330.pdf

A SunCam online continuing education course

Railroad Curves Simplified

by

William C. Dunn, P.E.

Table of Contents

I. Introduction 3
II. Superelevation (Cant) 4
Cant Deficiency 6
Gauge and Rail Spacing 7
U.S. Railroads 8
The Comparison 9
Superelevation Software 10
Sample problem #1 10
Warnings and prompts 12
Sample problem #2 13
Sample problem #3 15
Sample problem #4 18
III. Spiral Transition Curves 21
Spiral Transition Software 22
Run-in-Rate (rir) 24
Delta Angle (Δ) 24
Spiral Length (L_S) 24
Spiral Time (t) 25
Cant (E) 25
Maximum Velocity (V_{max}) 25
Spiral Tangent Offset (T_o) 25
Alpha angle (α) 26
Central Curve Angle (Δ-2α) 27
Central Curve Length (L_C) 27
Total Curve Length (L) 27
Spiral Tangent Extension (Se) 27
Spiral Tangent (Ts) 27
IV. Unlimited Speed 28
Sample problem #5 - Hyperloop 29

Sample problem #6 - Hyper Chariot 34
V. Appendix A - Cascading Look-Up Tables 37
VI. Appendix B - Principles of Physics formulas 38
VII. Appendix C – FRA Formulas 41

I. Introduction

If you ever received a model train-set as a child, you probably learned your first lesson about the perils of railroad curves before your train completed the first lap around that small oval track. It was a classic "train wreck" and a live demonstration of what we would come to know as **centrifugal force**.

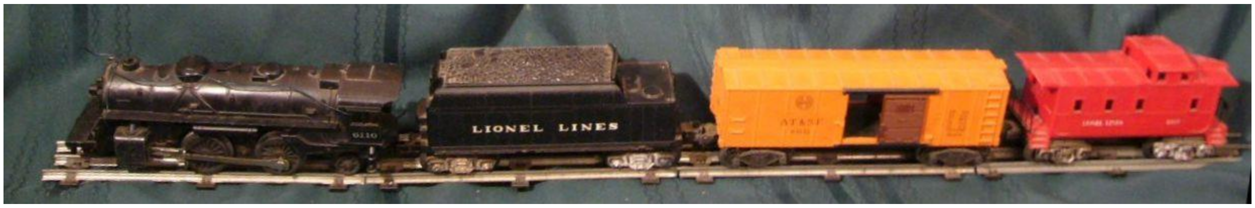


Figure 1 - We begin to understand the principles of physics long before we know it by that name.

Trains, even model trains, suffer from a high center of gravity and a narrow footprint, making curves uncomfortable for passengers and even unstable for the train in extreme cases.

Centrifugal force is a function of both train speed and track curvature. If trains operated at a low velocity or on a straight track, centrifugal force would not factor into a railway's engineering, but high speeds and curved track require an engineered solution. That solution is **"superelevation,"** which is also known interchangeably as **"cant."**

II. Superelevation (Cant)

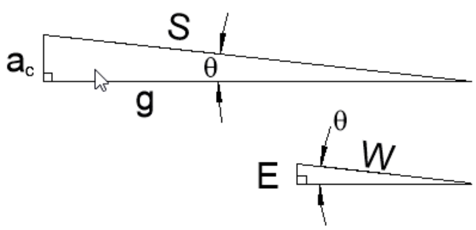
When a train passes through a curve in its alignment, centrifugal force acts on the train, the passengers, and the contents. The faster the train and the sharper the turn, the more pronounced the effect and the more significant the impact on passenger comfort and train stability. We use superelevation to counteract this effect by elevating the outside rail sufficiently to cause each rail to carry approximately the same wheel load. When the wheel loads are identical, we call the turn "balanced." Imagine a pendulum hung inside a level and motionless train car. The pendulum will hang vertically and perpendicular to the floor in that state but rounding a curve without superelevation will cause the pendulum to swing toward the outside of the curve. If we then tilt the car by raising the outer rail, we can bring the pendulum back perpendicular to the floor to create a balanced turn.

Figure 2 - A spirit level used to measure track superelevation (showing 5")

If we know the radius of the curve and the train's velocity, we can quantify the amount of superelevation needed for a balanced turn. We'll start with the formula for centrifugal acceleration, (a_c):

$$(1) \qquad a_c = \frac{v^2}{r}$$

When we examine the combined effects of the acceleration of gravity (g) with centrifugal acceleration (see Figure 3), we can calculate the amount to elevate the outside rail (Cant) to achieve a balanced turn using these similar right triangles.



$$(2) \qquad \theta = tan^{-1}\left(\frac{a_c}{g}\right)$$

$$(3) \qquad E = \mathrm{W} * \sin(\theta)$$

$$(4) \qquad E = W * sin\left(tan^{-1}\left(\frac{v^2}{r*g}\right)\right)$$

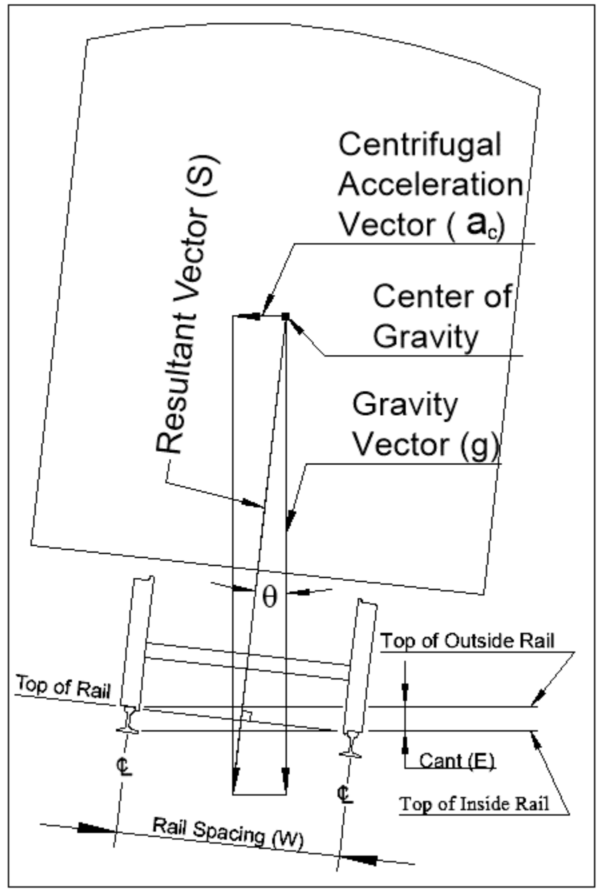


Figure 3 - The combined effects of gravity and centrifugal force

Where:	U.S. Customary Units	Metric
a_c =	Centrifugal acceleration (feet/sec^2)	Centrifugal acceleration (meters /sec^2)
v =	Velocity (feet/second)	Velocity (meters /second)
r =	Radius (feet)	Radius (meters)
E=	Superelevation (cant) (inches)	Superelevation (cant) (millimeters)
W=	Centerline spacing of the rails (inches)	Centerline spacing of the rails (millimeters)
g =	Acceleration of gravity (32.17405 feet/sec^2)	Acceleration of gravity (980.6650 cm/sec^2)

Cant Deficiency

When a train's speed does not match the balanced velocity of a superelevated curve, the pendulum used in the previous section will swing away from the center of the curve for speeds faster than balanced and just the opposite when the train is slower. This imbalance creates a condition called "cant deficiency." These conditions are generally unavoidable except on a dedicated line such as a transit system or dedicated high-speed rail line. When mixed traffic, operating at different speeds, use the same track, a cant deficiency will be present. This imbalance introduces a lateral acceleration that contributes to passenger discomfort, wheel/rail abrasion, and the accompanying noise called "flanging" when wheel flanges scrape against the rail.

NOTE

Trains operating below balanced speed are technically operating at “cant excess”. For simplicity, we will use negative cant deficiency to describe cant excess.

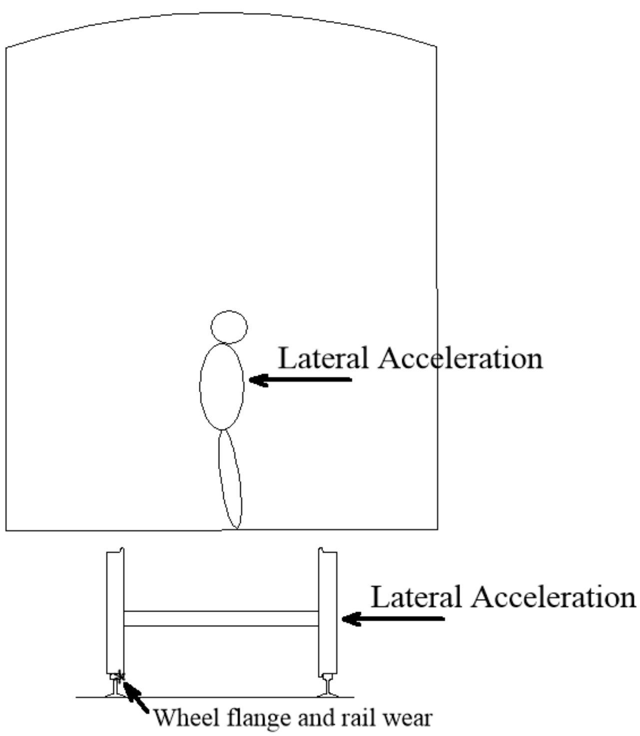


Figure 4 - Cant Deficiency introduces lateral Acceleration.

We calculate cant deficiency in much the same way as the superelevation for a balanced curve by finding the amount of superelevation required for the combination of train speed and radius and subtracting the actual cant. We can then rewrite equation (4) as:

$$(5)\qquad E \pm d = W * sin\left(tan^{-1}\left(\frac{v^2}{r*g}\right)\right)$$

Where:	U.S. Customary Units	Metric
v =	Velocity (feet/second)	Velocity (meters /second)
r =	Radius (feet)	Radius (meters)
E=	Superelevation (cant) (inches)	Superelevation (cant) (millimeters)
d=	Cant deficiency (inches)	Cant deficiency (millimeters)
W=	Centerline spacing of the rails (inches)	Centerline spacing of the rails (millimeters)
g =	Acceleration of gravity (32.17405 feet/sec^2)	Acceleration of gravity (980.6650 cm/sec^2)

See Appendix B for a complete listing of the principles of physics equations used in this course and the companion "Rail-Curve" software.

Gauge and Rail Spacing

The dimension "W" is the centerline to centerline distance between rails, so it is the gauge dimension plus the width of the rail head (half the rail head width on each side). Rail head widths vary from 2 $^{11}/_{16}$" for a 100-lb AREMA rail to 3 $^{1}/_{16}$" for a 141-lb AREMA rail. It gets more complicated because rail gauge is not measured at the widest point of the rail head but a point $^{5}/_{8}$" below the top of the rail. You can view detailed cross-sections of each of these U.S. rails at http://www.lbfoster-railproducts.com/newrail.asp

Don't get too caught up in trying to define the wheel spacing to a machinist's level of precision; here's why. In the U.S. and much of the world, "standard-gauge" for railroads is 4’-8½" (1524-mm). It's an odd dimension that some say dates to the Roman chariot wheel spacing. Regardless of its origin, it is commonly thought of as a single dimension when, in fact, it can range from 4’-8¼" to 4’-9¼" for the highest track classes and 4’-8" to 4’-10" for lower track classes.

This 1-2-inch tolerance in gauge width means it is unnecessary to fret over the small fractions of an inch of railhead width differences. We will be using 3" for all of our standard-gauge examples and test questions which gives us W = 59.5-inches.

U.S. Railroads

Engineers in the U.S. have long used a simplified formula to approximate the calculation on the previous pages. This slide-rule-friendly, legacy formula predates modern computers and calculators and simplifies calculations.

$$(6) \qquad E = 0.0007 * D * v^2$$

Where:

E = Cant (inches)
D = Degree of curvature (chord definition)
v = Train velocity (miles-per-hour)

A variation of this formula is adopted law in the U.S. Code of Federal Regulations, Title 49, Chapter II, Federal Railroad Administration, Department of Transportation, Part 213.

$$(7) \qquad V_{max} = \sqrt{\frac{E_a + E_u}{0.0007 * D}}$$

Where:	U.S. Customary Units
V_{max} =	Maximum unbalanced velocity (miles per hour)
E_a =	Actual superelevation (cant) (inches)
E_u=	Cant deficiency (inches)
D=	Degree of curvature (chord definition)

Notice that formulas (6) and (7) do not have a variable for rail spacing. That's because an approximation is built into the coefficient. The formula assumes that the horizontal component of the rail spacing is a constant 4.9-ft for cants up to 8".

See Appendix C for a complete listing of the U.S. Federal Railroad Administration equations used in this course and the companion "Rail-Curve" software.

The Comparison

The difference between the two methods is slight as illustrated in this table copied from the SunCam Rail-Curve software that accompanies this course. This table was modeled after the series of tables in Appendix A of the U.S. Code of Federal Regulations, Title 49, Chapter II, Federal Railroad Administration, Department of Transportation, Part 213. In the software, you will select and input *your* values for unbalance and rail spacing.

Variables	
4	Unbalance (in)
59.5	Rail Spacing (in)

A comparison of methods for calculating maximum allowable operating speed **(F.R.A. vs. Physics) (M.P.H.)**

Curve		Elevation of outer rail (inches)												
		0	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**	**5.5**	**6**
0°30'	FRA	107	113	120	125	131	136	141	146	151	156	160	165	169
	Physics	107	114	120	126	132	137	142	148	152	157	162	166	171
0°40'	FRA	93	98	104	109	113	118	122	127	131	135	139	143	146
	Physics	93	99	104	109	114	119	123	128	132	136	140	144	148
0°50'	FRA	83	88	93	97	101	106	110	113	117	121	124	128	131
	Physics	83	88	93	98	102	106	110	114	118	122	125	129	132
1°0'	FRA	76	80	85	89	93	96	100	104	107	110	113	116	120
	Physics	76	81	85	89	93	97	101	104	108	111	115	118	121
1°15'	FRA	68	72	76	79	83	86	89	93	96	99	101	104	107
	Physics	68	72	76	80	83	87	90	93	96	99	102	105	108
Portions omitted. For the complete table, use the "Comparison" tab on the Rail-Curve software.														
6°0'	FRA	31	33	35	36	38	39	41	42	44	45	46	48	49
	Physics	31	33	35	36	38	40	41	43	44	45	47	48	49
6°30'	FRA	30	31	33	35	36	38	39	41	42	43	44	46	47
	Physics	30	32	33	35	37	38	40	41	42	44	45	46	47
7°0'	FRA	29	30	32	34	35	36	38	39	40	42	43	44	45
	Physics	29	30	32	34	35	37	38	39	41	42	43	45	46
8°0'	FRA	27	28	30	31	33	34	35	37	38	39	40	41	42
	Physics	27	29	30	32	33	34	36	37	38	39	41	42	43
9°0'	FRA	25	27	28	30	31	32	33	35	36	37	38	39	40
	Physics	25	27	28	30	31	32	34	35	36	37	38	39	40
10°0'	FRA	24	25	27	28	29	30	32	33	34	35	36	37	38
	Physics	24	26	27	28	29	31	32	33	34	35	36	37	38
11°0'	FRA	23	24	25	27	28	29	30	31	32	33	34	35	36
	Physics	23	24	26	27	28	29	30	31	33	34	35	36	36
12°0'	FRA	22	23	24	26	27	28	29	30	31	32	33	34	35
	Physics	22	23	25	26	27	28	29	30	31	32	33	34	35

Table 1- Showing the slight differences between the two methods described in the previous pages.

Superelevation Software

The course materials include a download for the spreadsheet software that accompanies this course. "Rail-Curve" will handle most of the number crunching for the class so that we can concentrate on the principles of railroad curve design. We will be using the "Superelevation" section of the software for this part of the discussion. We will cover spiral transitions in the second half of this course.

Software Precautions

A professional engineer should have a healthy skepticism about all software outputs, even the software that you write yourself. Most state engineering boards have adopted rules to guide engineers like the Florida rule which states:

"61G15-30.008 Use of Computer Software and Hardware.

> *The engineer shall be responsible for the results generated by any computer software and hardware that he or she uses in providing engineering services.*"

At SunCam, we recommend the following best practices:

1. Test new software with known sets of data.
2. Always guess the outcome before you do the calculations (this is recommended for all calculations, not just software).
3. Ask yourself if the answer "looks right". (Also recommended for all calculations.)
4. When any of these cast doubt on the software output use hand calculations or alternative software to crosscheck and verify results.

Sample problem #1

Sample problem #1 illustrates rail curve selection in its most common form. The inputs are cant, cant deficiency, and curvature using the F.R.A. formula. The outputs are minimum, balanced, and maximum train speed and centrifugal acceleration values for each. Any train traversing this curve at a speed below the minimum or above the maximum will exceed the cant deficiency criteria. Such an imbalance could create an excessive wheel load on one of the rails and discomfort in the form of lateral acceleration for passengers. In this case, the centrifugal acceleration value measured parallel to the floor of the carbody is just 0.05g, well within the 0.15g allowed by F.R.A. regulations[1].

SunCam Rail-Curve v 1.0.0
Copyright© 2018 William C. Dunn

◉ FRA formula [E=0.0007DV²]
○ Principles of Physics

www.suncam.com

Clear Inputs

Superelevation

Function	Input			Output			Units
	V_{Min}	V_{Bal}	V_{Max}	V_{Min}	V_{Bal}	V_{Max}	
Train Speed (V)				53.45	75.59	92.58	mph
				78.40	110.87	135.79	fps
				86.02	121.65	149.00	kph
				23.90	33.79	41.39	mps
Centrifugal Acceleration (horizontal) (a_c)		(no input)		1.61	3.22	4.83	ft/sec²
				0.49	0.98	1.47	m/sec²
				0.050	0.100	0.150	g
Perpendicular ⊤ to floor		(no input)			1.005		g
Parallel ‖ with floor (Jerk)		(no input)		-0.050	0.000	0.050	g
Cant (E)		6.00			6.00		in
					152.40		mm
					5.74		angle°
Cant Deficiency (d)		3.00			3.00		in
					76.20		mm
Curvature (rr) simple curve					3,819.83		radius - ft
					0.72		radius - miles
					1,164.28		radius - m
					1.50000		° chord
	1	30		1	30	0.00	° chord (dms)
					1.49996		° arc
				1	29	59.85	° arc (dms)
Rail spacing (W) (NOT Gauge)					60.00		60-in. Fixed
					1524		mm

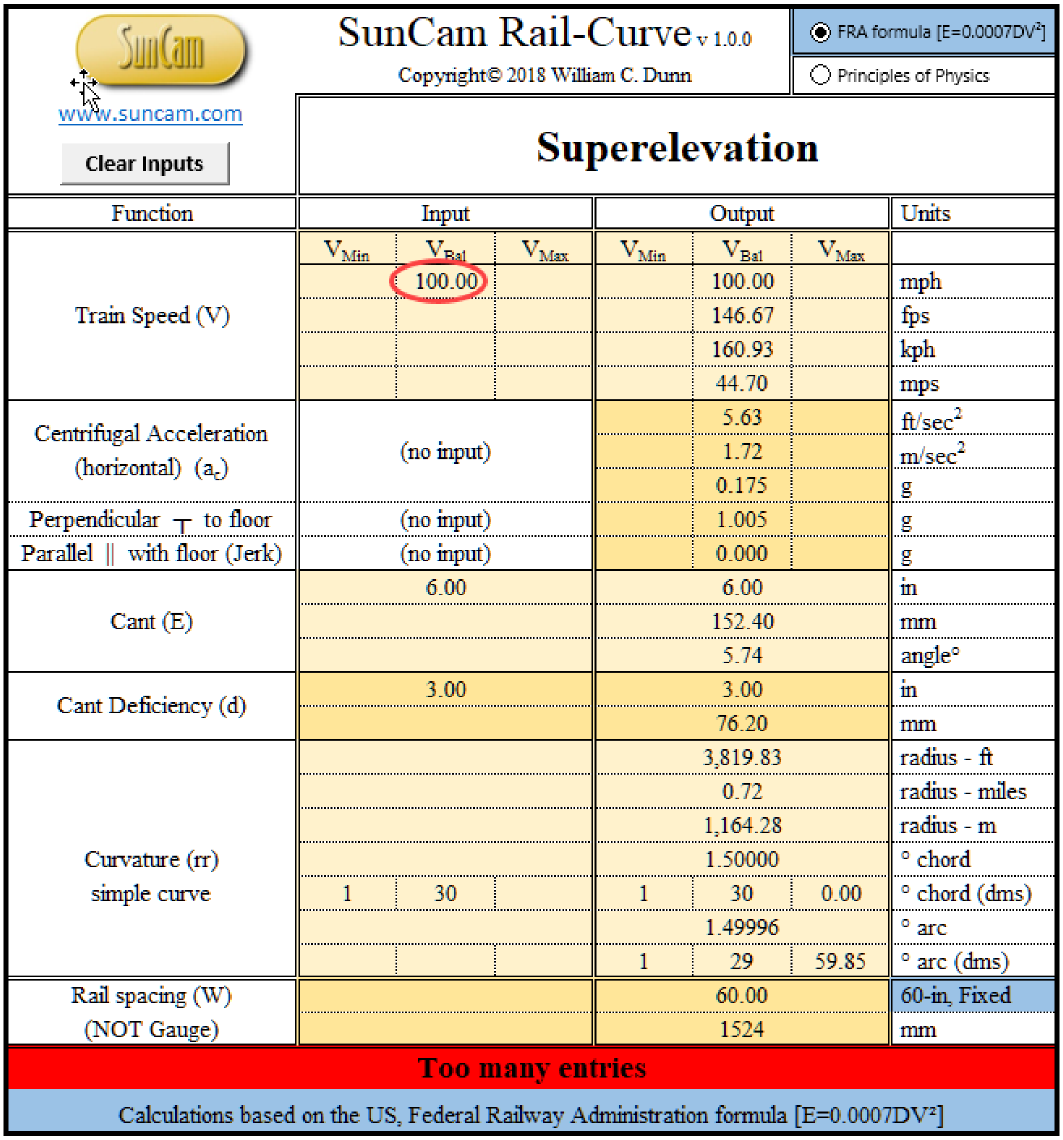
Calculations based on the US, Federal Railway Administration formula [E=0.0007DV²]

Figure 5 - Screenshot of "Rail-Curve" for sample problem #1

Note that for F.R.A. formula solutions, the rail spacing dimension "W" is fixed at 60-inches. This dimension is used only for the calculation of the cant angle.

[1] (FRA, Federal Railroad Administration, 2017)

Warnings and prompts


SunCam Rail-Curve v 1.0.0
Copyright© 2018 William C. Dunn
www.suncam.com
Clear Inputs
FRA formula [E=0.0007DV²]
Principles of Physics

Superelevation

Function	Input			Output			Units
	V_{Min}	V_{Bal}	V_{Max}	V_{Min}	V_{Bal}	V_{Max}	
Train Speed (V)		100.00			100.00		mph
					146.67		fps
					160.93		kph
					44.70		mps
Centrifugal Acceleration (horizontal) (a_c)	(no input)				5.63		ft/sec²
					1.72		m/sec²
					0.175		g
Perpendicular ┬ to floor	(no input)				1.005		g
Parallel ‖ with floor (Jerk)	(no input)				0.000		g
Cant (E)	6.00			6.00			in
				152.40			mm
				5.74			angle°
Cant Deficiency (d)	3.00			3.00			in
				76.20			mm
Curvature (rr) simple curve				3,819.83			radius - ft
				0.72			radius - miles
				1,164.28			radius - m
				1.50000			° chord
	1	30		1	30	0.00	° chord (dms)
				1.49996			° arc
				1	29	59.85	° arc (dms)
Rail spacing (W) (NOT Gauge)				60.00			60-in, Fixed
				1524			mm

Too many entries

Calculations based on the US, Federal Railway Administration formula [E=0.0007DV²]

Figure 6 - Warning Message

Warnings and prompts will appear to guide you in the use of the software. Here we have added a train speed to the entries in sample problem #1, making the calculation "Overdetermined."

The list of warnings and messages that you may see include:

- Balanced train speed cannot be greater than maximum train speed
- Minimum train speed cannot be greater than balanced train speed
- Minimum train speed cannot be greater than maximum train speed
- The value for 'E' is derived using a cascading look-up table
- Warning! Too many train speed entries
- Add cant or curvature
- Add train speed or curvature
- Add train speed or cant deficiency
- Warning! Cant deficiency for min & max speeds will be different values.
- Too many entries
- Enter a value for Rail Spacing (W)
- Need more data

Sample problem #2

Australian engineer Matilda Harper is planning a passenger rail line from Perth to Port Hedland. A portion of the route will share a track with the privately-owned B.H.P. freight line that hauls iron ore from the Yandi mine to the port in 7-km long trains with the heaviest wheel loads of any railroad in the world. Through most of its length, the freight line design is for a balanced speed of 80-kph, and the ore trains operate at that speed to avoid damage from unbalanced heavy wheel loads. The design criteria for the much lighter passenger train is 120-kph with a maximum cant deficiency of 90-mm.

In a meeting with Matilda, a B.H.P. engineer points out a 1½ degree curve with 65 mm of cant as a typical curve of concern. Matilda immediately produced the following Rail-Curve results and responded, "We can operate with 82 mm of cant deficiency and achieve our speed of 120-kph." By the end of the meeting, Matilda had run similar calculations for every curve in the B.H.P. line.

SunCam Rail-Curve v 1.0.0

Copyright© 2018 William C. Dunn

www.suncam.com

Clear Inputs

○ FRA formula [$E=0.0007DV^2$]

◉ Principles of Physics

Superelevation

Function	Input			Output			Units
	V_{Min}	V_{Bal}	V_{Max}	V_{Min}	V_{Bal}	V_{Max}	
Train Speed (V)				No Min.	49.45	74.56	mph
					72.53	109.36	fps
			120.00		79.58	120.00	kph
					22.11	33.33	mps
Centrifugal Acceleration (horizontal) (a_c)	(no input)				1.38	3.13	ft/sec^2
					0.42	0.95	m/sec^2
					0.043	0.097	g
Perpendicular ┬ to floor	(no input)				1.001		g
Parallel ‖ with floor (Jerk)	(no input)				0.000	0.054	g
Cant (E)				2.56			in
	65.00			65.00			mm
				2.45			angle°
Cant Deficiency (d)				3.24			in
				82.22			mm
Curvature (rr) simple curve				3,819.83			radius - ft
				0.72			radius - miles
				1,164.28			radius - m
				1.50000			° chord
	1	30		1	30	0.00	° chord (dms)
				1.49996			° arc
				1	29	59.85	° arc (dms)
Rail spacing (W) (NOT Gauge)				59.84			in
	1520.00			1520			mm

Calculations based on the principles of physics.

Figure 7 - Matilda Harper's proposal

Sample problem #3

Karl Nilsson is the chief planner for a new Swedish passenger rail line with two service classes, a high-speed train operating at 250-kph and a commuter line running at 120-kph. Karl wants to choose minimum curve criteria that will satisfy both train speeds and stay below a maximum cant deficiency of 90-mm. He enters his criteria into Rail-Curve with the following results.

SunCam Rail-Curve v 1.0.0
Copyright© 2018 William C. Dunn

○ FRA formula [E=0.0007DV²]
◉ Principles of Physics

www.suncam.com

Clear Inputs

Superelevation

Function	Input			Output			Units
	V_{Min}	V_{Bal}	V_{Max}	V_{Min}	V_{Bal}	V_{Max}	
Train Speed (V)				74.56	121.52	155.34	mph
				109.36	178.22	227.84	fps
	120.00		250.00	120.00	195.56	250.00	kph
				33.33	54.32	69.44	mps
Centrifugal Acceleration (horizontal) (a_c)	(no input)			1.16	3.08	5.03	ft/sec²
				0.35	0.94	1.53	m/sec²
				0.036	0.096	0.156	g
Perpendicular ┬ to floor	(no input)				1.005		g
Parallel ‖ with floor (Jerk)	(no input)			-0.059	0.000	0.060	g
Cant (E)				5.70			in
				144.69			mm
				5.46			angle°
Cant Deficiency (d)				3.54			in
	90.00			90.00			mm
Curvature (rr) simple curve				10,323.82			radius - ft
				1.96			radius - miles
				3,146.70			radius - m
				0.55499			° chord
				0	33	17.96	° chord (dms)
				0.55499			° arc
				0	33	17.95	° arc (dms)
Rail spacing (W) (NOT Gauge)				59.84			in
	1520.00			1520			mm

The value for 'E' is derived using a cascading lookup table (see tab below)

Calculations based on the principles of physics.

Figure 8 - Find the curve that meets one cant deficiency and two train speed requirements

Karl is satisfied with the results and sets 30-minutes as the minimum curve. He reruns Rail-Curve to verify his decision with the following results.

SunCam Rail-Curve v 1.0.0
Copyright© 2018 William C. Dunn
www.suncam.com
Clear Inputs
○ FRA formula [$E=0.0007DV^2$]
◉ Principles of Physics

Superelevation

Function	Input V_{Min}	Input V_{Bal}	Input V_{Max}	Output V_{Min}	Output V_{Bal}	Output V_{Max}	Units
Train Speed (V)				74.56	121.58	155.34	mph
				109.36	178.31	227.84	fps
	120.00		250.00	120.00	195.66	250.00	kph
				33.33	54.35	69.44	mps
Centrifugal Acceleration (horizontal) (a_c)	(no input)			1.04	2.77	4.53	ft/sec^2
				0.32	0.85	1.38	m/sec^2
				0.032	0.086	0.141	g
Perpendicular ┬ to floor	(no input)				1.004		g
Parallel ‖ with floor (Jerk)	(no input)			-0.054	0.000	0.054	g
Cant (E)					5.14		in
					130.60		mm
					4.93		angle°
Cant Deficiency (d)					3.20		in
					81.32		mm
Curvature (rr) simple curve					11,459.19		radius - ft
					2.17		radius - miles
					3,492.76		radius - m
					0.50000		° chord
		30		0	30	0.00	° chord (dms)
					0.50000		° arc
				0	29	59.99	° arc (dms)
Rail spacing (W) (NOT Gauge)					59.84		in
		1520.00			1520		mm

Calculations based on the principles of physics.

Figure 9 - Verifying the selected curve criteria

Karl rounds the cant to 130-mm and tests it against the 250-kph train speed with the following final results.

SunCam Rail-Curve v 1.0.0
Copyright© 2018 William C. Dunn

○ FRA formula [$E=0.0007DV^2$]
◉ Principles of Physics

www.suncam.com

Clear Inputs

Superelevation

Function	Input			Output			Units
	V_{Min}	V_{Bal}	V_{Max}	V_{Min}	V_{Bal}	V_{Max}	
Train Speed (V)				73.65	121.30	155.34	mph
				108.02	177.90	227.84	fps
			250.00	118.53	195.21	250.00	kph
				32.93	54.22	69.44	mps
Centrifugal Acceleration (horizontal) (a_c)		(no input)		1.02	2.76	4.53	ft/sec²
				0.31	0.84	1.38	m/sec²
				0.032	0.086	0.141	g
Perpendicular ┬ to floor		(no input)			1.004		g
Parallel ‖ with floor (Jerk)		(no input)		-0.054	0.000	0.055	g
Cant (E)					5.12		in
		130.00			130.00		mm
					4.91		angle°
Cant Deficiency (d)					3.23		in
					81.92		mm
Curvature (rr) simple curve					11,459.19		radius - ft
					2.17		radius - miles
					3,492.76		radius - m
					0.50000		° chord
		30		0	30	0.00	° chord (dms)
					0.50000		° arc
				0	29	59.99	° arc (dms)
Rail spacing (W) (NOT Gauge)					59.84		in
		1520.00			1520		mm

Calculations based on the principles of physics.

Figure 10 - Final Results

Sample problem #4

Brody Quinn, a railroad civil engineer, is tapped to select a new route for a high-speed rail line. Much of the new line will run through cornfields where the alignment is not constrained. Brody knows that the new train will have a design speed of 155-mph, but he also knows that the right-of-way that he selects will be around long after the 155-mph train is retired and forgotten. He wants to choose a route that will be as straight as possible to accommodate the next generation of high-speed travel. First, he examines the curve criteria for the 155-mph train.

SunCam Rail-Curve v 1.0.0
Copyright© 2018 William C. Dunn

◉ FRA formula [E=0.0007DV²]
○ Principles of Physics

www.suncam.com

Clear Inputs

Superelevation

Function	Input			Output			Units
	V_{Min}	V_{Bal}	V_{Max}	V_{Min}	V_{Bal}	V_{Max}	
Train Speed (V)			155.00	89.49	126.56	155.00	mph
				131.25	185.62	227.33	fps
				144.02	203.67	249.45	kph
				40.01	56.58	69.29	mps
Centrifugal Acceleration (horizontal) (a_c)	(no input)			1.61	3.22	4.83	ft/sec²
				0.49	0.98	1.47	m/sec²
				0.050	0.100	0.150	g
Perpendicular ┬ to floor	(no input)				1.005		g
Parallel ‖ with floor (Jerk)	(no input)			-0.050	0.000	0.050	g
Cant (E)	6.00			6.00			in
				152.40			mm
				5.74			angle°
Cant Deficiency (d)	3.00			3.00			in
				76.20			mm
Curvature (rr) simple curve				10,706.39			radius - ft
				2.03			radius - miles
				3,263.31			radius - m
				0.53516			° chord
				0	32	6.56	° chord (dms)
				0.53515			° arc
				0	32	6.56	° arc (dms)
Rail spacing (W) (NOT Gauge)				60.00			60-in, Fixed
				1524			mm

Calculations based on the US, Federal Railway Administration formula [E=0.0007DV²]

Figure 11 - Curve criteria for a 155-mph train

Next, he uses Rail-Curve to help him select the curve criteria for a 350-mph mag-lev train. Although rail spacing is meaningless for mag-lev trains, it is a useful proxy for cant angle. Keeping the same cant and cant deficiency results in a curve of slightly more than 6-minutes.

SunCam Rail-Curve v 1.0.0

Copyright© 2018 William C. Dunn

◉ FRA formula [$E=0.0007DV^2$]
○ Principles of Physics

www.suncam.com

Clear Inputs

Superelevation

Function	Input			Output			Units
	V_{Min}	V_{Bal}	V_{Max}	V_{Min}	V_{Bal}	V_{Max}	
Train Speed (V)			350.00	202.07	285.77	350.00	mph
				296.37	419.13	513.33	fps
				325.20	459.91	563.27	kph
				90.33	127.75	156.46	mps
Centrifugal Acceleration (horizontal) (a_c)	(no input)			1.61	3.22	4.83	ft/sec^2
				0.49	0.98	1.47	m/sec^2
				0.050	0.100	0.150	g
Perpendicular ┬ to floor	(no input)				1.005		g
Parallel ‖ with floor (Jerk)	(no input)			-0.050	0.000	0.050	g
Cant (E)	6.00			6.00			in
				152.40			mm
				5.74			angle°
Cant Deficiency (d)	3.00			3.00			in
				76.20			mm
Curvature (rr) simple curve				54,590.15			radius - ft
				10.34			radius - miles
				16,639.08			radius - m
				0.10496			° chord
				0	6	17.84	° chord (dms)
				0.10496			° arc
				0	6	17.84	° arc (dms)
Rail spacing (W) (NOT Gauge)				60.00			60-in, Fixed
				1524			mm

Calculations based on the US, Federal Railway Administration formula [$E=0.0007DV^2$]

Figure 12 - Curve criteria for a 350-mph mag-lev train

Brody's final minimum criteria is a 6-minute curve.

SunCam Rail-Curve v 1.0.0

Copyright© 2018 William C. Dunn

◉ FRA formula [E=0.0007DV²]
○ Principles of Physics

www.suncam.com

Clear Inputs

Superelevation

Function	Input V_{Min}	Input V_{Bal}	Input V_{Max}	Output V_{Min}	Output V_{Bal}	Output V_{Max}	Units
Train Speed (V)			350.00	221.20	292.77	350.00	mph
				324.42	429.40	513.33	fps
				355.98	471.17	563.27	kph
				98.88	130.88	156.46	mps
Centrifugal Acceleration (horizontal) (a_c)		(no input)		1.84	3.22	4.60	ft/sec²
				0.56	0.98	1.40	m/sec²
				0.057	0.100	0.143	g
Perpendicular ⊤ to floor		(no input)			1.005		g
Parallel ‖ with floor (Jerk)		(no input)		-0.043	0.000	0.043	g
Cant (E)		6.00			6.00		in
					152.40		mm
					5.74		angle°
Cant Deficiency (d)					2.58		in
					65.41		mm
Curvature (rr) simple curve					57,295.79		radius - ft
					10.85		radius - miles
					17,463.76		radius - m
					0.10000		° chord
		6		0	6	0.00	° chord (dms)
					0.10000		° arc
				0	5	60.00	° arc (dms)
Rail spacing (W) (NOT Gauge)					60.00		60-in. Fixed
					1524		mm

Calculations based on the US. Federal Railway Administration formula [E=0.0007DV²]

Figure 13 - Final results after rounding

III. Spiral Transition Curves

Except at very low speeds, a sudden change from straight track to curved would be damaging to equipment and uncomfortable to passengers. We use spiral transitions, also known as easement curves, to gradually change the curvature in the alignment simultaneously with the introduction of superelevation. (See figure 14 for a legend of spiral curve terms and dimensions.)

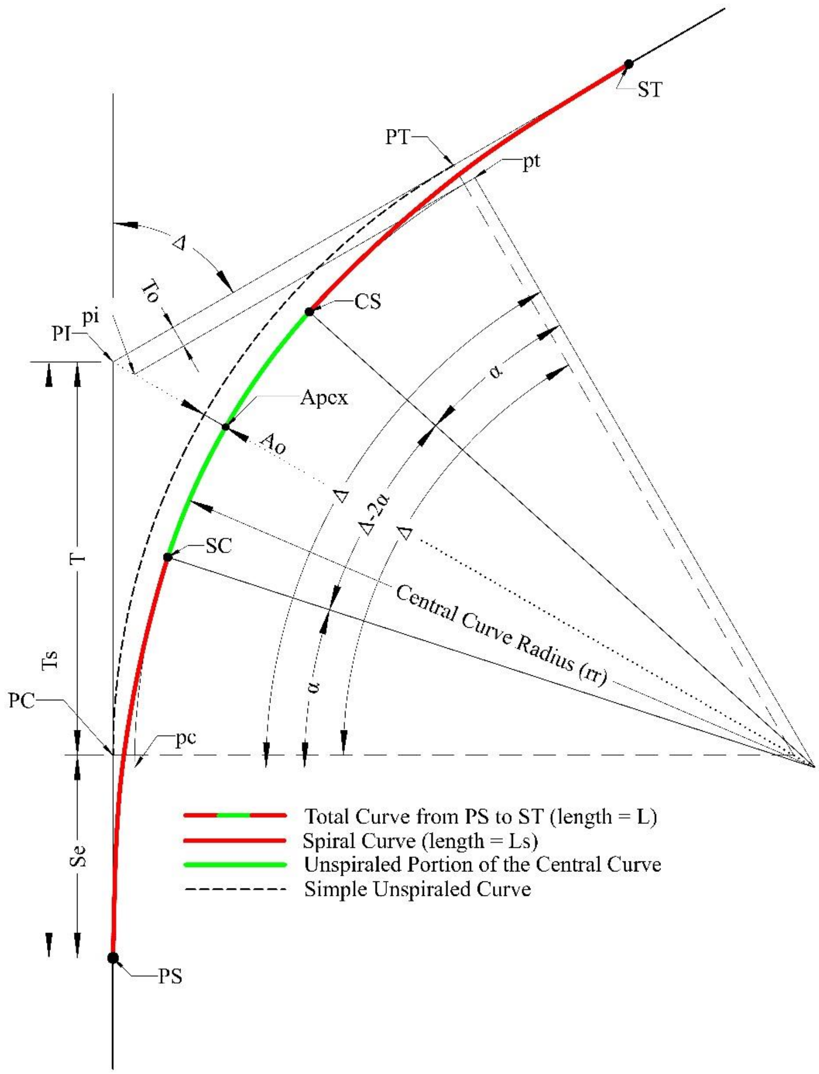


Figure 14 – The Spiral Curve Legend

Beginning at the P.S. or S.T., where the train enters the spiral, the curvature is zero. It gradually increases until it matches the central circular curve's radius at the end of the spiral (S.C. or C.S.). This gradual change in curvature matches the gradual increase in centrifugal acceleration and the superelevation that resists it. In an easement curve, the degree of curvature increases linearly along the length of the spiral. Thus, a 500-foot long spiral leading to a central curve of 5º and with 5-inches of cant would increase by 1º of curvature and 1-inch of superelevation for every 100-feet along its length. The "Run-in-Rate" would be 1.0-inches per second (explained below.)

Spiral Transition Software

We'll use the screenshot from Sample Problem #1 to begin the discussion of the Spiral Transition portion of the Rail-Curve software:

SunCam Rail-Curve v 1.0.0
Copyright© 2018 William C. Dunn
FRA formula [E=0.0007DV²]
Principles of Physics
www.suncam.com
Clear Inputs

Superelevation

Function	Input V_{Min}	Input V_{Bal}	Input V_{Max}	Output V_{Min}	Output V_{Bal}	Output V_{Max}	Units
Train Speed (V)				53.45	75.59	92.58	mph
				78.40	110.87	135.79	fps
				86.02	121.65	149.00	kph
				23.90	33.79	41.39	mps
Centrifugal Acceleration (horizontal) (a_c)		(no input)		1.61	3.22	4.83	ft/sec²
				0.49	0.98	1.47	m/sec²
				0.050	0.100	0.150	g
Perpendicular ┬ to floor		(no input)			1.005		g
Parallel ‖ with floor (Jerk)		(no input)		-0.050	0.000	0.050	g
Cant (E)		6.00			6.00		in
					152.40		mm
					5.74		angle°
Cant Deficiency (d)		3.00			3.00		in
					76.20		mm
Curvature (rr) simple curve					3,819.83		radius - ft
					0.72		radius - miles
					1,164.28		radius - m
					1.50000		° chord
	1	30		1	30	0.00	° chord (dms)
					1.49996		° arc
				1	29	59.85	° arc (dms)
Rail spacing (W) (NOT Gauge)					60.00		60-in, Fixed
					1524		mm

Calculations based on the US, Federal Railway Administration formula [E=0.0007DV²]

Figure 15- Screenshot from Sample Problem #1

We add a delta angle and any one of the remaining three variables in the "Spiral Transition" portion of the software. In this case, we'll use the run-in-rate. This gives us a full slate of values for the other variables, which we will explain below.

SunCam Rail-Curve v 1.0.0

Copyright© 2018 William C. Dunn

www.suncam.com

Clear Inputs

Spiral Transition

Function	Input	Output	Units
Spiral Run-in-Rate (rir)	1.125	1.125	in/sec
		28.6	mm/sec
	(no input)	0.0093	g/sec
Spiral Time (t)(@Vmax)		5.33	sec
Spiral Length (L_S)		724.20	ft
		220.74	m
Delta Angle (Δ)		15.00000	°
	15	15 0 0.00	° (dms)
Alpha angle (α)	(no input)	5.43148	°
		5 25 53.32	° (dms)
Central Curve Angle (Δ-2α)	(no input)	4.13704	°
		4 8 13.36	° (dms)
Central Curve Length (L_C)	(no input)	275.81	ft
		84.07	m
Total Curve Length (L)	(no input)	1,724.20	ft
		525.54	m
Spiral Tangent (T_s)	(no input)	865.63	ft
		263.84	m
Spiral Tangent Extension (S_s)	(no input)	362.74	ft
		110.56	m
Spiral Tangent Offset (T_o)	(no input)	5.72	ft
		1.74	m
Apex Offset (A_o)	(no input)	5.77	ft
		1.76	m

Figure 16 - Spiral Transition for sample problem #1

Run-in-Rate (rir)

The rate of change in superelevation (rir) as a train traverses an easement curve. We measure Run-in-Rate in inches/millimeters per second. (Hay, 1982)[2] suggests a run-in-rate of:

- 1¼ inches/sec for train speeds up to 60 mph
- $1^1/_6$ inches/sec for train speeds 60-80 mph
- $1^1/_8$ inches/sec for train speeds 80-100 mph

(8) $$rir = \frac{E}{t}$$

Where:	U.S. Customary Units	Metric
E =	Superelevation (cant) (inches)	Superelevation (cant) (millimeters)
t =	Spiral time (seconds) explained below	Spiral time (seconds) explained below

Delta Angle (Δ)

The delta angle is the deflection angle of the simple curve tangents and the central angle of the simple circular curve. The displaced circular curve mimics this curve and therefore has the same deflection angle, central angle, and radius. Delta angle is always an input value in Rail-Curve.

Spiral Length (L.S.)

As the name implies, spiral length is the length, as measured along the curve, of the spiral transition (P.S. to S.C. and S.T. to C.S., as shown in red in figure 14). Calculate the length of the spiral as follows:

(9) $$L_S = E * \frac{V_{max}}{rir}$$

(10) $$L_S = V_{max} * t$$

Space constraints may sometimes require shorter than desirable spirals; however, this could result in warping or twisting of the car body. There should be no more than a 1-inch difference between the front and rear diagonal corners of a car to prevent car body racking. To satisfy this requirement, the minimum length of the spiral must be: (Hay, 1982)[2]

(11) $$L_s = 62E$$

[2]Hay, W. H. (1982). Mgt., E., M.S., PhD. In Railroad Engineering, Second Edition (p. 606). Urbana, Illinois, U.S.: John Wiley and Sons.

If this minimum length requirement is not met, a warning message will appear in the Spiral Transition worksheet's footer.

WARNING! Short spiral length may cause diagonal warping of car bodies

Spiral Time (t)

The time required for a train to travel the length of the spiral is called spiral time (t), and it may be calculated as follows:

(12) $t = \frac{E}{rir}$

(13) $t = \frac{L_S}{V_{max}}$

Cant (E)

As discussed at the beginning of this course, cant is the amount that the outside rail is raised to resist centrifugal acceleration in a curve. Cant has the following relationship with run-in-rate and time.

(14) $E = rir * t$

Maximum Velocity (V_{max})

Maximum train speed is usually calculated using curvature, cant, and cant deficiency, but it also has a mathematical relationship with spiral length and spiral time.

(15) $V_{max} = \frac{L_S}{t}$

Spiral Tangent Offset (T_o)

In Figure 14, the central curve (green) is offset from the simple unspiraled curve (dashed black) to make room for the spirals. The two curves are not concentric so, the offset is not uniform throughout, but the offset of the tangent lines (T_o) is a constant. The AREMA[3] formula is:

[3] (AREMA, 2018)

$$(16)\quad T_o = 0.1454 * \alpha * S$$

HAY[4] achieves identical results with the following formula:

$$(17)\quad T_o = 0.0727 * k * S^3$$

Where:
S = length L_s in 100-foot stations
k = increase in the degree of curvature per 100-foot station along the spiral

A more precise formula and the one that we use in the "Rail-Curve" software is:

$$(18)\quad T_o = \frac{{L_S}^2}{24 * rr}$$

Apex Offset (A_o)
Borrowing a term from auto racing, we will call the midpoint of the unspiraled portion of the central curve the "Apex." The offset from the simple curve is the "Apex Offset." The Apex offset will always be slightly larger than the tangent spiral offset (T_o), and it will always be the point of greatest separation between the two curves.

$$(19)\quad T_0 = \frac{S_O}{cos\left(\frac{\Delta}{2}\right)}$$

Alpha angle (α)
The alpha angle is simply the portion of the central curve replaced by the spiral on each end of the curve. The least confusing way to understand the alpha angle is to review "Figure 14, The Spiral Curve Legend."

$$(20)\quad \alpha = DC * \frac{L_S}{200}$$

$$(21)\quad \alpha = DC * \frac{L_S}{60.96}\ (in\ metric\ units)$$

Where: U.S. Customary Units Metric

[4] (Hay, 1982)

DC =	Degree of Curvature (chord def.)	Degree of Curvature (chord def.)
L_s =	Spiral length (feet)	Spiral length (Meters)

Central Curve Angle (Δ-2α)

The central curve angle corresponds to the circular curve (S.C. to C.S.) between the two spiral curves.

Central Curve Length (L_{C})

Central Curve Length is the length of the unspiraled portion of the curve.

(22) $$L_c = \frac{\Delta - 2*\alpha}{360} * 2 * \pi * rr$$

Where:

rr = Central Curve Radius

Total Curve Length (L)

Total curve length runs from P.S. to S.T. and includes the central curve's length plus the two spirals.

(23) $$L = L_c + 2 * L_s$$

Spiral Tangent Extension (Se)

The spiral tangent extension is useful in establishing the P.S. and S.T. points of the curve.

(24) $$S_e = \frac{L_s}{2} - \frac{{L_s}^3}{240*rr^2} + T_o * TAN\left(\frac{\Delta}{2}\right)$$

(25) NOTE: The spiral extension in this formula is measured from (P.S.) to the point of curvature of the simple unspiraled curve (P.C.). This is the value that will appear on the "Rail-Curve" software. To calculate the shorter dimension from (P.S.) to the point of curvature of the central curve extended, (P.C.'), simply drop the last term leaving:

$$\frac{L_s}{2} - \frac{{L_s}^3}{240 * rr^2}$$

Spiral Tangent (Ts)

The tangent of the simple unspiraled curve is:

(26) $T = rr * TAN\left(\frac{\Delta}{2}\right)$

The spiral tangent is the sum of S_e and T:

(27) $T_s = S_e + T$

(28) $= \frac{L_s}{2} - \frac{{L_s}^3}{240 * rr^2} + T_o * TAN\left(\frac{\Delta}{2}\right) + rr * TAN\left(\frac{\Delta}{2}\right)$

This reduces to:

(29) $= \frac{L_s}{2} - \frac{{L_s}^3}{240 * rr^2} + (T_o + rr) * TAN\left(\frac{\Delta}{2}\right)$

IV. Unlimited Speed

The human body has a limited tolerance for acceleration. We can endure four to five times the force of gravity in a fighter jet or on a roller coaster for five to ten seconds at a time but only seated or reclining. A 60° banked turn in an aircraft will double a person's weight. In such a turn, the force will still be perpendicular to the floor so the drink on your tray table won't spill, but lifting it to your lips may be challenging, and standing to visit the lavatory would be like carrying another *you* on your shoulders. Acceleration, particularly *lateral* acceleration, will be a particular concern when traveling at very high speeds.

Vactrain (Vacuum Tube Train) concepts promise train speed of 760 mph (1,200 kph) for Elon Musk's Hyperloop to 4,000 mph (6,437 kph) for Nic Garzilli's Hyper Chariot. These unfathomable speeds are feasible because the vehicle operates in an evacuated tube that eliminates wind resistance, the number one obstacle to high-speed travel in the atmosphere. They bring the vacuum of space to the earth's surface, use frictionless magnetic levitation to support the spacecraft and a linear induction motor to propel it.

Yes, it's a spacecraft in every way except the absence of gravity. With no friction to resist it, a vehicle would use power to accelerate and then coast at its operating speed to the end of the trip, where it would use regenerative braking to recapture most of the energy used for its launch. Well, that's almost true. The tube will never be a perfect vacuum, so some power will be required to maintain speed, but very little compared to any other forms of travel.

Sample problem #5 - Hyperloop

Vactrain sounds perfect, but what about the civil engineering aspects of the design, the alignment of the tube. Because of the very high speeds involved, the ride could easily resemble a roller coaster, even though the alignment may seem relatively straight. Rail-Curve will help to evaluate the maximum curvature for traveling at these speeds. We'll start with the Hyperloop on a 6° cant angle and a 1.5° deflection angle (Delta). We'll set the spiral time to 1° per second = 6-seconds.

Enter the train speed, and cant angle on the "Superelevation" page (Figure 18), then enter the spiral time and Delta angle on the "Spiral Transition" page (Figure 19).

SunCam Rail-Curve v 1.0.0

Copyright© 2018 William C. Dunn

www.suncam.com

Clear Inputs

○ FRA formula [E=0.0007DV²]

◉ Principles of Physics

Superelevation

Function	Input V_{Min}	Input V_{Bal}	Input V_{Max}	Output V_{Min}	Output V_{Bal}	Output V_{Max}	Units
Train Speed (V)		760.00		760.00	760.00	760.00	mph
				1,114.67	1,114.67	1,114.67	fps
				1,223.10	1,223.10	1,223.10	kph
				339.75	339.75	339.75	mps
Centrifugal Acceleration (horizontal) (a_c)		(no input)		3.38	3.38	3.38	ft/sec²
				1.03	1.03	1.03	m/sec²
				0.105	0.105	0.105	g
Perpendicular ┬ to floor		(no input)			1.006		g
Parallel ‖ with floor (Jerk)		(no input)		0.000	0.000	0.000	g
Cant (E)							in
							mm
		6.00			6.00		angle°
Cant Deficiency (d)							in
							mm
Curvature (rr) simple curve					367,421.09		radius - ft
					69.59		radius - miles
					111,989.95		radius - m
					0.01559		° chord
				0	0	56.14	° chord (dms)
					0.01559		° arc
				0	0	56.14	° arc (dms)
Rail spacing (W) (NOT Gauge)							in
							mm

Calculations based on the principles of physics.

Figure 17 – Hyperloop on a 6° cant angle

SunCam Rail-Curve v 1.0.1

Copyright© 2018 William C. Dunn

www.suncam.com

Clear Inputs

Spiral Transition

Function	Input	Output	Units
Spiral Run-in-Rate (rir)			in/sec
			mm/sec
	(no input)	0.0000	g/sec
Spiral Time (t)(@Vmax)	6.00	6.00	sec
Spiral Length (L_S)		6688.00	ft
		2038.50	m
Delta Angle (Δ)		1.50000	°
	1 30	1 30 0.00	° (dms)
Alpha angle (α)	(no input)	0.52146	°
		0 31 17.27	° (dms)
Central Curve Angle (Δ-2α)	(no input)	0.45707	°
		0 27 25.45	° (dms)
Central Curve Length (L_C)	(no input)	2,931.06	ft
		893.39	m
Total Curve Length (L)	(no input)	16,307.06	ft
		4,970.39	m
Spiral Tangent (T_s)	(no input)	8,153.86	ft
		2,485.30	m
Spiral Tangent Extension (S_e)	(no input)	3,344.06	ft
		1,019.27	m
Spiral Tangent Offset (T_o)	(no input)	5.07	ft
		1.55	m
Apex Offset (A_o)	(no input)	5.07	ft
		1.55	m

Figure 18 - The Hyperloop spiral transition

The resulting curve has a radius of 70-miles and a total curve length of 3-miles.

Testing a normal railroad curve of 30-minutes is a comical exercise.

SunCam Rail-Curve v 1.0.1
Copyright© 2018 William C. Dunn

○ FRA formula [$E=0.0007DV^2$]
◉ Principles of Physics

www.suncam.com

Clear Inputs

Superelevation

Function	Input V_{Min}	Input V_{Bal}	Input V_{Max}	Output V_{Min}	Output V_{Bal}	Output V_{Max}	Units
Train Speed (V)		760.00		760.00	760.00	760.00	mph
				1,114.67	1,114.67	1,114.67	fps
				1,223.10	1,223.10	1,223.10	kph
				339.75	339.75	339.75	mps
Centrifugal Acceleration (horizontal) (a_c)		(no input)		108.43	108.43	108.43	ft/sec^2
				33.05	33.05	33.05	m/sec^2
				3.370	3.370	3.370	g
Perpendicular ⊤ to floor		(no input)			3.515		g
Parallel ‖ with floor (Jerk)		(no input)		0.000	0.000	0.000	g
Cant (E)							in
							mm
					73.47		angle°
Cant Deficiency (d)							in
							mm
Curvature (rr) simple curve					11,459.19		radius - ft
					2.17		radius - miles
					3,492.76		radius - m
					0.50000		° chord
		30		0	30	0.00	° chord (dms)
					0.50000		° arc
				0	29	59.99	° arc (dms)
Rail spacing (W) (NOT Gauge)							in
							mm

Calculations based on the principles of physics.

Figure 19 - Hyperloop in a 30-minute turn

The resultant cant angle of 73° would yield a fighter jet level of excitement at 3.5-times gravity. A 180-pound person would weigh 633 pounds. By comparison, astronauts experience an acceleration of 3-times the force of gravity during a space shuttle launch.

Testing at a 60° cant angle reduces the G-force to 2-times the force of gravity and doubles the radius to about four miles. A 2-G turn is probably acceptable for most people, given that passengers will be semi-reclining. Pregnant, elderly, and infirmed people may be at risk.

SunCam Rail-Curve v 1.0.1

Copyright© 2018 William C. Dunn

www.suncam.com

Clear Inputs

○ FRA formula [E=0.0007DV²]

◉ Principles of Physics

Superelevation

Function	Input			Output			Units
	V_{Min}	V_{Bal}	V_{Max}	V_{Min}	V_{Bal}	V_{Max}	
Train Speed (V)		760.00		760.00	760.00	760.00	mph
				1,114.67	1,114.67	1,114.67	fps
				1,223.10	1,223.10	1,223.10	kph
				339.75	339.75	339.75	mps
Centrifugal Acceleration (horizontal) (a_c)	(no input)			55.73	55.73	55.73	ft/sec²
				16.99	16.99	16.99	m/sec²
				1.732	1.732	1.732	g
Perpendicular ┬ to floor	(no input)				2.000		g
Parallel ‖ with floor (Jerk)	(no input)			0.000	0.000	0.000	g
Cant (E)							in
							mm
	60.00			60.00			angle°
Cant Deficiency (d)							in
							mm
Curvature (rr) simple curve				22,295.83			radius - ft
				4.22			radius - miles
				6,795.77			radius - m
				0.25698			° chord
				0	15	25.13	° chord (dms)
				0.25698			° arc
				0	15	25.13	° arc (dms)
Rail spacing (W) (NOT Gauge)							in
							mm

Calculations based on the principles of physics.

Figure 20 - Hyperloop in a 60 ° banked turn

One problem with high cant turns is the space required for spiral transitions. Executing a 10° deflection on this curve would require that spiral time be shortened to about 3-seconds to prevent the spirals from overlapping. Using high cant angles will not be possible at these speeds.

A 45° cant angle reduces the G-force further to 1.4-times the force of gravity and increases the radius to over seven miles but still leaves the spirals too short.

A 30° cant probably works best. At 1.15-G, and is generally the maximum bank used by commercial airline pilots.

SunCam Rail-Curve v 1.0.1
Copyright© 2018 William C. Dunn
○ FRA formula [E=0.0007DV²]
◉ Principles of Physics
www.suncam.com
Clear Inputs

Superelevation

Function	Input			Output			Units
	V_{Min}	V_{Bal}	V_{Max}	V_{Min}	V_{Bal}	V_{Max}	
Train Speed (V)		760.00		760.00	760.00	760.00	mph
				1,114.67	1,114.67	1,114.67	fps
				1,223.10	1,223.10	1,223.10	kph
				339.75	339.75	339.75	mps
Centrifugal Acceleration (horizontal) (a_c)		(no input)		18.58	18.58	18.58	ft/sec²
				5.66	5.66	5.66	m/sec²
				0.577	0.577	0.577	g
Perpendicular ┬ to floor		(no input)			1.155		g
Parallel ‖ with floor (Jerk)		(no input)		0.000	0.000	0.000	g
Cant (E)							in
							mm
		30.00			30.00		angle°
Cant Deficiency (d)							in
							mm
Curvature (rr) simple curve					66,887.49		radius - ft
					12.67		radius - miles
					20,387.31		radius - m
					0.08566		° chord
				0	5	8.38	° chord (dms)
					0.08566		° arc
				0	5	8.38	° arc (dms)
Rail spacing (W) (NOT Gauge)							in
							mm

Calculations based on the principles of physics.

Figure 21 - What minimum radius is required for Hyperloop?

A 30° cant angle requires a minimum turning radius of nearly 13-miles. Our 10° deflection would require 10-second transitions at that radius, still fast but probably comfortable because the passenger's head will be near the center of rotation of the vehicle.

www.suncam.com

Clear Inputs

SunCam Rail-Curve v 1.0.0

Copyright© 2018 William C. Dunn

Spiral Transition

Function	Input	Output	Units
Spiral Run-in Rate (rir)			in/sec
			mm/sec
	(no input)	0.0000	g/sec
Spiral Time (t)(@Vmax)	10.00	10.00	sec
Spiral Length (L_S)		11146.67	ft
		3397.50	m
Delta Angle (Δ)		10.00000	°
	10	10 0 0.00	° (dms)
Alpha angle (α)	(no input)	4.77411	°
		4 46 26.81	° (dms)
Central Curve Angle (Δ-2α)	(no input)	0.45177	°
		0 27 6.38	° (dms)
Central Curve Length (L_C)	(no input)	527.40	ft
		160.75	m
Total Curve Length (L)	(no input)	22,820.74	ft
		6,955.76	m
Spiral Tangent Extension (S_e)	(no input)	5,578.82	ft
		1,700.42	m
Spiral Offset (S_o)	(no input)	77.3987	ft
		23.59	m
Spiral Tangent (T_s)	(no input)	11,430.71	ft
		3,484.08	m

Figure 22 - Spiral transitions for Hyperloop

Note that smaller Delta Angles will only work by reducing the superelevation and flattening the curves; otherwise, the spirals will overlap. To illustrate this point, try entering an 8° cant angle, an 8.5-second spiral time, and a 2° Delta Angle.

Sample problem #6 - Hyper Chariot

The Hyper Chariot proposes to provide 4,000 mph transport from London to Edinburgh, making the 400-mile journey in just 8-minutes. The turning radius, assuming a 6° cant angle, would be nearly 2000-miles putting the center of the curve somewhere near Moscow.

SunCam Rail-Curve v 1.0.1

Copyright© 2018 William C. Dunn

○ FRA formula [E=0.0007DV²]

◉ Principles of Physics

www.suncam.com

Clear Inputs

Superelevation

Function	Input V_{Min}	Input V_{Bal}	Input V_{Max}	Output V_{Min}	Output V_{Bal}	Output V_{Max}	Units
Train Speed (V)		4000.00		4,000.00	4,000.00	4,000.00	mph
				5,866.67	5,866.67	5,866.67	fps
				6,437.38	6,437.38	6,437.38	kph
				1,788.16	1,788.16	1,788.16	mps
Centrifugal Acceleration (horizontal) (a_c)		(no input)		3.38	3.38	3.38	ft/sec^2
				1.03	1.03	1.03	m/sec^2
				0.105	0.105	0.105	g
Perpendicular ┬ to floor		(no input)			1.006		g
Parallel ‖ with floor (Jerk)		(no input)		0.000	0.000	0.000	g
Cant (E)							in
							mm
		6.00			6.00		angle°
Cant Deficiency (d)							in
							mm
Curvature (rr) simple curve					10,177,869.48		radius - ft
					1,927.63		radius - miles
					3,102,214.62		radius - m
					0.00056		° chord
				0	0	2.03	° chord (dms)
					0.00056		° arc
				0	0	2.03	° arc (dms)
Rail spacing (W) (NOT Gauge)							in
							mm

Calculations based on the principles of physics.

Figure 23 - What minimum radius is required for Hyper Chariot?

The vertical alignment is even more demanding. Hyper Chariot will be traveling faster than the muzzle velocity of any modern rifle firing high-performance cartridges. Building the tube to follow the contours of the land will be impractical. The route will encounter terrain varying in elevation by 1000-2000 feet, and even the slightest crest curve could leave passengers weightless.

References

AREMA. (2018). In *Manual for Railway Engineering* (pp. 5-3-1 to 5-3-5). American Railway Engineering and Maintenance-of-Way Association.

F.R.A., Federal Railroad Administration. (2017). U.S. Code of Federal Regulation, Title 49. U.S.

Hay, W. H. (1982). Mgt., E., M.S., Ph.D. In *Railroad Engineering, Second Edition* (p. 606). Urbana, Illinois, U.S.: John Wiley and Sons.

Seely, E. E. (1945, 1951, 1960). Design, Volume One. John Wiley & Son.

V. Appendix A - Cascading Look-Up Tables

When equations cannot be readily reduced algebraically, we use a technique called Cascading Look-Up Tables to produce exceptionally precise table extractions. The VLOOKUP function in Excel is used to extract a value for cant (E) when inputs include only cant deficiency (d) and any two of the train speed variables. The technique uses a series of 15 VLOOKUP tables each containing 10 rows of data, to make ever increasingly accurate estimates of the cant variable. Each iteration adds one digit to the results. Results will be accurate to an (admittedly excessive) 15 significant digits.

Here are three examples.

Variable			
d (in)	3.00	3.00	3.00
Vmin (fps)	90.00	X	90.00
Vbal (fps)	120.00	120.00	X
Vmax (fps)	X	150.00	150.00
Iteration			
1	6	5	6
2	6.8	5.4	6.4
3	6.89	5.42	6.43
4	6.898	5.422	6.434
5	6.8981	5.4227	6.4340
6	6.89817	5.42270	6.43402
7	6.898171	5.422702	6.434029
8	6.8981713	5.4227027	6.4340294
9	6.89817139	5.42270274	6.43402948
10	6.898171395	5.422702749	6.434029480
11	6.8981713955	5.4227027495	6.4340294803
12	6.89817139554	5.42270274952	6.43402948032
13	6.898171395542	5.422702749522	6.434029480328
14	6.8981713955423	5.4227027495225	6.4340294803282
15	6.89817139554233	5.42270274952255	6.43402948032821

For live data use the Cascading Lookup Tables tab on the Rail-Curve software.

VI. Appendix B - Principles of Physics formulas

Equations based on the P**rinciples of Physics** used in the Rail-Curve spreadsheet, the companion software to this course.

All calculations in the software are performed using U.S. Customary Units then converted to metric units.

Where:

Vmin =	Minimum velocity (feet/sec)
Vbal =	Balanced velocity (feet/sec)
Vmax =	Maximum velocity (feet/sec)
rr =	Radius (feet)
E =	Superelevation (cant) (inches)
W =	Centerline spacing of the rails (inches)
d =	Cant deficiency (in)
g =	Acceleration of gravity (32.17405 feet/sec^2)

(30) $$Vmin = \sqrt{\left(\left(TAN\left(ASIN\left(\frac{E-d}{W}\right)\right) * g\right) * rr\right)}$$

(31) $$Vbal = \sqrt{\left(\left(TAN\left(ASIN\left(\frac{E}{W}\right)\right) * g\right) * rr\right)}$$

(32) $$Vmax = \sqrt{\left(\left(TAN\left(ASIN\left(\frac{E+d}{W}\right)\right) * g\right) * rr\right)}$$

(33) $$E = SIN\left(ATAN\left(\frac{Vmin^2}{rr*g}\right)\right) * W + d$$

(34) $$E = SIN\left(ATAN\left(\frac{Vbal^2}{rr*g}\right)\right)*W$$

(35) $$E = SIN\left(ATAN\left(\frac{Vmax^2}{rr*g}\right)\right)*W-d$$

(36) $$\sin\left(\operatorname{atan}\left(\frac{Vbal^2}{rr*g}\right)\right)*W = \sin\left(\operatorname{atan}\left(\frac{Vmax^2}{rr*g}\right)\right)*W-d$$

(37) $$E = \left(W*\frac{\frac{Vmax^2}{rr}}{\sqrt{\left(\left(\frac{Vmax^2}{rr}\right)^2+g^2\right)}}\right) - \frac{\left(W*\frac{\frac{Vmax^2}{rr}}{\sqrt{\left(\left(\frac{Vmax^2}{rr}\right)^2+g^2\right)}}\right)-\left(W*\frac{\frac{Vmin^2}{rr}}{\sqrt{\left(\left(\frac{Vmin^2}{rr}\right)^2+g^2\right)}}\right)}{2}$$

NOTE: We use Cascading Lookup Tables to solve the following three equations for the variable 'E.'

(38) $$\frac{Vbal^2}{TAN\left(ASIN\left(\frac{E}{W}\right)\right)*g}-\frac{Vmax^2}{TAN\left(ASIN\left(\frac{E+d}{W}\right)\right)*g}=0$$

(39) $$\frac{Vbal^2}{TAN\left(ASIN\left(\frac{E}{W}\right)\right)*g}-\frac{Vmin^2}{TAN\left(ASIN\left(\frac{E-d}{W}\right)\right)*g}=0$$

(40) $$\frac{Vmax^2}{TAN\left(ASIN\left(\frac{E+d}{W}\right)\right)*g}-\frac{Vmin^2}{TAN\left(ASIN\left(\frac{E-d}{W}\right)\right)*g}=0$$

(41) $$d = \frac{\left(W * \frac{\frac{Vmax^2}{rr}}{\sqrt{\left(\left(\frac{Vmax^2}{rr}\right)^2 + g^2\right)}}\right) - \left(W * \frac{\frac{Vmin^2}{rr}}{\sqrt{\left(\left(\frac{Vmin^2}{rr}\right)^2 + g^2\right)}}\right)}{2}$$

(42) $$d = E - \left(W * SIN\left(ATAN\left(\frac{\frac{Vmin^2}{rr}}{g}\right)\right)\right)$$

(43) $$d = \left(W * SIN\left(ATAN\left(\frac{\frac{Vmax^2}{rr}}{g}\right)\right)\right) - E$$

(44) $$rr = \frac{Vmin^2}{TAN\left(ASIN\left(\frac{E-d}{W}\right)\right) * g}$$

(45) $$rr = \frac{Vbal^2}{TAN\left(ASIN\left(\frac{E}{W}\right)\right) * g}$$

(46) $$rr = \frac{Vmax^2}{TAN\left(ASIN\left(\frac{E+d}{W}\right)\right) * g}$$

VII. Appendix C – F.R.A. Formulas

Equations based on the **U.S. Federal Railway Administration formula** used in the Rail-Curve spreadsheet, the companion software to this course.

All calculations are performed using U.S. Customary Units then converted to metric units.

Where:	
DC =	Degrees of Curvature (° chord definition)
Vmin =	Minimum velocity (miles per hour)
Vbal =	Balanced velocity (miles per hour)
Vmax =	Maximum velocity (miles per hour)
rr =	Radius (feet)
E =	Superelevation (cant) (inches)
d =	Cant deficiency (in)

(47) $$Vmin = \sqrt{\frac{E-d}{0.0007*DC}}$$

(48) $$Vbal = \sqrt{\frac{E}{0.0007*DC}}$$

(49) $$Vmax = \sqrt{\frac{E+d}{0.0007*DC}}$$

(50) $$E = 0.0007 * DC * Vmin^2 + d$$

(51) $$E = 0.0007 * DC * Vmax^2 - d$$

(52) $$E = \frac{0.0007*DC*(Vmax^2+Vmin^2)}{2}$$

(53) $$d = E - 0.0007 * DC * Vmin^2$$

(54) $d = 0.0007 * DC * Vmax^2 - E$

(55) $D = \left(\frac{E-d}{0.0007*Vmin^2}\right)$

(56) $D = \left(\frac{E}{0.0007*Vbal^2}\right)$

(57) $D = \left(\frac{E+d}{0.0007*Vmax^2}\right)$

(58) $D = \frac{d}{0.0007*(Vbal^2-Vmin^2)}$

(59) $D = \frac{d}{0.0007*\left(\frac{Vmax^2-Vmin^2}{2}\right)}$

(60) $D = \frac{d}{0.0007*(Vmax^2-Vbal^2)}$

(61) $rr = \frac{50}{SIN\left(\frac{D}{2}\right)}$

