A SunCam online continuing education course

Structural Nonlinearity: Defining Nonlinearity

by

John Klein, P.E., M.L.S.E.

Copyright© 2022 Klein

Table of Contents

Copyright© 2022 Klein

405.pdf

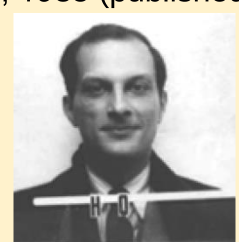
Introduction

Many parts of our physical world can be modeled as acting linearly without much error. Real-world behavior is usually nonlinear but even so, the engineering industry broadly adopts linearity making most calculations manageable enough to be practical. This adoption of linearity does have an unintended side effect; the uninitiated can be intimidated and confused by the notion of nonlinearity.

It is understandable that nonlinearity can be daunting and mysterious at first. By definition nonlinearity is the null set of linearity. It is defined by what it is not. The term *nonlinear* is a hypernym, or a word that acts as an umbrella term for many subcategories of behaviors. Behaviors that are only related by being not-linear.

> "Using a term like nonlinear science is like referring to the bulk of zoology as the study of non-elephant animals."
>
> — Dr. Stanislaw Ulam, 1985 (published)

Los Alamos National Laboratory ID badge photo during Manhattan Project



Quick notes

This course does not intend to identify every situation in structural engineering that involves nonlinearity. This course only covers the nonlinearity that can occur in the structural analysis of whole members or multi-member structures. Mathematical nonlinearity occurs in many other sectors of structural engineering that will not be included in this course.

Structural movement includes displacements, rotations, deflections, and slopes of structures and their members. The term *distort* will be used as appropriate in this document to substitute for structural movement.

Copyright© 2022 Klein

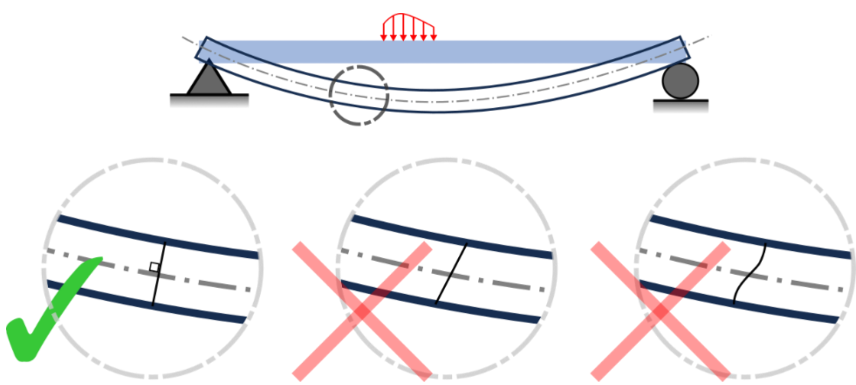
Linearity

Nonlinearity is merely the lack of linearity, so linearity must first be defined to show the bounds of nonlinearity. *Linearity* is defined by a set of simplifying fundamental assumptions that are listed below.

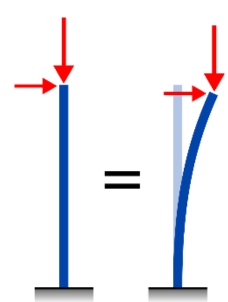
- Euler-Bernoulli beam theory (EBT), sometimes referred to as *small deflection* theory:

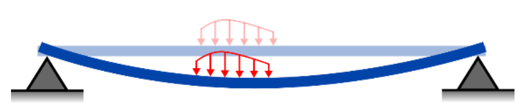
Fashionable engineering superstars of the 18th century

  - Plane sections through members remain plane after deflecting and sloping.

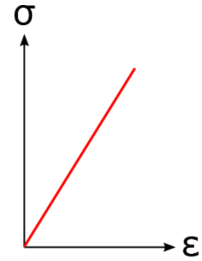


  - Structural movement is infinitesimal. Deflections, slopes, displacements, and rotations are so slight that the distorted shape of the structure can be ignored in the load-stiffness continuum, and for the bounds and length of the elastic curve. Though movement magnitude can be calculated using EBT, <u>the distorted shape of the structure or its members will not be included within the analysis nor the change in member length due to transverse deflections and rotations</u>.





 Copyright© 2022 Klein



- The materials will deform linearly in proportion to their loading (linear stress vs strain), and the material remains in a/the linear portion of the material's stress-strain throughout the load range.
- Supports are always reactive and constant, and equally in opposite directions.
- Members are straight, uniform in material and cross-section (prismatic) over their length and react equally in opposite directions.
- The structure is unloaded, and stress and strain-free before being loaded.

- Forces are applied and act in the global reference frame, retaining their initial global vector direction regardless of distorted structure geometry.

- Time is irrelevant. The structure instantaneously responds to forces, and consideration of any time after loading is irrelevant as the distorted shape of the structure is not included within the analysis.

Linear Analysis

The industry standard tool for structural engineers linearly analyzing buildings and bridges is the displacement method of analysis (stiffness analysis, matrix stiffness analysis, direct stiffness analysis). This method of structural analysis requires that a stiffness matrix [K] and a load vector {F} for the overall structural system be assembled with numerical values only. The *linear analysis* then includes solving for the displacement vector {u} using [K] and {F}, and using that displacement vector to provide results (reactions, internal forces, member deflections and slopes, etc.).

$$\{F\} = [K]\{u\}$$

$$\begin{bmatrix} F_{x1} \\ \vdots \\ F_{yn} \end{bmatrix} = \begin{bmatrix} K_{11} & \cdots & K_{1n} \\ \vdots & \ddots & \vdots \\ K_{n1} & \cdots & K_{nn} \end{bmatrix} \begin{bmatrix} u_{x1} \\ \vdots \\ u_{yn} \end{bmatrix}$$

 Copyright© 2022 Klein

405.pdf

Nonlinearity

We can consider nonlinearity now that the bounds of linearity have been defined. Nonlinearity results when a situation violates one or more of the fundamental assumptions of linearity. These nonlinearities mathematically manifest as variables or expressions in the stiffness matrix or load vector in place of numerical values.

There is no universally accepted definition for structural nonlinearity, but for our purposes it will be defined thus: structural nonlinearity is a structural system that results in having a stiffness matrix or load vector that is not constant. Here, a structural system includes the members, elements, joints, components, and supports making up the structure, the geometry of their initial arrangement and their distorted geometry, and the loads applied to the structure.

A structural system is nonlinear if an accurate stiffness matrix or load vector would contain expressions instead of numerical values, and the expressions include variable(s) such as:

- member or joint deflection, slope, displacement, or rotation
- the location along the length of a member
- direction of force(s)
- extent of strain
- magnitude and/or direction of reaction
- time

Most of these variables can only be defined as the potential results of a linear analysis. Simply stated, the results would be a function of the results. Therefore, nonlinear structural systems cannot be linearly analyzed, and:
A structural system with one or more instances of acting or engaged structural nonlinearity cannot be analyzed by only a single linear stiffness analysis.

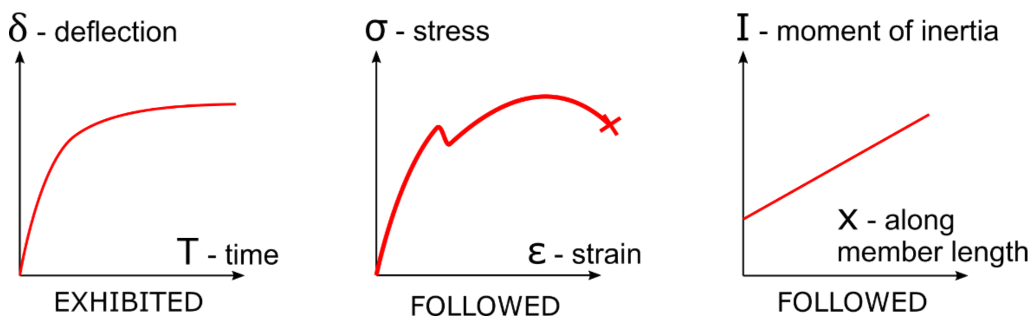
More viscerally, nonlinearity means an equilibrium state is something that must be arrived at through recursive calculations and convergence instead of being directly solved for as with linear analysis. Methods for analyzing nonlinearity are presented in the next course.

 Copyright© 2022 Klein

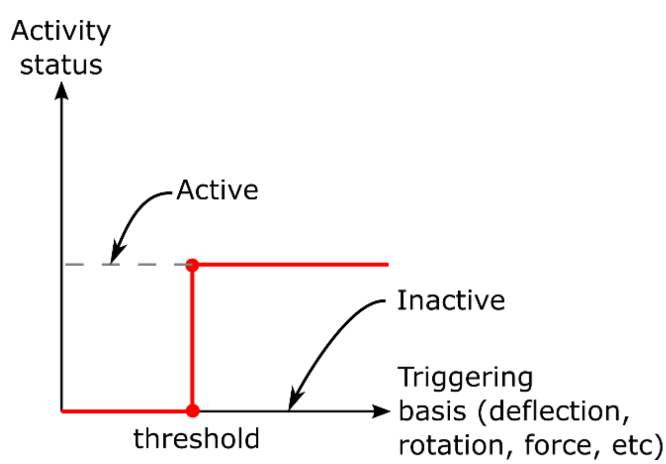
Behavior profiles

Structural nonlinearities can be roughly grouped into two categories when thinking of how they relate to structural analyses. Nonlinearities (1) occur as a continuum or (2) act in phases.

When continuum-type nonlinear relationships are graphed they form curves, discontinuous multi-segment lines, or a single sloped line. Continuum-type nonlinearities either **exhibit** behavior that can be graphed as a curve, or they have a variable **follow** along a given curve or line(s). Note that there are no continuum-type nonlinearities that exhibit single sloped line behavior as that would indicate linearity.



Phase-type nonlinear activity can be graphed as a step-function, where the active/inactive status of a member, joint, support, or similar is either on or off, depending on a given trigger or threshold. Once triggered, the status remains active unless conditions change that causes a reversal of the triggering. This discontinuous behavior extends to nonlinearities that combine phase-type behavior with time where loads and/or stiffness are applied or changed in phases, which cannot be reversed.



Copyright© 2022 Klein

Time

Time is not considered in linear structural analyses as the application of loads and the reaction of the structure to the loads are thought to occur instantaneously. To expressly include time in an analysis constitutes *structural dynamics*, where motion and the causing/resulting forces would be included. Structural dynamics is beyond the scope of this course. However, time must be regarded or at least acknowledged to accommodate some types of nonlinearity that require considering a structure while it is in an already-loaded and/or distorted state. There is a compromise between statics and dynamics where time can be regarded as a pseudo-variable that can be used to visualize and evaluate nonlinearity. This compromise is known as *kinematics* and allows taking snapshots of structural response and movement forward in time without explicitly regarding time as a variable.

Image A: "Horse gallop"

Types of nonlinearity

Up to this point we have discussed nonlinearity broadly. Let's pull the curtain back a bit and get more specific. The following are the different types of nonlinearity that can occur in a structural system.

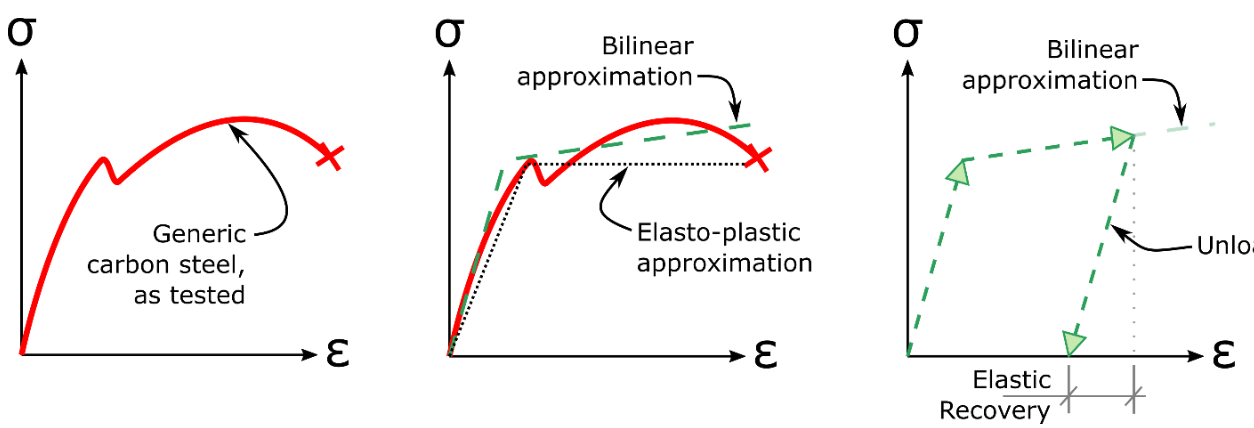
- The subsections named "variability in stiffness analysis" explain how entries in the stiffness matrix or load vector would not be constant if a linear stiffness analysis was attempted for a structural system containing the nonlinearity.
- Some nonlinearities can create or remove a degree-of-freedom (DOF), and the ability to affect DOF is listed for each type of nonlinearity.

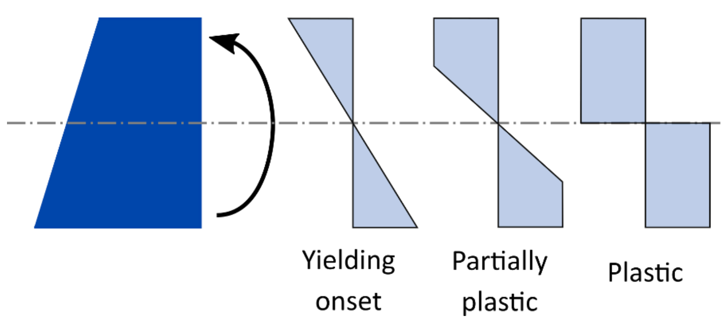
 Copyright© 2022 Klein

Material Nonlinearity: typically indicates that a material has been stressed beyond its yield point or proportional limit into the plastic region of the material's stress-strain curve but can also include materials that never exhibit a near-linear stress-strain curve (*nonlinear elasticity*) like rubber and most plastics.

Nonlinear behavior: continuum-type, following along a stress-strain curve or discontinuous polyline where a stress-strain curve is approximated by multiple linear segments (not along a single sloped line, which would be linear material behavior)

Variability in stiffness analysis: modulus of elasticity, E, in the stiffness matrix is a function of strain





DOF abilities: can create one or more DOF as an internal member release (hinge, etc.) upon yielding if an elasto-plastic approximation for stress-strain is used, or upon failure (ultimate rupture/crushing) using any stress-strain relationship

Copyright© 2022 Klein

Nonlinear Boundary Conditions: structural supports that provide discontinuous support depending on reaction direction and/or magnitude.

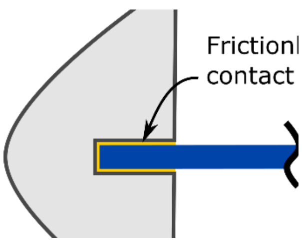
Variability in stiffness analysis: support activity or stiffness of supports in the stiffness matrix depends one or more of reaction direction, movement magnitude, or a given stiffness function

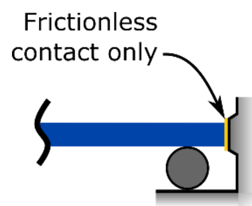
Nonlinear boundary conditions include:

- *Directional Support*: where displacements or rotations are supported or partially resisted in one direction and either not supported or differently resisted in the opposite direction. This behavior is also commonly called *contact*.

  Nonlinear behavior: phase-type triggered by force direction

  Baseplate bearing where compressive reactions are supported but no resistance is provided for tension is a classic example of contact.





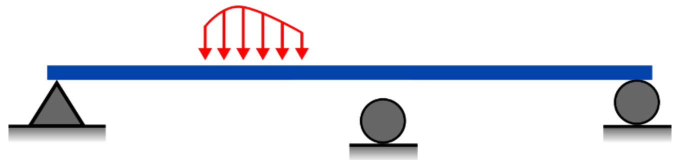
  DOF abilities: creates a directional DOF by definition

- *Gap-Type Supports*: where no support reaction is provided until a translational or rotational gap is closed, at which point the structure is then supported for an increasing reaction in that direction.

  Nonlinear behavior: phase-type triggered by a translational or rotational movement magnitude in a given direction

  This is an extension of the concept of *contact* but switching of the contact behavior is controlled by a predefined deflection/displacement or slope/rotation magnitude.

 Copyright© 2022 Klein

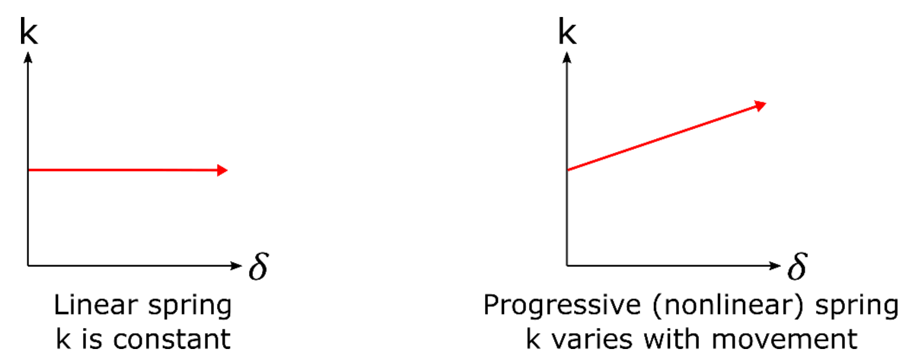


DOF abilities: removes a DOF for a certain location in the structure upon closing of the gap

- *Nonuniform Supports*: where a support provides differing rates of resistance based on movement

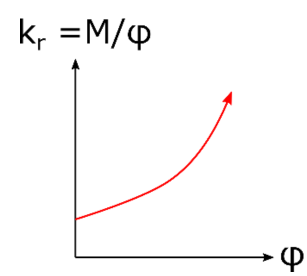
  Nonlinear behavior: continuum-type, following a linear, discontinuous multi-linear, or curved spring constant vs displacement (or rotation)

  Note that a spring with a constant stiffness value, such as 20 kips/inch or 3000 k-ft/radian (3500 kN/m or 4100 kN-m/radian), corresponds to linearly proportional results and does not exhibit nonlinearity. A spring with stiffness that varies with movement, such as a progressive spring, constitutes nonlinearity even if the stiffness can be graphed as a single sloped line.



  Springs used to model soil-structure interaction often have nonuniform or nonlinear stiffness.

  DOF abilities: can technically create a DOF if a spring curve's stiffness decreased and approached zero as movement increased, and conversely could remove a DOF by increasing stiffness to approach infinity, but these situations are not commonly encountered in practice



Nonlinear rotational spring with increasing stiffness

 Copyright© 2022 Klein

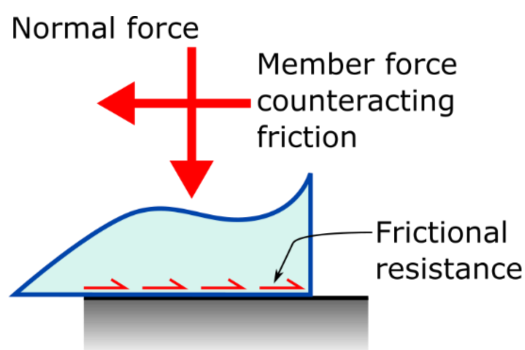
Friction: the interface of two surfaces with friction is treated as a translational or rotational locking or coupling between the two surfaces until static frictional resistance is exceeded, and then the locking/coupling is released and replaced with a constant force (or moment) parallel to the friction surface.

Nonlinear behavior: phase-type, triggered by a coupled pair of forces – the member force along the friction surface and the member force normal to the friction surface

Variability in stiffness analysis: the frictional resistance represents a support in the stiffness matrix, and its activity status is a function of the member forces acting along and perpendicular to the friction surface

Friction is an extension of *contact* and is predominantly used at supports rather than between two members. However, friction could occur between members, or elements, or a mixture of components.

DOF abilities: creates a DOF by definition when static friction is overcome

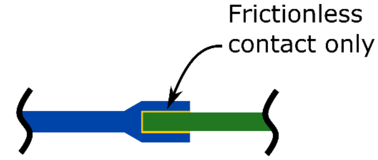


Copyright© 2022 Klein

405.pdf

Directional Members: members within a structure that have stiffness in one direction, but either have no stiffness or different stiffness in the opposite direction.

Nonlinear behavior: phase-type, triggered by internal member force direction, step function of active/inactive status vs member internal force direction

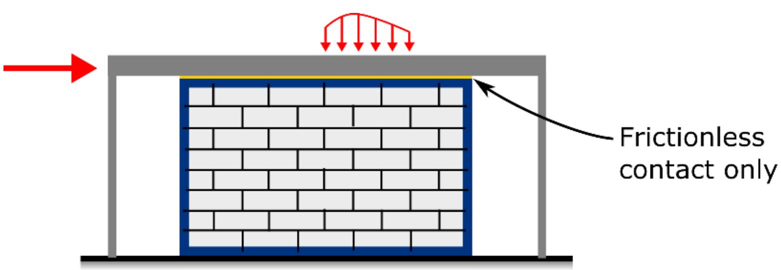


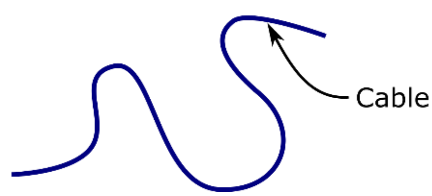
Compression-only member

Variability in stiffness analysis: the stiffness of a directional member in the stiffness matrix is a function of the sign, or direction, of one of the member's internal forces

This is another application of *contact*, but with the interface occurring within or along the member instead of at a joint or support location.

A beam with a one-way bending stiffness qualifies, but directional members are usually limited to tension-only members and compression-only members in practice (e.g. cable bracing and infill walls, respectively).





DOF abilities: Is a directional DOF by definition when the member is disengaged

Horizontally oriented cables can only be analyzed as directional members when used for direct axial tension over short spans. A long span cable that is oriented horizontally would classify as a transversely loaded cable just for cable self-weight.

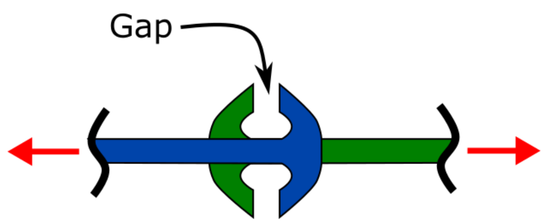
→ Cables with transverse loads exhibit a different type of nonlinearity that will be discussed below.

 Copyright© 2022 Klein

- *Gap-Type Elements/Members*: a connecting part situated in between two objects (members, or joints, or member and joint) where stiffness engagement and force transferal is interrupted between the two connected objects until a translational or rotational gap is closed, at which point the two objects are connected for that stiffness (axial tension, +X shear, etc) for increasing forces in that direction.

  Nonlinear behavior: phase-type, triggered by a known displacement or rotation magnitude, step function of active/inactive status vs displacement or rotation in a given direction

  A gap-type element between members can account for slip from through-bolts or nail-type fastenings prior to initial member engagement.

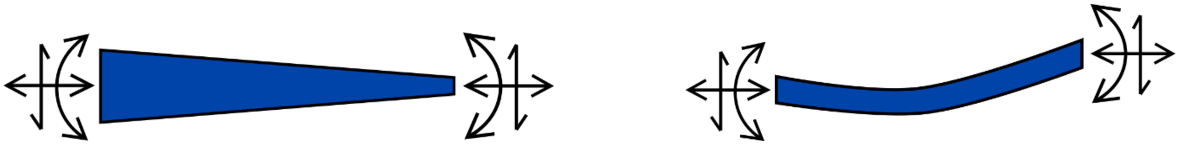


Tension-only member with gap

  DOF abilities: is a DOF by definition for the disengaged direction, and is a DOF by definition until the gap is closed in the engaged direction

Copyright© 2022 Klein

Member Nonlinearity by Shape: both nonprismatic members and curved members are essentially nonlinear as defined by physical shape, and both introduce nonlinearity into the theoretical structural analyses.



Nonlinear behavior: continuum-type, following along the sloped line, multi-linear, or curve defined by member stiffness as a function of length, and/or following along the curve formed by the centroidal axis of a curved member. This type of nonlinearity is also referred to as *physical nonlinearity*.

Variability in stiffness analysis: the stiffness properties in the stiffness matrix for any members that are nonlinear by shape change along the length of the member, and are a function of the distance along the member

Types of nonlinearity by shape include:

- *Nonprismatic Members*: Beam shear and flexure can directly interact in members that taper, are haunched, or arched, as opposed to prismatic members where shear and flexure are often decoupled.
- *Curved Members*: Axial tension/compression can be developed for members curved in elevation that are subject to transverse, beam-type gravity forces, and torsion and radial forces can develop for members curved in plan.
- *Nonuniform Material Stiffness*: It is possible but unlikely to have members where the material stiffness (modulus of elasticity, E) changes along their length. A hypothetical scenario could have a member that was subjected to a temperature gradient along its length that modified the initial E, such as a fire at one end.

Nonlinearity by shape only includes members where the stiffness or centroidal axis varies continuously along the member length, and not members where stiffness or centroidal axis is constant with discontinuities (e.g. stepped member cross section, or kinked member with multiple straight, connected segments), as these can be subdivided into multiple prismatic and straight members and analyzed linearly.

DOF abilities: unrelated, does not create or remove DOFs

 Copyright© 2022 Klein

405.pdf

TIME: The types of nonlinearity listed below include time as a pseudo-variable, either through the notion of kinematics or to accommodate changes that occur in phases

Geometric Nonlinearity: involves a structure where the distorted geometry of the structure and/or its individual members has a non-negligible effect on the stiffness, and hence on the internal members forces and on stability thresholds such as global member buckling. i.e. As a structure distorts under load it behaves differently than it did before moving.

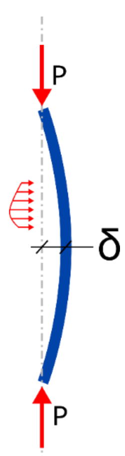
Nonlinear behavior: continuum-type, exhibiting asymptotic, multiple-extrema, or simple curves of stiffness vs movement (displacement, rotation, slope, deflection), and/or for the affected forces and stability thresholds vs movement

Variability in stiffness analysis: the stiffnesses of affected members in the stiffness matrix are unknown functions of movement

Transversely loaded cables exhibit very high geometric nonlinearity, as small changes in the geometry of a loaded cable can cause large changes in cable tension.

DOF abilities: geometric nonlinearity (GN) generally cannot create or remove DOFs except for extreme movement cases

P-δ and P-Δ effects are the most famous and perhaps notorious of the many subdivisions of geometric nonlinear effects, and both have a softening effect on the structure in that they have the tendency to self-magnify (for axial compression). This self-magnification process is referred to as a *mechanism*.



Shown are the initial, linearly-obtained equilibrium states, and below demonstrates the mechanism of the P-δ effect shown to the left progressing through kinematic time steps:

$$M(x)_{linear}\ \&\ \delta(x)_{linear} = flexure\ \&\ deflection\ from\ linear\ analysis$$

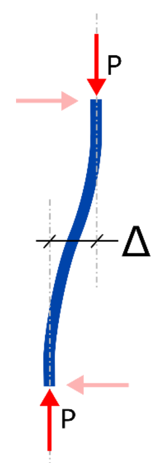
Kinematic step#1: $M(x)_{add'l} = P \cdot \delta(x)_{linear}$

$\delta(x)_{add'l}\ arises\ from\ M(x)_{add'l}$

Kinematic step #2: $M(x)_{add'l-2} = P \cdot \delta(x)_{add'l}$

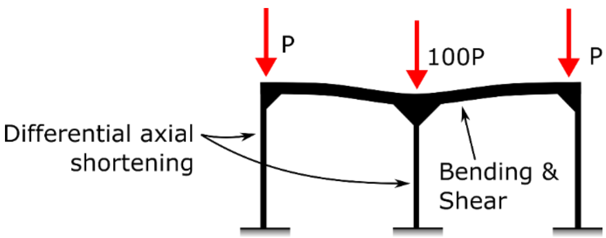
$\delta(x)_{add'l-2}\ arises\ from\ M(x)_{add'l-2}\ \ldots$

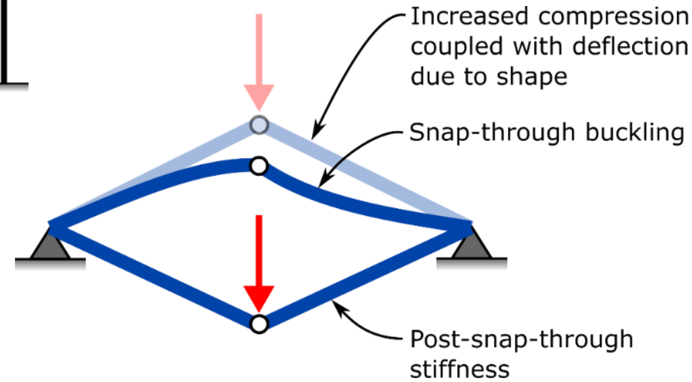
and so on with more kinematic steps, continuing until $\delta(x)_{add'l-n} \rightarrow 0$ or ∞



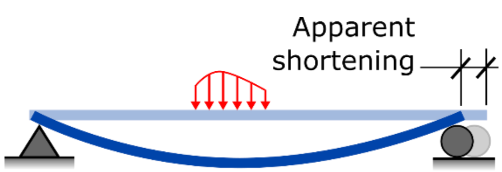
 Copyright© 2022 Klein

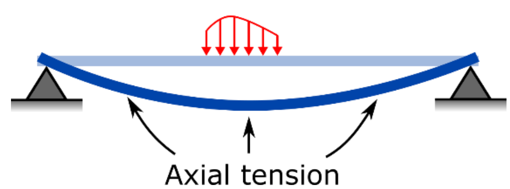
Discussions of GN are often limited to P-δ and P-Δ effects (collectively called *P-delta effects*) but GN extends to many other behaviors such as differential column shortening and snap-through of shallow vaulted structures.





The most elusive and arcane of GN is *large deflection*. Many geometric nonlinear effects can be considered to fall within the realm of small deflection theory with reasonable accuracy, but once those limits are exceeded the complexity of the calculus involved in fundamental beam mechanics significantly increases. The same strain and stress tensors can no longer be used, and severe mathematical nonlinearity will develop when the length of elastic curves and nonlinear beam curvature are both regarded.





Geometric nonlinearity (GN) is a broad, complex, and nuanced topic that deserves a more detailed treatment than can be squeezed into this course. The traits that are common to all GN effects are (1) the structure and/or its members being flexible, and (2) the structural system behaving and reacting differently in its distorted geometry than it did in its initial, undistorted geometry. Geometric nonlinearity is in play when the structural system moves if there is a change in the equilibrium, the internal forces, or the stiffness of the structure or its members.

GN will be covered in depth in the last course in this series.

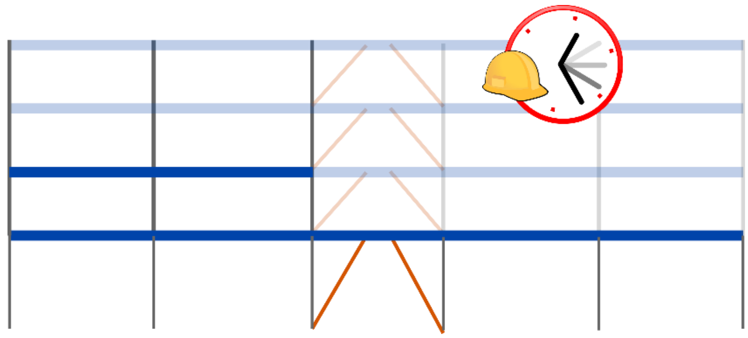
Copyright© 2022 Klein

Initial Load/Stress State(s): pre-loading or other source of initial stress and internal forces in a structure.

Nonlinear behavior: phase-type, where loads or stress states are phased over time

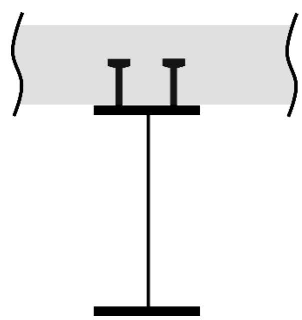
Variability in stiffness analysis: the load vector varies; the stiffness matrix may not be nonlinear but it may change from load state to load state and it cannot store displacements from earlier states

Can include simple pre-loading where prestress is to be combined with stress from subsequent loads (e.g. prestressed concrete), or staged construction or sequential loadings where results consist of an envelope of response



- *Special case*: changing the structural stiffness (adding/removing members or supports) while a structure is under an initial stress state. This combines initial stress state behavior with gap-type nonlinear behavior.

An example is unshored composite construction. For the initial phase steel beams with shear studs on the top flange are placed, without shoring, and support their self-weight and the weight of a concrete deck pour. The following phase occurs after the concrete cures, where the stiffness has increased and the steel has a “locked in” state of stress while the concrete is (longitudinally) unloaded and unstressed.



DOF abilities: can only create or remove DOFs if that is the change that is being made to the structural system, e.g. adding a support to an existing, initially-stressed structure would remove a DOF at certain location in the structure

 Copyright© 2022 Klein

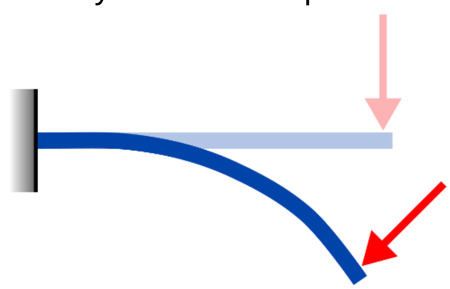
405.pdf

Forces that Change Over Time: At its simplest, there is light nonlinearity in considering moving loads. Unless there are additional nonlinearities at play the structural system can be evaluated linearly but post-processing of response envelopes is required. Other times a load can be specified as a spectrum or load curve, where the magnitude of load changes over time.

Variability in stiffness analysis: the load vector varies over time; the stiffness matrix does not necessarily vary and could be constant over the duration of the load event

DOF abilities: unrelated, does not create or remove DOFs

- *Special case:* forces that can change direction over time or as a structure distorts, such as a *follower force* that remains normal to a member as it significantly rotates or slopes. Follower forces that are oriented to the member reference frame significantly affect geometrically nonlinear response.



Sources of Nonlinearity

The sources of structural nonlinearity can be summarized as:

Distorted Geometry	Contact	Time
Material elasticity	Member shape	

All of these nonlinearities can be present in a singular structural system. No one nonlinearity supercedes, invalidates or precludes any other. This possibility is an important facet of consideration and analyzing nonlinearities in structural systems.

 Copyright© 2022 Klein

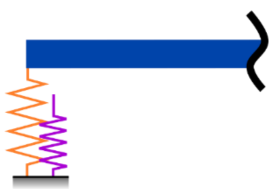
<u>Nonlinearity or Special Condition?</u>

There are a few conditions that can occur in a structural system that may seem like they could be nonlinearities that aren't. These conditions can complicate the stiffness analysis and slow a structural analysis solver down a bit, but these can be analyzed using a single linear analysis and are not nonlinearities:

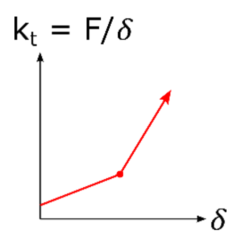
- Indeterminacy: indeterminacy and nonlinearity are separate and mutually inclusive. Determinate structures can contain a nonlinearity or behave nonlinearly, and highly indeterminate structures can display or contain no nonlinearities.
- Internal member releases: a hinge in the middle of a member, or an axial force release between beam endpoints could render a structure unstable by adding a DOF, but member releases fall within the reach of linear analyses
- Elastic supports: as long as they provide constant and directionally-equal support they can be analyzed linearly
- Forced displacements/rotations as loads

Nonlinearity in play

Some nonlinearities can be combined. An example of gap-type and nonuniform support nonlinearity combined is a "nested" progressive two-spring support with a bilinear stiffness response that transitions stiffness at a certain displacement magnitude.



"Nested" progressive translational springs

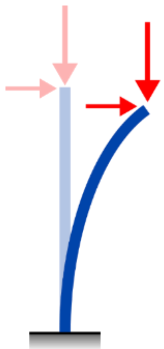


<u>Nonlinearity causing nonlinearity</u>

One of the most fascinating aspects of geometric nonlinearity (GN) is that it often exhibits another type of nonlinearity within itself. Look at the diagram below of a cantilevered column highlighting P-Δ effects (with exaggerated distortion for clarity).

 Copyright© 2022 Klein

405.pdf

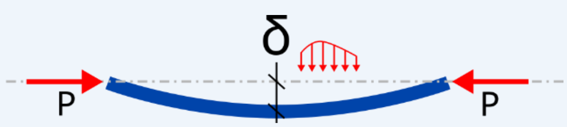


A geometrically nonlinear analysis of the P-Δ effect on the column shown (left) requires structurally analyzing the distorted shape while the member is held in that distorted position. Structural analysis of this deflected and sloped member shape means including *nonlinearity by shape* since it is curved.

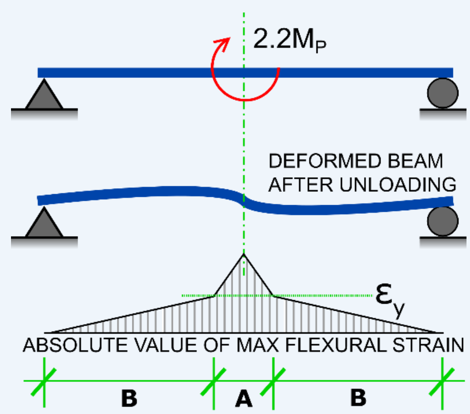
Nonlinear by shape, by proxy

Both GN and material nonlinearity can cause or produce members that are *nonlinear by shape* (physical nonlinearity). Physical nonlinearity is a direct, in-progress effect of GN, and a result and remainder of material nonlinearity.

GN concerns the effects of the distorted geometry of a structure while it is loaded, and curved (nonlinear by shape) members are actively analyzed while the structure and its members are held in the distorted position.



Material nonlinearity consisting of yielding type inelastic behavior involves permanently deforming a structure. Once or if the load is removed only a portion of the movement will recover, and some permanent deformations will remain. This can result in residual stresses in the structure due to the physical nonlinearity introduced from the deformed change in member shape. Refer to the example to the right. The two portions ("B" portions) of the member outside of the plastic deformation region (portion "A") would be trying to elastically return to the undistorted position, and the deformed portion of the member (portion "A") prevents full return, equating to residual strain (stress).



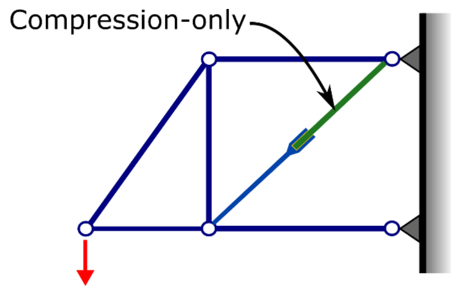
A: PLASTIC HINGE
B: RESIDUAL STRAIN (STRESS)

Inelastic material nonlinearity leaves a structural system with an *initial stress state* and member *nonlinearity by shape* for any further analyses.

 Copyright© 2022 Klein

Degrees-of-Freedom

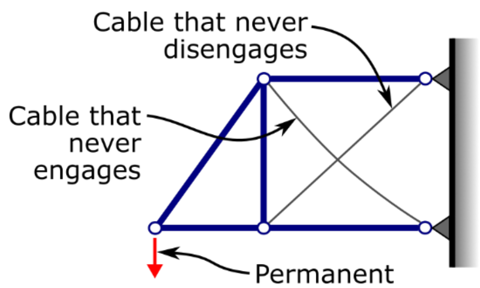
Some nonlinearities, particularly phase-type nonlinearities, can create a DOF for a member or the structure as a whole. The DOF that is created by a nonlinearity could render a determinate structure internally unstable, as shown in the truss containing a compression-only directional member. Therefore, a structural system that is indeterminate to the n^{th} degree could be rendered internally unstable with (n+1) nonlinearities such as directional members or supports.



Directional member creating DOF

When should nonlinearity be included?

Note that there is a difference between consideration and inclusion. Nonlinearity may be considered during the engineering process and found to be inconsequential and hence not included in any analyses. For instance, a structural system with a directional member that never disengages shouldn't need to explicitly include the directional nonlinearity of that member in the analysis. All possible nonlinearities should be considered for every structural system.



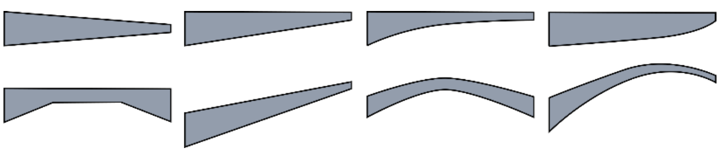
Directional members that are ensured to not trigger or untrigger their directionality may be able to be analyzed as linear members

Simply stated, nonlinearity should be included when its effects will be non-negligible and consequential or significantly influential to the capacity, stability, or serviceability of the structure or its members. This answer is indefinite and can be frustrating to the inexperienced, as it is essentially a more detailed way of saying "it depends". However, here are a few general guidelines:

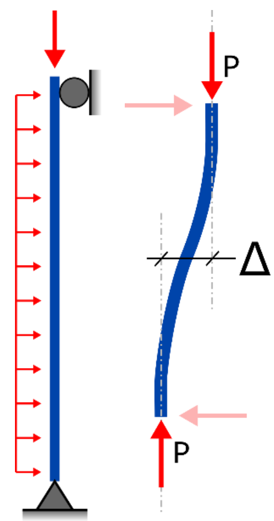
- Initial load/stress states should usually be included if they are present. Directionality should be included if the directional component(s) could be engaged or disengaged during loading.
- If material nonlinearity needs to be included then it is usually known beforehand as an initial scope parameter or analysis requirement, such as performing forensic analysis of failure or as a requirement for seismic design.

Copyright© 2022 Klein

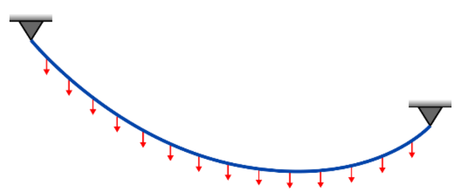
- Friction in its nonlinear format is analyzed with the possibility of the static frictional resistance being broken during the analysis, leading to that DOF being released and a coupled, opposing kinetic friction force being applied. Friction is rarely analyzed this way as it is not typically relied on in conventional building structures to resist superimposed forces or to support reactions. Frictional resistance is usually analyzed by structural engineers after-the-fact, where normal forces determined by linear analysis are used to find and check member forces needed to break static frictional resistance at bearing-type supports.
- *Nonlinearity by shape* should usually be included though there is a way to simplify most analyses of curved and nonprismatic members that is covered in the next course.



- GN will be covered in detail in a later course but the following can be used to anticipate whether or not GN should be included in analyses:
  - Slender columns or other compression members that are both transversely loaded and highly loaded in axial compression are susceptible to P-δ effects.
  - Heavy structures that are laterally flexible are prone to P-Δ effects.
  - *Small deflection theory* continues to apply with reasonable accuracy and GN *large deflections* don't occur until members that are supported at both ends transversely deflect to approximately half the member depth (L/40 to L/60).
  - Cable structures are highly geometrically nonlinear and should almost always be nonlinearly analyzed.



P-δ susceptible (left)
P-Δ effect (right)



It is reassuring to learn that performing analyses that incorporate and accommodate nonlinearity will not hurt. <u>If a nonlinear analysis is performed then it will not produce misleading or erroneous results if those nonlinearities don't occur, aren't engaged, or are found to be inconsequential</u>. However, nonlinear analyses can be more demanding

Copyright© 2022 Klein

of time and expertise, and software with nonlinear capabilities can be relatively expensive so performing unnecessary nonlinear analyses can be inefficient.

Rules of the game – what makes nonlinear different?

Two of the most powerful and beneficial features of linear structural analyses are not valid for nonlinear analyses. The two following rules prohibit these features.

The first rule is that nonlinear analysis results cannot be extrapolated or scaled. If only one load is applied to a nonlinear structural system, and nonlinear analysis of that

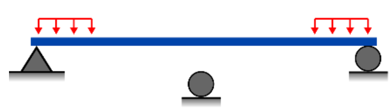
structure with a load of magnitude Q produces an internal member force or deflection/rotation of R, then the nonlinear results for 2Q will not be 2R. Whether the nonlinear results are greater or less than 2R depends on the specific structural system, and the types and characteristics of the nonlinearities.

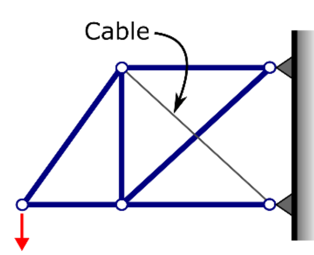
> "Nonlinear Response – Structural behavior in which the deflections are not directly proportional to the loads..."
>
> — AASHTO

Sometimes the inclusion of one type of nonlinearity can lead to a need for including one or several more additional types. For instance, a pushover analysis (material nonlinearity) could require also including directional members such as compression-only infill walls to obtain accurate and useful results. Concurrently acting nonlinearities need to be known because the second rule states that results from separate nonlinear analyses cannot be combined and superimposed. If several types of nonlinearity are present and consequential then they must all be analyzed concurrently.

Exceptions to the rules:

- Rules #1 & #2 – phase-type nonlinearities that have not been triggered or engaged (by loading up to 2Q)





- Rule #2 – phase-type nonlinearities that occur over time are essentially analyzed by superimposing the internal force/stress response for the different phases, but still may not be validly combined with results from another, separate analysis of any type

Copyright© 2022 Klein

Conclusion

Being able to identify nonlinearities in structural systems is an important skill for a practicing engineer. Once nonlinearities are identified they can be isolated, avoided, or analyzed to help provide safe, understood, and predictable structural systems.

The next course focuses on analysis methods for solving structural nonlinearity.

References

1. AASHTO LRFD bridge design specifications, customary U.S. units. Washington, DC: American Association of State Highway and Transportation Officials, 2012.
2. McGuire W, Gallagher RH, Ziemian RD. Matrix structural analysis. 2nd ed. John Wiley & Sons; 1999.
3. Nayfeh AH, Pai PF. Linear and nonlinear structural mechanics. Weinheim, Germany: Wiley-VCH Verlag; 2008.
4. Gere JM. Mechanics of Materials. Belmont, CA: Wadsworth Publishing; 2005.
5. Fertis DG. Nonlinear mechanics, 2nd ed. Boca Raton, FL: CRC Press; 1998.
6. Fertis DG. Advanced mechanics of structures. Boca Raton, FL: CRC Press; 1996.
7. Williams A. Structural analysis: in theory and practice. Burlington: Butterworth-Heinemann; 2009.
8. Bathe KJ. Finite Element Procedures. 2nd ed., 2014. ISBN: 9780979004957
9. Bathe KJ. RES.2-002 Finite Element Procedures for Solids and Structures. Spring 2010. Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu. License: Creative Commons BY-NC-SA.
10. Wierzbicki T. 2.080J Structural Mechanics. Fall 2013. Massachusetts Institute of Technology: MIT OpenCourseWare, https://ocw.mit.edu. License: Creative Commons BY-NC-SA.

Images

All unlisted images were created by the author or are in the public domain.

A. "Horse gallop.gif" by Alice Gee, License: CC BY 3.0
   a. https://upload.wikimedia.org/wikipedia/commons/c/cb/Horse_gallop.gif
   b. https://en.wikipedia.org/wiki/User:GeeAlice

 Copyright© 2022 Klein