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# Convection Heat Transfer Coefficient Estimation

by

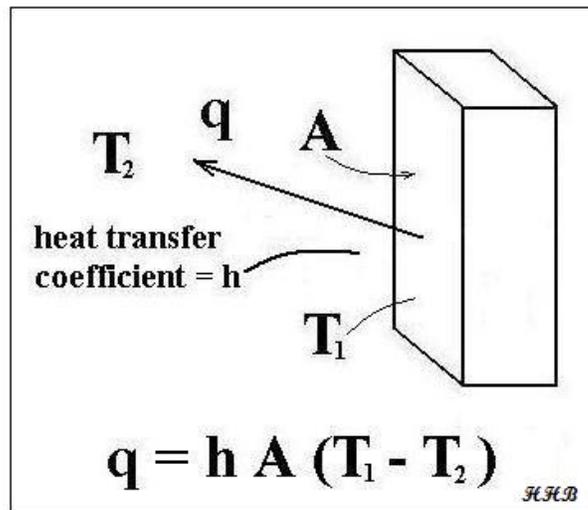
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## 1. Introduction

Convection heat transfer takes place whenever a fluid is in contact with a solid surface that is at a different temperature than the fluid. If the fluid is moving past the solid surface because of an external driving force, like a pump or blower, then it is called forced convection. If fluid motion is due to density differences caused by temperature variation in the fluid, then it is called natural convection or free convection. A major component of most convection heat transfer calculations is obtaining a good estimate for a convection heat transfer coefficient. This course gives some background on Newton's law of cooling, a discussion of the dimensionless numbers used in convection heat transfer correlations, and a description of some of the typical configurations of interest for convection heat transfer. Then the rest of the course is devoted to presentation of correlations and example calculations for estimating natural convection and forced convection heat transfer coefficients. A spreadsheet that will assist in making these calculations for either turbulent or laminar pipe flow is included with the course.



Convection Heat Transfer



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## 2. Learning Objectives

At the conclusion of this course, the student will

- Be able to use Newton's Law of Cooling for forced convection and natural convection heat transfer calculations.
- Be able to calculate the Reynolds number, Prandtl number, Grashof number and Rayleigh number if given suitable information about a fluid flow situation.
- Be able to use the correlations discussed in this course to calculate a value for the convection heat transfer coefficient for turbulent pipe flow.
- Be able to use the correlations discussed in this course to calculate a value for the convection heat transfer coefficient for laminar pipe flow.
- Be able to use the correlations discussed in this course to calculate a value for the convection heat transfer coefficient for turbulent flow through non-circular ducts.
- Be able to use the correlations discussed in this course to calculate a value for the convection heat transfer coefficient for cross flow over a circular cylinder.
- Be able to use the correlations discussed in this course to calculate a value for the convection heat transfer coefficient for flow parallel to a flat plate.
- Be able to use the correlations discussed in this course to calculate a value for the convection heat transfer coefficient for natural convection from a vertical surface.
- Be able to use the correlations discussed in this course to calculate a value for the convection heat transfer coefficient for natural convection from a horizontal surface.
- Be able to use the correlations discussed in this course to calculate a value for the convection heat transfer coefficient for natural convection from an inclined flat surface.



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- Be able to use the correlations discussed in this course to calculate a value for the convection heat transfer coefficient for natural convection from a horizontal cylinder.
- Be able to use the correlations discussed in this course to calculate a value for the convection heat transfer coefficient for natural convection from a sphere.
- Be able to use S.I. units in convective heat transfer coefficient calculations.

### **3. Topics Covered in this Course**

I. Newton's Law of Cooling

II. Dimensionless Numbers for Convection Heat Transfer

III. The Spreadsheet that Came with this Course

IV. Forced Convection Heat Transfer Configurations

V. Turbulent Pipe Flow Correlations

VI. Laminar Pipe Flow Correlations

VII. Turbulent Flow Through Non-Circular Ducts

VIII. Cross Flow Over a Circular Cylinder

IX. Flow Parallel to a Flat Plate

X. Natural Convection Heat Transfer Configurations

XI. Heat Transfer from a Vertical Surface

XII. Heat Transfer from a Horizontal Surface

XIII. Heat Transfer from an Inclined Flat Surface

XIV. Heat Transfer from a Long Horizontal Cylinder

XV. Heat Transfer from a Sphere



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XVI. Use of S.I. units in Heat Transfer Coefficient Calculations

XVII. Summary

XVIII. References

#### 4. Newton's Law of Cooling

Newton's Law of Cooling is an equation that is widely used for both forced convection and natural convection calculations. The equation for Newton's Law of Cooling is:

$Q = h A \Delta T$ . The parameters in the equation and their typical U.S. and S.I. units are as follows:

- **Q** is the rate of heat transfer between the fluid and the solid surface (Btu/hr - U.S.; W - S.I.)
- **A** is the area of the surface that is in contact with the fluid (ft<sup>2</sup> - U.S.; m<sup>2</sup> - S.I.)
- **$\Delta T$**  is the temperature difference between the fluid and the solid surface (°F - U.S.; °C or K - S.I.)
- **h** is the convective heat transfer coefficient (Btu/hr-ft<sup>2</sup>-°F - U.S.; W/m<sup>2</sup>-K - S.I.)

**Example #1:** What is the natural convection heat transfer coefficient for heating the air around a 3 ft diameter sphere if the surface of the sphere is at 150°F, the air temperature is 80°F, and the rate of heat loss from the sphere is 1320 Btu/hr.

**Solution:** Newton's Law of Cooling can be solved for the heat transfer coefficient, **h**, to give  $h = Q/(A \Delta T)$ . The area of the sphere is  $4 \pi r^2 = (4)(\pi)(1.5^2) = 28.3 \text{ ft}^2$ . Substitution into the equation for **h** gives:  $h = (1320)/(28.3)(150 - 80) = \mathbf{0.67 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}$ .



## 5. Dimensionless Numbers for Convection Heat Transfer

When making convection heat transfer calculations, it is well to keep in mind that estimating values for convection heat transfer coefficients is not an exact science. The value of a convection heat transfer coefficient depends upon the physical configuration as well as upon several properties of the fluid involved. Empirical correlations are available to estimate heat transfer coefficients for a variety of natural convection and forced convection heat transfer configurations and will be presented and discussed in this course.

Those correlations are typically expressed in terms of dimensionless numbers. The dimensionless numbers used for forced convection heat transfer coefficients are the Nusselt number (Nu), Prandtl number (Pr), and Reynolds number (Re). Definitions of Nu, Pr, and Re are shown below. The heat transfer coefficient,  $h$ , appears in the Nusselt number, so the correlations are typically in the form of an equation for Nu in terms of Re and Pr.

$$\text{Nu} = \frac{h D}{k} \quad \text{Re} = \frac{D V \rho}{\mu}$$
$$\text{Pr} = \frac{v}{\alpha} = \frac{\mu C_p}{k}$$

Dimensionless Numbers used in  
Forced Convection Heat Transfer  
Coefficient Correlations

The parameters appearing in the Nusselt, Prandtl, and Reynolds numbers and their U.S. and S.I. units are as follows:



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- $k$  is the thermal conductivity of the fluid in Btu/sec-ft-°F (U.S.) or kJ/s-m-K (S.I.)
- $D$  is a characteristic length parameter, such as diameter for flow through a pipe or flow around a circular cylinder in ft (U.S.) or m (S.I.)
- $V$  is a characteristic velocity, such as the average velocity for pipe flow in ft/sec (U.S.) or m/s (S.I.)
- $\rho$  is the density of the fluid in slugs/ft<sup>3</sup> (U.S.) or kg/m<sup>3</sup> (S.I.)
- $\mu$  is the viscosity of the fluid in lb-sec/ft<sup>2</sup> (U.S.) or N-s/m<sup>2</sup> (S.I.)
- $C_p$  is the heat capacity of the fluid in Btu/slug-°F (U.S.) or kJ/kg-K (S.I.)

The dimensionless numbers typically used for natural convection heat transfer coefficient correlations are the Nusselt number (Nu), Prandtl number (Pr), Rayleigh number (Ra), and Grashof number (Gr). Nu and Pr were defined above. Equations for the Grashof number and Rayleigh number are:

$$\mathbf{Gr = D^3\rho^2g\Delta T\beta/\mu^2} \quad \text{and} \quad \mathbf{Ra = (Gr)(Pr)}$$

The additional parameters that weren't defined above are:

- $g$  is the acceleration due to gravity(32.17 ft/sec<sup>2</sup> - U.S. Or 9.81 m/s<sup>2</sup> - S.I.)
- $\beta$  is the coefficient of volume expansion of the fluid in °R (U.S.) or K (S.I.)

## 6. Forced Convection Heat Transfer Configurations

Forced convection heat transfer takes place when a fluid is pumped or blown past a solid surface that is at a different temperature than the fluid. The forced convection



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configurations for which heat transfer coefficient correlations will be presented and discussed in this course are:

- Turbulent flow inside a circular pipe
- Laminar flow Inside a circular pipe
- Turbulent Flow through a non-circular duct
- Flow across a single circular cylinder
- Flow parallel to a flat plate

## **7. The Spreadsheet that Came with this Course**

A spreadsheet to assist with estimation of forced convection heat transfer coefficients for pipe flow is included with this course. The spreadsheet includes a tab for each of the first two forced convection heat transfer configurations just listed (turbulent pipe flow and laminar pipe flow). For each configuration there is provision for input of information such as pipe diameter, wall temperature, fluid bulk temperature and fluid properties. The spreadsheet then calculates heat transfer coefficients using one or more of the correlations discussed in the next two sections.

A Note on References: The first two references at the end of this course are heat transfer textbooks. One of them is available for free download. Both of them have a lot of theory and background to go with the convective heat transfer coefficient correlations that are presented and discussed in this course. Most of the convective heat transfer correlations discussed in this course are given in one or both of these two textbooks. A primary reference is also given for each of the correlations when it is introduced in the body of the course.



## 8. Forced Convection for Turbulent Flow inside a Circular Tube

There are several correlations available for calculation of the convective heat transfer coefficient for turbulent flow of a fluid in a pipe, with the fluid and pipe at different temperatures. The temperature of the pipe may be either hotter or colder than the fluid. Or in other words, the fluid may be either heated or cooled by the pipe.

**Classic Correlations:** - A classic correlation for the convection heat transfer for turbulent flow in a pipe is the Dittus-Boelter equation (ref #3), which was published in 1930, as:

$$\text{Nu} = 0.0243 \text{Re}^{0.8} \text{Pr}^{0.4} \quad \text{for heating of the fluid } (T_{\text{wall}} > T_{\text{fluid}})$$

and 
$$\text{Nu} = 0.0265 \text{Re}^{0.8} \text{Pr}^{0.3} \quad \text{for cooling of the fluid } (T_{\text{wall}} < T_{\text{fluid}})$$

In subsequent years, the equations have been revised somewhat and the equation,  $\text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4}$ , has come to be known as the Dittus-Boelter equation, with a note that the exponent on Pr should be 0.3 if the fluid is being cooled. See Witherton (Ref #4) for discussion of this change in the equation that has come to be known as the Dittus-Boelter equation.

The Dittus-Boelter equation is valid for smooth pipes and for:

$$0.6 \leq \text{Pr} \leq 160$$

$$\text{Re}_D \geq 10,000$$

$$L/D \geq 10$$

The Dittus Boelter equation is recommended only for rather small temperature differences between the bulk fluid and the pipe wall.

A few years later, in 1936, Sieder and Tate (ref #5) proposed the following equation to accommodate larger temperature differences:



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$$\text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^{1/3} (\mu_b/\mu_w)^{0.14}$$

Where:  $\mu_b$  is the fluid viscosity at the fluid bulk temperature  
 $\mu_w$  is the fluid viscosity at the pipe wall temperature

The fluid bulk temperature at any cross section in the pipe is the average fluid temperature over that cross section. The Sieder-Tate equation is valid for smooth pipes and for:

$$0.7 \leq \text{Pr} \leq 16,700$$

$$\text{Re}_D \geq 10,000$$

$$L/D \geq 10$$

Both the Dittus-Boelter and Sieder-Tate equations are still in widespread use.

**Example #2:** Calculate the convection heat transfer coefficient for flow of water at an average bulk temperature of 85°F through a 2 inch diameter pipe that is at 120°F using the Dittus-Boelter equation and using the Sieder-Tate equation. The average velocity of the water in the pipe is 1.8 ft/sec.

**Solution:** Values for the density, viscosity, specific heat, and thermal conductivity of water are needed at the fluid bulk temperature (85°F) and the viscosity is also needed at the wall temperature (120°F). Tables and/or graphs of fluid properties are available in many fluid mechanics, thermodynamics, and heat transfer textbooks and handbooks. They can also be obtained from various websites through a search for “viscosity of water,” “thermal conductivity of air,” etc.

For water at 85°F: density = 1.93 slugs/ft<sup>3</sup>, viscosity = 1.64 x 10<sup>-5</sup> lb-sec/ft<sup>2</sup> (slug/ft-sec), specific heat = 32.2 Btu/slug-°F, and thermal conductivity = 0.33 Btu/hr-ft-°F.



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Calculation of Re and Pr: The Reynolds number (Re) and Prandtl number (Pr) are needed for both equations:

$$Re_D = DV\rho/\mu = (2/12)(1.8)(1.93)/(1.64 \times 10^{-5}) = 35,305$$

$$Pr = \mu C_p/k = 3600(1.64 \times 10^{-5})(32.2)/(0.33) = 5.8$$

(the 3600 factor is needed to convert sec to hr)

Dittus-Boelter equation:  $Nu = 0.023 Re^{0.8} Pr^{0.4}$

$$= 0.023(35305)^{0.8}(5.8)^{0.4} = 201$$

Calculate the heat transfer coefficient, h, from the definition of Nu ( $Nu = hD/k$ )

$$h = (Nu)(k)/D = 201*0.33/(2/12) = \underline{\underline{399 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}}$$

Sieder-Tate equation:  $Nu = 0.023 Re^{0.8} Pr^{1/3} (\mu_b/\mu_w)^{0.14}$

For this equation, the viscosity of the water at the wall temperature (120°F) is also needed. ( $\mu_w = 1.16 \times 10^{-5}$  lb-sec/ft<sup>2</sup> (slug/ft-sec))

$$\text{Thus: } Nu = 0.023(35305)^{0.8}(5.8)^{1/3}(1.64 \times 10^{-5}/1.16 \times 10^{-5})^{0.14} = 188$$

Calculate the heat transfer coefficient, h, from the definition of Nu ( $Nu = hD/k$ )

$$h = (Nu)(k)/D = 188*0.33/(2/12) = \underline{\underline{372 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}}$$

NOTE: The calculations using the Dittus-Boelter and Sieder-Tate correlations can be conveniently done with the spreadsheet that was included with this course. The figure on the next page shows the first page of the turbulent pipe flow spreadsheet with the input values for Example #1 and the calculation of Nu and h using the Dittus-Boelter



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correlation and the Sieder-Tate correlation. You can change the input values to calculate the forced convection heat transfer coefficient using the Dittus-Boelter and Sieder-Tate equations for any turbulent or laminar pipe flow situation.

Forced Convection Heat Transfer Coefficient Correlations					
1. Turbulent Flow Inside a Circular Tube - U.S. units					
<b>Instructions:</b> Enter values in blue boxes. Spreadsheet calculates values in yellow boxes					
fluid:	water				
<b>Inputs</b>			<b>Calculations</b>		
Ave. Fluid Temp, $T_b =$	85	°F	Reynolds No., $Re =$	35,305	
Pipe Diam., $D =$	2	in	Prandtl No., $Pr =$	5.8	
Pipe Diam., $D =$	0.167	ft	<u>Correlation #1, the Dittus-Boelter Correlation</u>		
(calculated)					
Ave. Velocity, $V =$	1.8	ft/sec	$T_{wall} > T_{fluid}$ :	$Nu =$	201
Fluid Density, $\rho =$	1.93	slugs/ft <sup>3</sup>		$h =$	399 Btu/hr-ft <sup>2</sup> -°F
Fluid viscosity, $\mu =$	0.0000164	lb-sec/ft <sup>2</sup>	$T_{wall} < T_{fluid}$ :	$Nu =$	169
Fluid Sp. Heat, $C_p =$	1	Btu/lb-°F		$h =$	335 Btu/hr-ft <sup>2</sup> -°F
Fluid Sp. Heat, $C_p =$	32.2	Btu/slug-°F			
(calculated)			<u>Correlation #2, the Sieder-Tate Correlation:</u>		
Fluid Thermal					
Conductivity, $k =$	0.33	Btu/hr-ft-°F		$Nu =$	188
Pipe Length, $L =$	15	ft		$h =$	372 Btu/hr-ft <sup>2</sup> -°F
Ave. Wall Temp, $T_w =$	120	°F			
Fluid Visc. at $T_w$ , $\mu_w =$	1.164E-05	lb-sec/ft <sup>2</sup>			



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**More Recent Correlations** – In 1970 Petukhov (ref #6) presented the following equation, which is somewhat more complicated, but provides greater accuracy than the Dittus-Boelter and Sieder-Tate equations. The  $f$  in the following equations is the Moody friction factor. The equation given here is from ref #1, for  $10^4 < Re < 5 \times 10^6$ , with smooth pipe (independent of  $\epsilon/D$ ).

$$\text{Nu}_o = \frac{\left(\frac{f}{8}\right) (\text{Re Pr})}{1.07 + 12.7 \left(\frac{f}{8}\right)^{0.5} (\text{Pr}^{\frac{2}{3}} - 1)}$$

where  $f = (0.790 \ln \text{Re} - 1.64)^{-2}$

for:  $0.5 < \text{Pr} < 2000$   
 $10^4 < \text{Re} < 5 \times 10^6$        $L/D \geq 10$

In 1976, Gnielinsky (ref #7) came out with the following slight revision, which extends coverage into the transition region, down to a Reynolds number of 3000.

$$\text{Nu}_o = \frac{\left(\frac{f}{8}\right) (\text{Re} - 1000) \text{Pr}}{1 + 12.7 \left(\frac{f}{8}\right)^{0.5} (\text{Pr}^{\frac{2}{3}} - 1)}$$

where  $f = (0.790 \ln \text{Re} - 1.64)^{-2}$

for:  $0.5 < \text{Pr} < 2000$   
 $3000 < \text{Re} < 5 \times 10^6$        $L/D \geq 10$



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**Correction for Variations in Fluid Properties** – Lienhardt & Lienhardt (ref #2) give the following update of the Sieder-Tate viscosity factor to correct for variations in fluid properties.

After calculating  $Nu_o$  at the fluid bulk temperature, using either the Petukhov correlation or the Gnielinsky correlation, that value of Nusselt number should be corrected for fluid property variations with one of the following equations:

For **liquids** with  $0.025 \leq \mu_b/\mu_w \leq 12.5$ :

$$Nu_D = Nu_o(\mu_b/\mu_w)^n, \text{ with } n = 0.11 \text{ for fluid heating and } n = 0.25 \text{ for fluid cooling}$$

For **gases** with absolute temperature ratio:  $0.27 \leq T_b/T_w \leq 2.7$

$$Nu_D = Nu_o(T_b/T_w)^n, \text{ with } n = 0.47 \text{ for fluid heating and } n = 0 \text{ for fluid cooling}$$

**Example #3:** Calculate the convective heat transfer coefficient using the Petukhov and Gnielinsky correlations, for the same conditions as Example #1 (water at an average bulk temperature of 85°F through a 2 inch diameter pipe that is at 120°F, with the water velocity in the pipe being 1.8 ft/sec.

**Solution:** As calculated in Example #2:  $Re = 35,305$  and  $Pr = 5.8$ . Substituting into the equation for  $f$ :

$$f = (0.790 \ln(35,305) - 1.64)^{-2} = 0.02273$$

Substituting values for  $f$ ,  $Re_D$  and  $Pr$  into either the Petukhov correlation or the Gnielinsky correlation gives  $Nu_o = 225$ . (Both give the same result in this case.)

Fluid property variation factor:  $\mu_b/\mu_w = 1.64/1.16 = 1.41$ , thus

$$Nu_D = Nu_o(\mu_b/\mu_w)^{0.11} = (225)(1.41^{0.11}) = 245$$



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$$h = (\text{Nu})(k)/D = 245*0.33/(2/12) = \underline{\underline{485 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}}$$

This value of  $h$  is about 20% higher than the Dittus-Boelter estimate and about 30% higher than the Sieder-Tate estimate.

The heat transfer coefficient spreadsheet that came with this course is set up to calculate the forced convection coefficient for turbulent pipe flow using the Gielinsky correlation on the “turb. pipe flow” tab in the section following that in which Dittus-Boelter and Sieder-Tate calculations are made. The calculations in Example #3 can be confirmed with the spreadsheet and the heat transfer coefficient for other turbulent pipe flow cases can be calculated using the Gielinsky correlation.

## 9. Forced Convection for Laminar Flow inside a Circular Tube

Convection heat transfer associated with laminar flow in a circular tube ( $Re < 2300$ ) is less common than with turbulent flow. The correlations are rather simple, however, with the Nusselt number being constant for fully developed flow. The  $L/D$  ratio for the entrance length required to reach fully developed flow is larger for laminar flow than for turbulent flow, however, so practical situation with entrance region flow are possible. Two correlations for use in the entrance region are included in this discussion.

**Fully Developed Flow** - Both of the first two references at the end of this course give the following expressions for the Nusselt number in fully developed laminar flow in a pipe (i.e.  $L \gg L_e$ ):

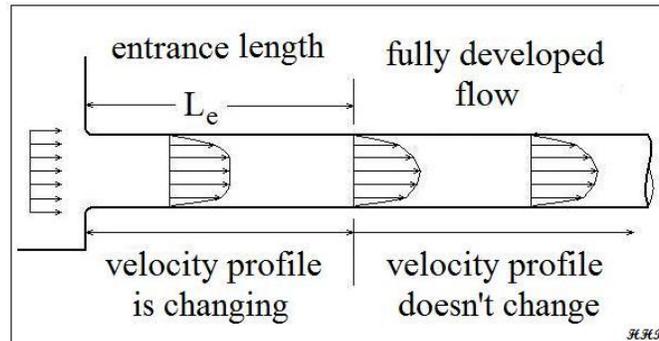
For uniform wall heat flux:  **$Nu_D = 4.36$**

For uniform wall temperature:  **$Nu_D = 3.66$**

The entrance length for pipe flow ( $L_e$ ) is the portion of the pipe in which the velocity profile is changing. The velocity profile remains the same, however, throughout the fully developed flow portion of the pipe, as illustrated in the diagram below.



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For laminar flow, the entrance length can be estimated from the equation:

$$L_e/D = 0.06 Re_D$$

**Laminar Entry Region Flow** – Incropera et al (ref #1) gives the following two correlations for use in estimating convection heat transfer coefficients for laminar entry region flow.

Laminar Entry Region Correlation #1 ( $L < L_e$ ):

Thermal or Thermal/Hydrodynamic Entrance Region,  
Constant Wall Temperature,  $Pr \geq 5$   
Laminar Pipe Flow,  $Re < 2300$

$$Nu_D = 3.66 + \frac{0.0668 Re Pr (D/L)}{1 + 0.04 [Re Pr (D/L)]^{2/3}}$$



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Laminar Entry Region Correlation #2 ( $L < L_e$ ):

Thermal and Hydrodynamic Entrance Region  
Constant Wall Temperature - Laminar Flow ( $Re < 2300$ )  
 $0.6 < Pr < 5$        $0.0044 < \frac{\mu_b}{\mu_w} < 9.75$

$$Nu_D = 1.86 \left( \frac{Re Pr}{L/D} \right)^{1/3} \left( \frac{\mu_b}{\mu_w} \right)^{0.14}$$

**Example #4:** Estimate the convection heat transfer coefficient for water flowing through a 2 inch diameter pipe at a velocity of 0.1 ft/sec. The average bulk temperature of the water is 85°F and the pipe wall temperature is constant at 120°F. Estimate the heat transfer coefficient for **a)** the case where  $L \gg L_e$  (fully developed flow) and **b)**  $L < L_e$  (entry region)

**Solution:** The properties of water at 85°F that were used in Example #2 and #3 can be used here (density = 1.93 slugs/ft<sup>3</sup>, viscosity = 1.64 x 10<sup>-5</sup> lb-sec/ft<sup>2</sup> (slug/ft-sec), specific heat = 32.2 Btu/slug-°F, and thermal conductivity = 0.33 Btu/hr-ft-°F.) The viscosity of water at 120°F is 1.16 x 10<sup>-5</sup> lb-sec/ft<sup>2</sup> (slug/ft-sec).

The Prandtl number will be the same as in Example #2:  $Pr = 5.8$

The Reynolds number can be calculated as:

$$Re_D = DV\rho/\mu = (2/12)(0.1)(1.93)/(1.64 \times 10^{-5}) = 1961$$

For fully developed flow ( $L \gg L_e$ ) with uniform wall temperature:  $Nu_D = 3.66$

Thus:  $h = (Nu)(k)/D = 3.66 * 0.33 / (2/12) = \underline{7.2 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}$



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For laminar entry region flow ( $L < L_e$ ), correlation #1 should be used here, because  $Pr > 5$ .  
From that correlation:  **$Nu_D = 7.8$**

Thus:  $h = (Nu)(k)/D = 7.8 * 0.33 / (2/12) = \underline{\underline{15.5 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}}$

These calculations can also be conveniently made with the course spreadsheet, as shown in the screenshot on the next page. This tab on the spreadsheet has provision for entering the following input parameters: the pipe diameter,  $D$ ; the pipe length,  $L$ ; the entrance length,  $L_e$ ; the average velocity of the fluid in the pipe,  $V$ ; the average bulk fluid temperature,  $T_b$ ; and the following fluid properties at the average bulk fluid temperature: density, viscosity, specific heat, and thermal conductivity. The spreadsheet then makes some unit conversions, calculates the Reynolds number and Prandtl number, and then calculates the Nusselt number and heat transfer coefficient using each of the correlations discussed above in this section.



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Forced Convection Heat Transfer Coefficient Correlations					
2. Laminar Flow Inside a Circular Tube - U.S. or S.I. units					
<b>Instructions:</b> Enter values in blue boxes. Spreadsheet calculates values in yellow boxes					
Click on the blue cell below and on the arrow to the right of it. Then use the drop down list to select the units you want to use:					
U.S. Units					
<b>Inputs</b>			<b>Calculations</b>		
Fluid =	water		Reynolds Number, Re =	1961	
			Prandtl Number, Pr =	5.8	
Ave. Fluid Temp, $T_b$ =	85	deg F			
Pipe Diam., D =	2	in	<b>Correlations #1, Fully Developed Flow ( <math>L \gg Le</math> )</b>		
Pipe Diam., D =	0.167	ft			
(calculated)			Const. $T_{wall}$	Nu = 3.66	
Ave. Velocity, $V$ =	0.1	ft/sec		h = 7.2	Btu/hr-sq ft-deg F
Fluid Density, $\rho$ =	1.93	slugs/cu ft			
Fluid viscosity, $\mu$ =	0.0000164	lb-sec/sq ft		Nu = 4.36	
Fluid Thermal			Const. heat flux	h = 8.6	Btu/hr-sq ft-deg F
Conductivity, k =	0.33	Btu/hr-ft-deg F			
Fluid Sp. Heat, $C_p$ =	1	Btu/lb-deg F	<b>Correlation #2 ( <math>Pr &gt; 5 ; Le &gt; L</math> ):</b>		
Fluid Sp. Heat, $C_p$ =	32.2	Btu/slug-deg F			
(calculated)				Nu = 7.8	
Pipe Length, L =	15	ft		h = 15.5	Btu/hr-sq ft-deg F
Entrance Length, $Le$ =	19.6	ft			
(calculated)			<b>Correlation #3 ( <math>0.6 &lt; Pr &lt; 5 ; Le &gt; L</math> ):</b>		
Ave. Wall Temp, $T_w$ =	120	deg F			
Fluid Visc. at $T_w$ , $\mu_w$ =	0.0000116	lb-sec/sq ft		Nu = 9.8	
				h = 19.3	Btu/hr-sq ft-deg F
NOTE: The spreadsheet calculates the forced convection heat transfer coefficient, h, for each of the three correlations. You need to choose the correct one to use.					



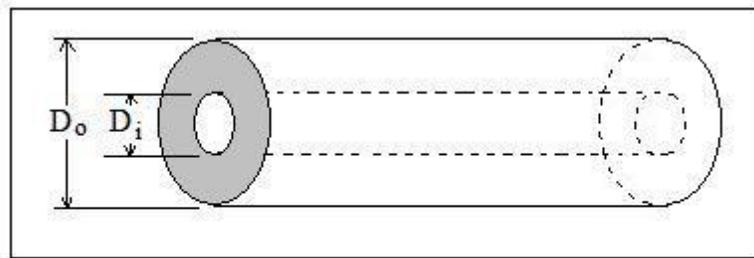
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## 10. Forced Convection for Turbulent Flow Through a Non-Circular Duct

For turbulent flow through a non-circular duct, the turbulent flow, circular pipe correlations discussed in Section 8 can be used with diameter,  $D$ , replaced by the hydraulic diameter,  $D_H$ , where:

$$D_H = 4(A/P), \text{ with } A = \text{cross-sectional area of flow and } P = \text{wetted perimeter}$$

**Flow Through a Circular Annulus** – A particular non-circular cross section of some interest is the circular annulus. An example would be flow through the shell side of a double-pipe heat exchanger, i.e. through the shaded area in the diagram below.



Flow Through a Circular Annulus

H.H.B

The hydraulic diameter for flow through the circular annulus shown above is:

$$D_H = 4(A/P) = 4[ (\pi/4)(D_o^2 - D_i^2) ] / [ \pi(D_o + D_i) ]$$

Simplifying the expression:  $D_H = (D_o - D_i)$  ( for flow through a circular annulus )

**Example #5:** Use the Gnielinski correlation to estimate the convection heat transfer coefficient for flow of water at 85°F through a circular annulus (wall temperature = 120°F) with an outside diameter of 4 inches and an inside diameter of 3 inches. The average velocity of the water in the annulus is 1.3 ft/sec.



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**Solution:** The hydraulic diameter for this flow is:  $D_H = D_o - D_i = 4'' - 3'' = 1''$ . So 1'' will be used for the diameter in these calculations. The properties of water at 85°F will be the same as those used for the previous examples. (density = 1.93 slugs/ft<sup>3</sup>, viscosity = 1.64 x 10<sup>-5</sup> lb-sec/ft<sup>2</sup> (slug/ft-sec), specific heat = 32.2 Btu/slug-°F, and thermal conductivity = 0.33 Btu/hr-ft-°F.) Using these water property values together with  $D_H = 1''$  and  $V = 1.3$  ft/sec, the Reynolds number and Prandtl number can be calculated.

$$Re_D = DV\rho/\mu = (1/12)(1.3)(1.93)/(1.64 \times 10^{-5}) = 12,749$$

$$Pr = \mu C_p/k = 3600(1.64 \times 10^{-5})(32.2)/(0.33) = 5.8$$

Substituting the value of Re into the equation for f:

$$f = (0.790 \ln(12,749) - 1.64)^{-2} = 0.02944$$

Substituting values of f, Re, and Pr into the Gnielinsky equation gives:

$Nu_o = 92$  with fluid properties at the average bulk fluid temperature

The viscosity of water at the wall temperature (120°F) is 1.16 x 10<sup>-5</sup> lb-s/ft<sup>2</sup>

Fluid property variation factor:  $\mu_b/\mu_w = 1.64/1.16 = 1.414$ , thus

$$Nu_D = Nu_o(\mu_b/\mu_w)^{0.11} = (92)(1.414^{0.11}) = 96$$

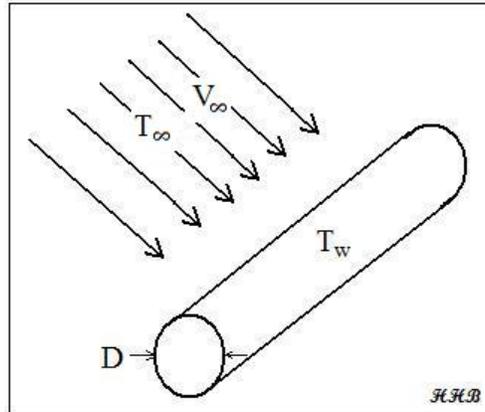
$$h = (Nu)(k)/D = 96*0.33/(1/12) = \underline{\underline{379 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}}$$

**NOTE:** In his 1950 book, Process Heat Transfer (Ref #16), Kern recommended using  $D_H = (D_o^2 - D_i^2)/D_i$  as the hydraulic radius for convective heat transfer calculations for flow through a circular annulus. If this expression is used instead of  $D_H = (D_o - D_i)$  in Example #5, the results are as follows:  $D_H = 2.33$  in = 0.1944 ft,  $Re = 29,748$ ,  $f = 0.0237$ ,  $Nu_o = 193.7$ ,  $Nu_D = 201.3$ , and  $h = 341.6$ . For design, this is a more conservative estimate.



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## 11. Forced Convection for Cross Flow over a Circular Cylinder



An experimental correlation with widespread application for cross flow over a circular cylinder, prepared by Churchill and Bernstein (ref #8), is given in both ref #1 and ref #2 as the following equation:

$$\text{Nu}_D = 0.3 + \frac{0.62 \text{Re}_D^{1/2} \text{Pr}^{1/3}}{\left[1 + (0.4/\text{Pr})^{2/3}\right]^{1/4}} \left[1 + \left(\frac{\text{Re}_D}{282,000}\right)^{5/8}\right]^{4/5}$$

for all Re and  $\text{Pr} \geq 0.2$

Note that i) the characteristic length,  $D$ , for  $\text{Re}$  and  $\text{Nu}$  is the diameter of the cylinder, ii) the characteristic velocity,  $V$ , for the Reynolds number is the approach velocity,  $V_\infty$ , and iii) the fluid properties are to be determined at the approach temperature,  $T_\infty$ , as shown in the diagram above.

**Example #6:** Use the Churchill and Bernstein correlation to estimate the convective heat transfer coefficient for cross flow of water at 85°F and an approach velocity of 1.3 ft/sec over a 3 inch diameter cylinder.



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**Solution:** Using the same properties of water at 85°F that we've been using in the other examples the Prandtl number will be 5.8 and the Reynolds number will be:

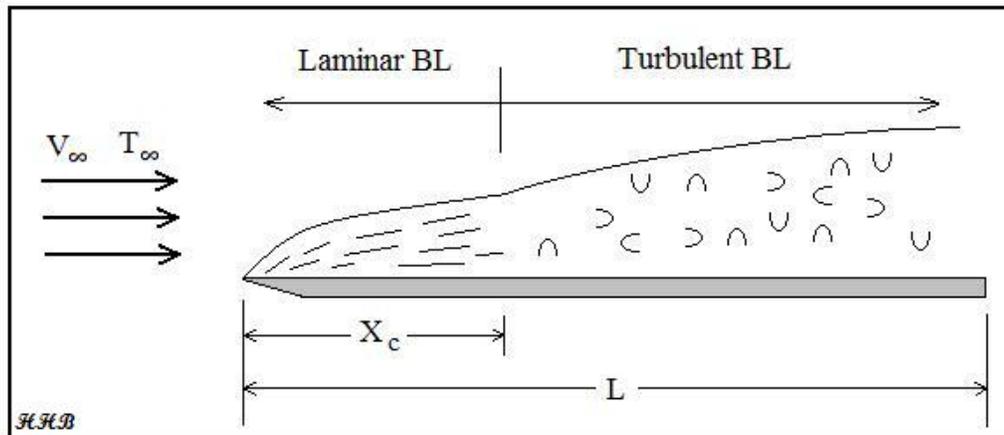
$$Re_D = DV\rho/\mu = (3/12)(1.3)(1.93)/(1.64 \times 10^{-5}) = 38,247$$

Substituting values for  $Re_D$  and  $Pr$  into the Churchill and Bernstein correlations gives:

$Nu_D = 256$ , and the heat transfer coefficient will be:

$$h = (Nu_D)(k)/D = 256*0.33/(3/12) = \underline{\underline{338 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}}$$

## 12. Forced Convection for Flow Parallel to a Flat Plate



Boundary Layer Flow Over a Flat Plate

Equations for heat transfer coefficients for forced convection in fluid flow parallel to a flat plate, can be obtained by consideration of the boundary layer flow adjacent to the plate.



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Laminar boundary layer flow and turbulent boundary layer flow will each be considered separately.

Laminar Boundary Layer Flow: The Reynolds number for flow parallel to a flat plate is defined as:  $Re_L = LV_\infty\rho/\mu$ , where L is the length of the plate in the direction of flow and  $V_\infty$  is the approach velocity. The flow in a boundary layer for flow parallel to a flat plate will be laminar from the leading edge of the plate up to the point where the Reynolds number reaches a critical value (typically in the range from  $3 \times 10^5$  to  $3 \times 10^6$ ), at which the flow in the boundary layer changes to turbulent flow. The critical length,  $X_c$ , is the term used for the distance from the leading edge of the flat plate to the point at which the flow in the boundary layer changes from laminar to turbulent, as shown in the diagram at the beginning of this section.

Lienhard and Lienhard (ref #2) show the derivation of the following correlation for laminar boundary layer flow over a flat plate, using the Blasius similarity solution:

$$Nu_L = hL/k = 0.664 Re_L^{1/2} Pr^{1/3}$$

$$\text{Subject to: } Pr \geq 0.6 \quad \text{and} \quad Re_L < 200,000$$

Turbulent Boundary Layer Flow: Lienhard & Lienhard also give the following equation (attributed to Zukauskas & Slanciauskas (ref #9)), for turbulent boundary layer flow over a flat plate:

$$Nu_L = hL/k = (0.037 Re_L^{4/5} - A) Pr^{1/3}$$

$$A = 0.037 Re_{x,c}^{4/5} - 0.664 Re_{x,c}^{1/2}$$

$$\text{Subject to: } 0.6 \leq Pr \leq 60$$

$$Re_{x,c} \leq Re_L < 10^8$$

Where,  $Re_{x,c}$  is the Reynolds number at the transition from laminar to turbulent flow.



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**Example #7:** Estimate the heat transfer coefficient for water at 85°F flowing with an approach velocity of 2.5 ft/sec over a 6 inch long flat plate at 120°F.

**Solution:** The needed properties of water at 85°F are: density = 1.93 slugs/ft<sup>3</sup>, viscosity = 1.64 x 10<sup>-5</sup> lb-sec/ft<sup>2</sup> (slug/ft-sec), specific heat = 32.2 Btu/slug-°F, and thermal conductivity = 0.33 Btu/hr-ft-°F.

The Prandtl number will be the same as in the previous Examples: Pr = 5.8

The Reynolds number can be calculated as:

$$Re_L = LV\rho/\mu = (6/12)(2.5)(1.93)/(1.64 \times 10^{-5}) = 147,104$$

Since  $Re_L$  is less than 200,000, the correlation for laminar flow should be used:

$$Nu_L = 0.664 Re_L^{1/2} Pr^{1/3}$$
$$= 0.664(147,104)^{1/2}(5.8)^{1/3} = 456, \quad \text{and the heat transfer coefficient will be:}$$

$$h = (Nu_D)(k)/D = 456*0.33/(6/12) = \underline{\underline{301 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}}$$

### 13. Natural Convection Heat Transfer Configurations

Natural convection heat transfer takes place when a fluid is in contact with a solid surface that is at a different temperature than the fluid and fluid motion is not caused by an external driving force such as a pump or blower. With natural convection, fluid motion is caused by fluid density differences due to temperature variation within the fluid. The natural convection solid surface configurations for which heat transfer coefficient correlations will be presented and discussed in this course are:

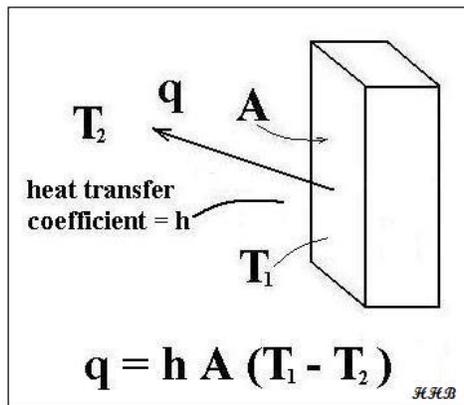
- Heat transfer from a vertical surface



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- Heat transfer from a horizontal surface
- Heat transfer from an inclined flat surface
- Heat transfer from a horizontal cylinder
- Heat transfer from a sphere

#### 14. Natural Convection from a Vertical Surface



Convection Heat Transfer

Natural convection heat transfer between a fluid and a vertical surface will take place whenever a fluid is in contact with a vertical surface that is at a temperature different from the fluid, as shown in the figure above. If the solid surface is hotter than the fluid, then the fluid adjacent to the surface will be heated, its density will decrease, and it will rise, causing a natural circulation flow. Hence the name “natural convection” for this type of heat transfer.



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The two correlations below, from Churchill and Chu (ref # 10), are given in Incropera et al (ref #1). Keep in mind that the two new dimensionless numbers introduced for natural convection correlations are the Grashof number (  $Gr = D^3\rho^2g\Delta T\beta/\mu^2$  ) and the Rayleigh number (  $Ra = GrPr$  ). The thermal expansion coefficient,  $\beta$ , is just the inverse of the absolute temperature for an ideal gas, so it will have units of  $^{\circ}R^{-1}$  for U.S. units or  $K^{-1}$  for S.I. units. The two correlations for Nu are:

for all values of Ra:

$$Nu = \left\{ 0.825 + \frac{0.387 Ra^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}} \right\}^2$$

Slightly better for laminar flow (  $Ra \leq 10^9$  ):

$$Nu = 0.68 + \frac{0.670 Ra^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}}$$

Note that for this natural convection configuration, the length parameter, D, in the Nusselt number and in the Grashof number is the height of the vertical surface.

The temperature to be used for fluid properties for natural convection is typically the film temperature,  $T_f$ , defined as follows:

$$T_f = (T_{\infty} + T_w)/2, \text{ where}$$

$T_{\infty}$  = the temperature of the fluid far from the vertical surface

$T_w$  = the temperature of the vertical surface



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**Example #8:** Estimate the heat transfer coefficient for natural convection between a 5 ft high vertical surface at 120°F and air at 85°F that is in contact with that surface.

**Solution:** The film temperature is equal to  $(85 + 120)/2 = 103^\circ\text{F}$ . The needed properties of air at 103°F are:

- Density,  $\rho = 0.00221 \text{ slugs/ft}^3$
- Viscosity,  $\mu = 3.94 \times 10^{-7} \text{ lb-sec/ft}^2$
- Specific Heat,  $C_p = 7.7 \text{ Btu/slug-}^\circ\text{F}$
- Thermal conductivity,  $k = 0.0157 \text{ Btu/hr-ft-}^\circ\text{F}$

The thermal expansion coefficient is calculated as follows:

$$\beta = 1/T_f = 1/(103 + 459.67) = 0.001779 \text{ }^\circ\text{R}^{-1}$$

The dimensionless numbers, Pr, Gr, & Ra, are then calculated as follows:

$$\text{Pr} = \mu C_p / k = 3600(3.94 \times 10^{-7})(7.7)/(0.0157) = 0.70$$

$$\text{Gr} = D^3 \rho^2 g \Delta T \beta / \mu^2 = (5^3)(0.00221^2)(32.17)(120 - 85)(0.001779)/((3.94 \times 10^{-7})^2)$$

$$\text{Gr} = 7.88 \times 10^9$$

$$\text{Ra} = \text{GrPr} = (7.88 \times 10^9)(0.70) = 5.50 \times 10^9$$

Since  $\text{Ra} > 10^9$ , the first correlation above should be used. Substituting  $\text{Pr} = 0.7$  and  $\text{Ra} = 5.50 \times 10^9$ , into the equation for the first correlation above gives

$\text{Nu} = 208$ , The convection heat transfer coefficient,  $h$  is then:

$$h = (\text{Nu}_D)(k)/D = 208 * 0.0157 / (5) = \underline{\underline{0.65 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}}$$



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## 15. Natural Convection from a Horizontal Surface

Convection heat transfer to or from a horizontal surface is complicated by the fact that the heat transfer may be with a fluid either above or below a horizontal plane and the surface may be either hotter or colder than the fluid. These factors are important because heated fluid will rise and cooled fluid will sink due to the fluid temperature change.

The correlations below were presented by McCabe (ref #11) and modified by Lloyd & Moran (ref #12) by introducing the use of  $D = A/P$  for the characteristic length, where A is the area of the horizontal surface and P is its perimeter. The correlations and the conditions for the use of each are as follows:

- I. For heating a fluid from the upper surface of a plate or cooling a fluid from the lower surface of a plate:

$$\mathbf{Nu = 0.54 Ra^{1/4}} \quad \text{for } 10^4 \leq Ra \leq 10^7$$

$$\mathbf{Nu = 0.15 Ra^{1/3}} \quad \text{for } 10^7 \leq Ra \leq 10^{11}$$

- II. For heating a fluid from the lower surface of a plate or cooling a fluid from the upper surface of a plate:

$$\mathbf{Nu = 0.27 Ra^{1/4}} \quad \text{for } 10^5 \leq Ra \leq 10^{10}$$

The film temperature should be used for fluid properties in this case also, the same as with convection from a vertical plate.



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**Example #9:** Estimate the heat transfer coefficient for convection from the upper surface of a plate at 120°F to air at 85°F above that surface. The surface area of the plate is 10 ft<sup>2</sup> and its perimeter is 14 ft.

**Solution:** The characteristic length for this case is  $D = A/P = 10/14 = 0.714$  ft.

The film temperature is  $(120 + 85)/2 = 103^\circ\text{F}$ , so the fluid properties are the same as in Example #7, which also had a film temperature of 103°F.

The dimensionless numbers, Pr, Gr, & Ra, are thus calculated as follows:

$$\text{Pr} = \mu C_p / k = 3600(3.94 \times 10^{-7})(7.7)/(0.0157) = 0.70$$

$$\text{Gr} = D^3 \rho^2 g \Delta T \beta / \mu^2 = (0.714^3)(0.00221^2)(32.17)(120 - 85)(0.001779)/((3.94 \times 10^{-7})^2)$$

$$\text{Gr} = 2.3 \times 10^7$$

$$\text{Ra} = \text{GrPr} = (2.3 \times 10^7)(0.70) = 1.6 \times 10^7$$

Since  $\text{Ra} > 10^7$ , and the fluid is being heated from an upper horizontal surface, the correlation to use is:

$$\text{Nu} = 0.15 \text{Ra}^{1/3} = 0.15 (1.6 \times 10^7)^{1/3} = 37.8,$$

The convection heat transfer coefficient,  $h$  is then:

$$h = (\text{Nu}_D)(k)/D = 37.8 * 0.0157 / (.714) = \mathbf{0.83 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}$$



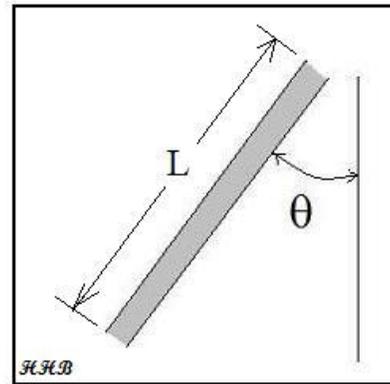
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## 16. Natural Convection from an Inclined Surface

As proposed by Rich (ref #13), the heat transfer coefficient for natural convection from an inclined surface can be estimated using the correlation given above for a vertical surface with  $g$  replaced by  $g(\cos\theta)$  in calculating the Grashof number. The equations for  $Nu$ ,  $Pr$ ,  $Gr$  and  $Ra$  are shown below along with a diagram showing the definition of  $\theta$  as the angle of the inclined surface from the vertical. Limitations of this method are that it can only be used for convection from the lower surface of a heated plate or from the upper surface of a cooled plate and that the angle  $\theta$  must be less than  $60^\circ$ .

$$\begin{aligned}Nu &= \frac{h L}{k} & Pr &= \frac{\mu C_p}{k} \\Gr &= \frac{L^3 \rho^2 g \cos \theta \Delta T \beta}{\mu^2} \\Ra &= GrPr\end{aligned}$$

Natural Convection from  
an Inclined Plane



Natural Convection from  
an Inclined Plate

The characteristic length,  $L$ , in the expressions for  $Nu$  and  $Gr$  is the height of the inclined surface measured along the surface as shown in the diagram. The correlation to be used for estimating  $Nu$  from the  $Gr$  and  $Pr$  values is as follows:

For heating a fluid from the upper surface of a plate or cooling a fluid from the lower surface of a plate inclined at an angle  $\theta$  from the vertical:



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$$\text{for all values of Ra:} \quad 0 \leq \theta \leq 60^\circ$$
$$\text{Nu} = \left\{ 0.825 + \frac{0.387 \text{ Ra}^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}} \right\}^2$$

**Example #10:** Estimate the heat transfer coefficient for natural convection between a 5 ft high surface at 120°F and air below the surface at 85°F, if the surface is tilted at an angle of 30 degrees from the vertical .

**Solution:** The film temperature is 103°F, so the fluid properties for use in this calculation will be the same as in Examples #8 and #9, which also had a film temperature of 103°F. The Prandtl number, Grashof number and Rayleigh number are thus calculated as follows:

$$\text{Pr} = \mu C_p / k = 3600(3.94 \times 10^{-7})(7.7)/(0.0157) = 0.70$$

$$\begin{aligned} \text{Gr} &= D^3 \rho^2 g (\cos \theta) (\Delta T) \beta / \mu^2 \\ &= (5^3)(0.00221^2)(32.17)(\cos(30^\circ))(120 - 85)(0.001779)/((3.94 \times 10^{-7})^2) \end{aligned}$$

$$\text{Gr} = 6.83 \times 10^9$$

$$\text{Ra} = \text{GrPr} = (6.83 \times 10^9)(0.70) = 4.76 \times 10^9$$

Substituting  $\text{Ra} = 4.76 \times 10^9$  and  $\text{Pr} = 0.79$  into the correlation above gives:

$\text{Nu} = 199$ , The convection heat transfer coefficient,  $h$  is then:



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$$h = (Nu_D)(k)/D = 199 * 0.0157 / (5) = \underline{\underline{0.63 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}}$$

## 17. Natural Convection from a Long Horizontal Cylinder

Churchill and Chu (ref #14) give the correlating equation below for natural convection from a long, isothermal horizontal cylinder, with applicability over a wide range for Rayleigh number. As might be expected, the characteristic length,  $D$ , is the diameter of the cylinder.

$$Nu = \left\{ 0.60 + \frac{0.387 Ra^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2$$
$$Gr = \frac{D^3 \rho^2 g \Delta T \beta}{\mu^2} \quad Nu = \frac{h D}{k} \quad Pr = \frac{\mu C_p}{k}$$

for  $10^{-6} \leq Ra \leq 10^{12}$       ( $Ra = Gr Pr$ )

Natural Convection Heat Transfer from an  
Isothermal Horizontal Cylinder

**Example #11:** Estimate the heat transfer coefficient for natural convection from a long, 24 inch diameter pipeline with an external surface temperature of 120°F to surrounding air at 85°F.

**Solution:** The fluid properties should be evaluated at the film temperature (103°F), the same as in the last several examples, so the needed fluid properties will be:



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- Density,  $\rho = 0.00221$  slugs/ft<sup>3</sup>
- Viscosity,  $\mu = 3.94 \times 10^{-7}$  lb-sec/ft<sup>2</sup>
- Specific Heat,  $C_p = 7.7$  Btu/slug-°F
- Thermal conductivity,  $k = 0.0157$  Btu/hr-ft-°F

The dimensionless numbers can then be calculated as follows:

$$Pr = \mu C_p / k = 3600(3.94 \times 10^{-7})(7.7) / (0.0157) = 0.70$$

$$Gr = D^3 \rho^2 g \Delta T \beta / \mu^2 = (2^3)(0.00221^2)(32.17)(120 - 85)(0.001779) / ((3.94 \times 10^{-7})^2)$$

$$Gr = 5.05 \times 10^8$$

$$Ra = GrPr = (5.05 \times 10^8)(0.70) = 3.52 \times 10^8$$

Substituting  $Pr = 0.7$  and  $Ra = 3.52 \times 10^8$ , into the correlation for  $Nu$  above gives:

$Nu = 83$ , The convection heat transfer coefficient,  $h$  is then:

$$h = (Nu_D)(k) / D = 83 * 0.0157 / (2) = \underline{\underline{0.65 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}}$$

## 18. Natural Convection from a Sphere

The correlation below was provided by Churchill (ref # 15) for natural convection from a sphere, for the Prandtl number and Rayleigh number ranges shown. The characteristic length,  $D$ , is of course, the diameter of the sphere.



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$$\text{Nu} = 2 + \frac{0.589 \text{ Ra}^{1/4}}{[1 + (0.469/\text{Pr})^{9/16}]^{4/9}}$$
$$\text{Gr} = \frac{D^3 \rho^2 g \Delta T \beta}{\mu^2} \quad \text{Nu} = \frac{h D}{k} \quad \text{Pr} = \frac{\mu C_p}{k}$$

for  $\text{Ra} \leq 10^{11}$ ,  $\text{Pr} \geq 0.7$  ( $\text{Ra} = \text{Gr Pr}$ )

Natural Convection Heat Transfer from a Sphere

**Example #12:** Estimate the heat transfer coefficient for natural convection from a 2 ft diameter sphere with a surface temperature of 120°F to surrounding air at 85°F.

**Solution:** The fluid properties should be evaluated at the film temperature (103°F), the same as in the last several examples, so the needed dimensionless numbers will be:

$$\text{Pr} = \mu C_p / k = 3600(3.94 \times 10^{-7})(7.7)/(0.0157) = 0.70$$

$$\text{Gr} = D^3 \rho^2 g \Delta T \beta / \mu^2 = (2^3)(0.00221^2)(32.17)(120 - 85)(0.001779)/((3.94 \times 10^{-7})^2)$$

$$\text{Gr} = 5.05 \times 10^8$$

$$\text{Ra} = \text{GrPr} = (5.05 \times 10^8)(0.70) = 3.52 \times 10^8$$

Substituting  $\text{Pr} = 0.7$  and  $\text{Ra} = 3.52 \times 10^8$ , into the correlation for  $\text{Nu}$  above gives:

$\text{Nu} = 64$ , The convection heat transfer coefficient,  $h$  is then:

$$h = (\text{Nu}_D)(k)/D = 64 * 0.0157 / (2) = \underline{\underline{0.50 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}}}$$



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## 19. Use of S.I. Units in Heat Transfer Coefficient Calculations

The correlations presented for all of the forced convection and natural convection configurations are in terms of dimensionless numbers (Reynolds, Prandtl, Nusselt, Grashof, and Rayleigh numbers), so the equations in those correlations remain the same for any set of units. Whatever set of units is being used, you need to be sure, however, that the units on the parameters used to calculate any of those dimensionless numbers are consistent, so that the units all “cancel out” and the number is indeed dimensionless.

**Example #13:** If a convection heat transfer coefficient is to be determined from a calculated value of Nusselt number together with known fluid thermal conductivity,  $k$ , in W/m-K and characteristic length,  $D$ , in m, what will be the units of the heat transfer coefficient?

**Solution:** The definition of Nusselt number ( $Nu = hD/k$ ) can be rearranged to:  $h = Nu k/D$ . Since  $Nu$  is dimensionless, the units of  $h$  will be the units of  $k$  divided by the units of  $D$ , or simply: units of  $h = (W/m-K)/m = \underline{W/m^2-K}$ .

**Example #14:** Use the Dittus-Boelter correlation to estimate the convective heat transfer coefficient in  $kJ/hr-m^2-k$ , for flow of water at  $30^\circ C$ , with velocity equal  $0.55$  m/s, through a  $50$  mm diameter pipe that has a surface temperature of  $50^\circ C$ .

**Solution:** The needed properties of water at  $30^\circ C$  are: density =  $995$   $kg/m^3$ , viscosity =  $0.000785$  N-s/ $m^2$ , specific heat =  $4.19$   $kJ/kg-K$ , thermal conductivity =  $0.58$  W/m-K.

Calculation of Reynolds number:



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$$\begin{aligned} \text{Re} &= \frac{DV\rho}{\mu} = \frac{(0.05 \text{ m})(0.55 \text{ m/s})(995 \text{ kg/m}^3)}{0.000785 \text{ N}\cdot\text{s/m}^2} \\ &= 34,860 \frac{(\text{m})(\text{m})(\text{kg})(\text{m}^2)}{(\text{s})(\text{m}^3)(\text{N})(\text{s})} = 34,860 \frac{(\text{kg})(\text{m})}{(\text{N})(\text{s}^2)} \end{aligned}$$

But  $1 \text{ N} = 1 \text{ kg}\cdot\text{m/s}^2$ , so the units do indeed cancel out, leaving it dimensionless, so the Reynolds number is **34,860 = Re**.

Calculation of Prandtl number:

$$\begin{aligned} \text{Pr} &= \frac{\mu C_p}{k} = \frac{(0.000785 \text{ N}\cdot\text{s/m}^2)(4.19 \text{ kJ/kg}\cdot\text{K})}{0.58 \text{ W/m}\cdot\text{K}} \\ &= 0.000567 \frac{(\text{N})(\text{s})(\text{kJ})(\text{m})(\text{K})}{(\text{m}^2)(\text{kg})(\text{K})(\text{W})} = 0.000567 \frac{\text{kJ}}{\text{J}} \end{aligned}$$

The relationships,  $1 \text{ N} = 1 \text{ kg m/s}^2$  and  $1 \text{ W} = 1 \text{ J/s}$ , were used to simplify the units in the above expression, leading to the result that  $\text{Pr} = 0.000567 \text{ kJ/J}$ . Multiplying by 1000 J/kJ will make Pr dimensionless:

$$\text{Pr} = (0.000567 \text{ kJ/J})(1000 \text{ J/kJ}) = \mathbf{0.567}$$

Since the fluid is being heated, the Dittus-Boelter equation to use is:

$$\text{Nu} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.4} = 0.023 (34,860^{0.8})(0.567^{0.4}) = 198$$

$$h = \text{Nu } k/D = (198)(0.58 \text{ W/m}\cdot\text{K})/(0.05 \text{ m}) = 2297 \text{ J/s}\cdot\text{m}^2\cdot\text{K}$$

To convert to the required units, multiply by 3600 s/hr and divide by 1000 J/kJ to give:

$$\mathbf{h} = 2297 \cdot 3600 / 1000 = \mathbf{8270 \text{ kJ/hr}\cdot\text{m}^2\cdot\text{K}}$$



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## 20. Summary

A major part of most convection heat transfer calculations is determination of a value for the heat transfer coefficient. The equations for forced convection coefficients are typically in the form of a correlation for Nusselt number in terms of Reynolds number and Prandtl number, while natural convection correlations give Nusselt number in terms of Prandtl number and either Grashof number or Rayleigh number. This course includes correlations for five different forced convection configurations and five different natural convection configurations. A spreadsheet included with the course can be used to calculate heat transfer coefficients for either laminar or turbulent flow through pipes.

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