	400

R3 + 5 R4 = 100

Now subtract the first equation from the second equation (changing signs in the first equation) and solve for R4.

R3 + 5 R4 = 100
<u>(-)R3 (-)R4 = (-)40</u>
4 R4 = 60
<u>R4 = 15 kips</u>

Now solve for R3 by substituting into the first equation.

$$R3 + R4 = 40$$
$$R3 + 15 = 40$$
$$\underline{R3 = 25 \text{ kips}}$$

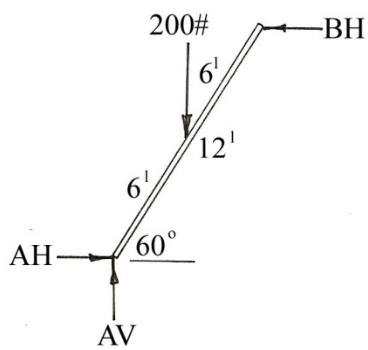
EQUILIBRIUM OF NONCONCURRENT, NONPARALLEL FORCES

Three Unknown Forces The majority of real-life problems involve three unknown forces. They are also typically coplanar (2-dimensional), nonconcurrent, and nonparallel force problems. Because it is 2-dimensional, there are only three equilibrium equations available, and, because there are three unknowns, all three equilibrium equations must be used: $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M = 0$.

The following three examples illustrate the procedure for solving problems with three unknowns. Notice in the following examples, each support is identified by a letter – A, B, C, etc. – and each force acting on that support is further identified by either an H or a V which represents the horizontal and vertical. Hence EV is the vertical force at support E, and DH is the horizontal force at support D.

In the example below, a free-body diagram represents a 200 pound person on a step ladder leaning against a wall. Often the vertical force at B is assumed to be frictionless, i.e., BV = 0.

Example: Find the magnitude of the three unknown forces of the step ladder:



Notice that by selecting which equilibrium equations will be solved first, second, and third, all unknown forces can be found without using simultaneous equations.

First, sum forces vertically equals zero. This will determine AV.

$$\sum F_y = 0$$

↑	↓
AV	200

AV= 200 #

Second, sum moments about A equals zero to find BH. Use the sine and cosine functions to determine the horizontal and vertical distances necessary to sum the moments about point A.

$$\sum M @ A = 0$$

↶	↷
BH (12 sin 60)	200 (6 cos 60)
10.4 BH =	600

BH = 58 #

And, third, sum forces horizontally equals zero to find AH.

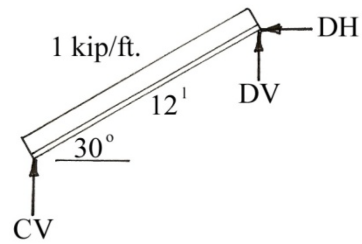
$$\sum F_x = 0$$

←	→
58	AH
58 =	AH

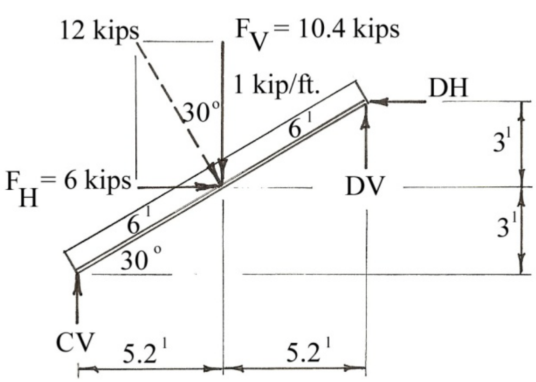
AH = 58 #

The next example is a variation on the ladder problem. In this case, the load on the sloping beam is a uniformly distributed load acting perpendicular to the beam.

Example: Find the magnitude of the three unknown forces of the sloped beam.



The first step in solving this problem is to resolve the angled uniformly distributed load along the sloped beam into its resultant. The magnitude of the resultant is equal to the area under the load diagram (1 kip/ft x 12 feet = 12 kips). It is shown as a dashed line and it is located at the mid-point of the uniformly distributed load. It is parallel to the distributed load. The 12 kip load now takes the place of the uniformly distributed load for the remainder of the solution. Then, determine the vertical and horizontal components of the 12 kip load (F_V and F_H) and show them as solid lines so they won't be confused with the 12 kip load. Also determine the horizontal and vertical dimensions to all forces. Doing this will simplify the arithmetic in the solution



$F_H = 12 (\sin 30) = 6$ kip
$F_V = 12 (\cos 30) = 10.4$ kip

Horizontal distance = 6 (cos 30) = 5.2 ft
Vertical distance = 6 (sin 30) = 3 ft

Next sum forces horizontally equals zero to determine DH.

$\sum F_x = 0$

←	→
DH	6

DH =6 kip

Then sum the moments at C equals zero to find DV. Then sum the moments at D equal zero to find CV. In these two tally's, use the resultant 12 kip force times half the length of the beam, 6 feet. It is not necessary to use the components of the 12 kip force. Using the 12 kip force requires only one calculation. Using the components requires two calculations for the same answer. Using two calculations instead of one offers one more chance to make an error.

$\sum M @ C = 0$

↶	↷
DH (6) DV (10.4)	12 (6)
6 DH + 10.4 DV =	72

6 (6) + 10.4 DV = 72

DV = 36 / 10.4

DV = 3.5 kips

$\sum M @ D = 0$

↶	↷
12 (6)	CV (10.4)
72 =	10.4 CV

CV = 6.9 kips

Finally, check the work by summing forces vertically.

$\sum F_y$

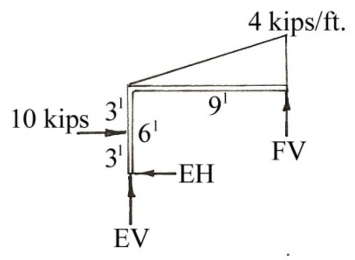
↑	↓
3.5 6.9	10.4

10.4 = 10.4

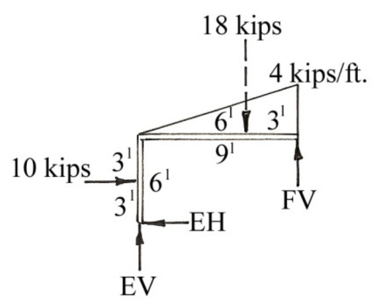
Check ✓

In the example below, the free-body diagram could represent a rigid frame with a uniformly varying distributed load. The horizontal member could be supporting a roof over a drive, itself supported by a column on the left end and attached to the building on the right end. The triangular load could represent snow drifted against the second story of the building.

<u>Example:</u> Find the magnitude of the three unknown forces.



Begin by finding the resultant of the triangular load. The resultant has a magnitude equal to the area under the load diagram; is located at the centroid of the load diagram, which is at the two-thirds point; is parallel to the load diagram; and is shown as a dashed line.



Since there is only one unknown in the horizontal direction, begin by summing the forces horizontal equals zero to determine EH

$$\sum F_x = 0$$

←	→
EH	10

<u>EH = 10 kip</u>

Then sum the moments at E equals zero to find FV, and sum the moments at F equals zero to find EV.

$\sum M @ E = 0$

↶	↷
FV (9)	10 (3)
	18 (6)
9 FV =	138

<u>FV = 15.3 kips</u>

$\sum M @ F = 0$

↶	↷
18 (3)	EV (9)
10 (3)	EH (6)
84 =	9 EV + 6 EH

84 = 9 EV + 6 (10)

9 EV = 84 – 60

<u>EV = 2.7 kips</u>

Now check the work by summing forces vertically.

$\sum F_y$

↑	↓
15.3	18
2.7	
18 =	18

Check ✓

This completes the first course in the series, **What Every Engineer Should Know About Structures Part A – Statics Fundamentals.** A wide range of the most common statics problems can now be solved using the fundamentals presented in this course.

The second course in the series, **What Every Engineer Should Know About Structures Part B – Statics Applications** will cover solutions to several specific statics problems. Some of the topics covered include the friction forces of an object with impending motion; finding the tipping load for construction cranes; determining forces in the cables used to support traffic lights hanging over intersections; determining the safe angle for a ladder leaning against a building; and seeing why many cranes have a large piece of equipment - such as an air compressor or an electric generator – hanging from them overnight and on the weekends. It's not to deter theft!