Fundamentals of Post-Tensioned Concrete Design for Buildings

Part Two

by

John P. Miller

Overview of This Course

This is Part Two of a three-part course that covers the fundamentals of post-tensioned concrete design for building structures using unbonded tendons. This course is intended as an introductory course for structural engineers new to post-tensioned concrete design and is a refresher for experienced structural engineers. It is assumed that Part One of this course has been successfully taken by the user of this Part Two. By successfully completing this three-part course, you should be comfortable performing a preliminary design by hand and be able to quickly check a computer generated design or an existing design by hand or by simple analysis techniques.

Part One gave a brief historical background and post-tensioned members were differentiated from pre-tensioned members. You learned about the load balancing concept, hyperstatic moments, pre-stress losses, the basic requirements of ACI 318-08 (Building Code Requirements for Structural Concrete), and nominal flexure and shear capacities of post-tensioned members.

In Part Two, we will now turn our attention to working examples of two of the structural systems commonly used in buildings and parking structures, namely a one-way continuous slab and a two-span beam. Part Two is an example-intensive course, with key concepts introduced along the way.

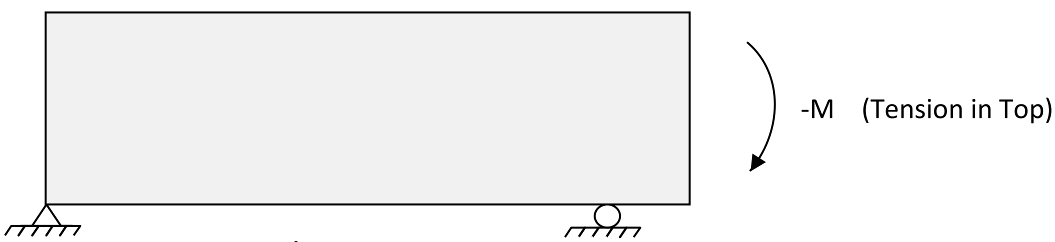
Part Three will continue with the study of two-way, post-tensioned slab systems, including a design example using the Equivalent Frame concept. Part Three also covers related topics such as punching shear for two-way slabs and moment transfer at the column. Part Three is an example-intensive course, with key concepts introduced along the way.

The user of this course material must recognize that pre-stressed concrete design is a very broad topic and that only certain fundamentals in a specific area are covered here in this course. It is not intended, nor is it possible within the confines of this course, to cover all aspects of pre-stressed concrete design. It is not intended that the material included in this course be used for design of facilities by an engineer who is inexperienced in pre-stressed concrete design without oversight and guidance from someone more experienced in this field. The author of this course has no control or review authority over the subsequent use of this course material, and thus the author accepts no liability for damages that may result from its use.

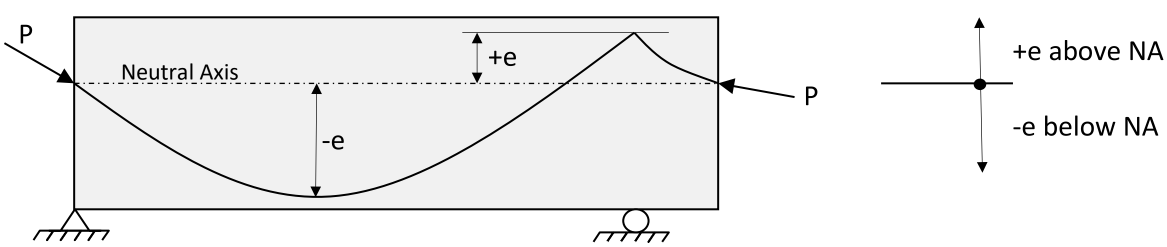
A Word About Sign Conventions

In this course, moments causing tension in the bottom fiber are considered positive. Moments causing tension in the top fiber are negative. Eccentricities below the neutral axis are negative and above the neutral axis they are positive. These sign conventions are illustrated below.

Positive Moment



Negative Moment



Eccentricity

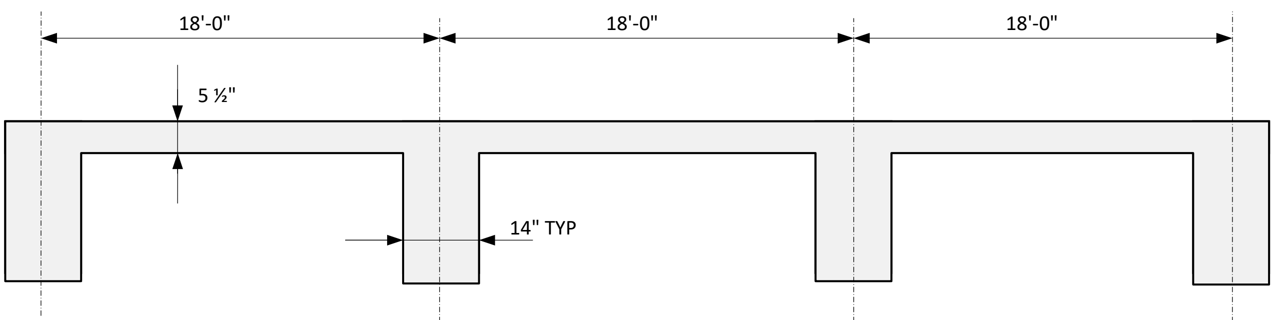
Continuous One-Way Slabs

It is assumed that the reader of this Part Two has taken Part One and passed the test, or is someone experienced in post-tensioned concrete design. Sometimes during this course, it is helpful to refer to material previously covered in Part One, and in those cases the information will be briefly repeated. We will now immediately turn our attention to working a one-way, continuous, post-tensioned slab example, using basic structural analysis techniques and lessons from Part One.

Continuous One-Way Slab Design Example

Find:

Choose an effective post-tensioning force, F_e, and determine the mild reinforcing required for the following three-span one-way slab.



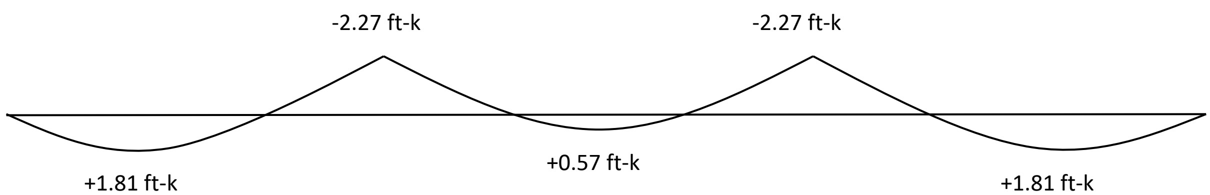
Given:

- f_{PU} = 270 ksi, ½"ϕ unbonded tendons, f_{SE} = 160 ksi
- F_{SE} = 0.153 x 160 = 24.5 kips/tendon
- f'_c = 4,000 psi, normal weight concrete (150 pcf), f'_{ci} = 0.75 f'_c = 3,000 psi
- Live Load w_L = 40 psf, Dead Load w_D = 70 psf
- w_{BAL} = 65% x w_D = 0.65 x 70 psf = 45 psf
- Use span centerline dimensions (ignore support widths)

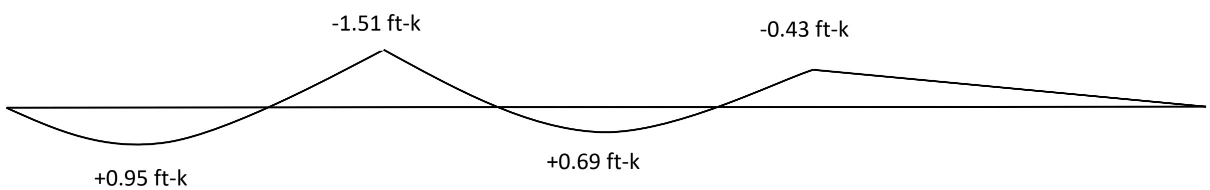
Solution:

First, we will determine the service load (unfactored) bending moment demands for a one-foot wide strip of slab. The demands can be computed using coefficients, beam diagrams, hand methods, or two-dimensional software. In this case, the beam diagrams from the Manual of Steel Construction published by the American Institute of

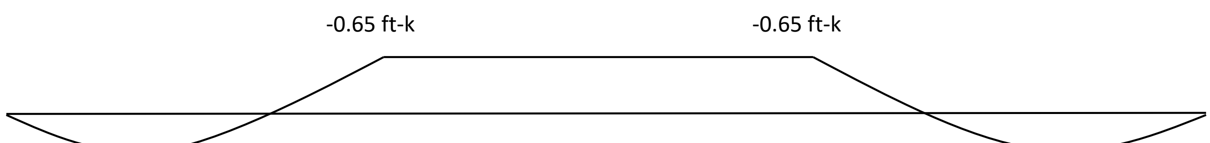
Steel Construction are used. For example, assuming all supports are pinned and using centerline dimensions, for a uniform load on all three spans, the maximum negative moment at the interior support is $-0.1wL^2$ and the maximum positive moment in the end spans and center span are $+0.08wL^2$ and $+0.025wL^2$, respectively. In a similar fashion, using the AISC diagrams for live load on alternate spans, the following service load moment diagrams can be generated. Note that although the exact location of the maximum positive moment in the end spans varies slightly, we will use the maximum values in the subsequent combinations.



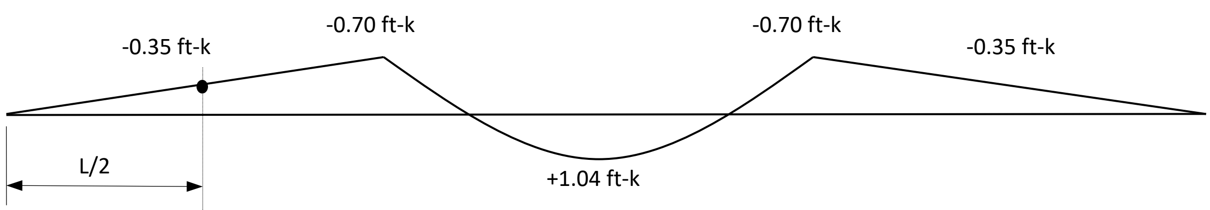
Service Dead Load Moment Diagram



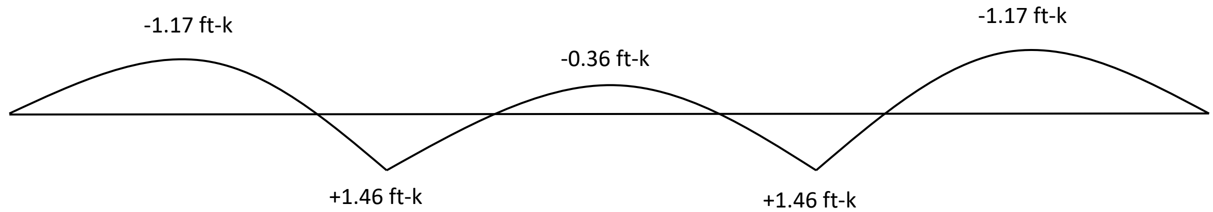
Service Live Load Moment Diagram
First Two Spans Loaded



Service Live Load Moment Diagram
Two End Spans Loaded



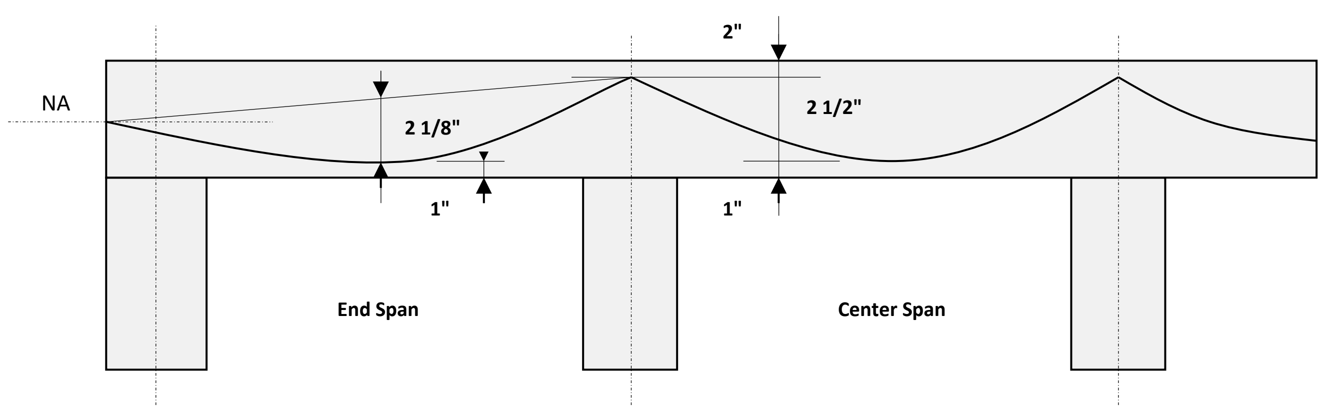
Service Live Load Moment Diagram
Middle Span Loaded



Balanced Load Moment Diagram

Note that the balanced load due to the tendon is acting upward, and hence the moments are in the opposite direction of the gravity load moments.

The next step is to determine the available drape. We will use a concrete cover of 1.75 inches and 0.75 inches for the top and bottom, respectively. These concrete covers are typical for parking structures. Therefore, the distance to the centroid of the ½"ϕ unbonded tendons is 2.0 inches at the top and 1.0 inch at the bottom. From the diagram below, placing the centroid of the tendon at mid-depth of the slab at the exterior ends (i.e. at the Neutral Axis), the available drapes are 2 1/8 inches and 2 1/2 inches in the end spans and the center span, respectively. Refer to the following diagram.



Next, we will determine the effective post-tensioning force required to balance 65% of the dead load, or 45 psf. Recall from Part One of this course that the effective post-tensioning force, F_e, is the force in the tendons after all losses and is computed as:

$$F_e = \frac{wL^2}{8a}$$

$$End\ Span\ F_e = \frac{wL^2}{8a} = \frac{0.045(18)^2}{8(2.125)}(12) = 10.3\ k/ft$$

$$Center\ Span\ F_e = \frac{wL^2}{8a} = \frac{0.045(18)^2}{8(2.5)}(12) = 8.7\ k/ft$$

Since it is practical to add post-tensioning force in end spans, we will use the above effective pre-stress forces in the following analysis. Let's first check to make sure that we meet the minimum pre-stress force and that we do not exceed a reasonable maximum, per ACI 318.

$$Minimum\ \frac{P}{A} = \frac{8.7(1000)lb/ft}{5.25(12)} = 132\ psi > 125\ psi\ \boldsymbol{OK}$$

$$Maximum\ \frac{P}{A} = \frac{10.3(1000)lb/ft}{5.25(12)} = 163\ psi < \sim 250\ psi\ \boldsymbol{OK}$$

Now let's calculate the concrete stresses at transfer and at service loads. If the concrete stresses exceed the allowable values set by ACI 318, then we will need to revise the balanced load, adjust the drape, or add mild reinforcing steel. First of all, let's summarize the service load moments. Since our three-span structure is symmetrical about the middle of the center span, we can conveniently tabulate the maximum service load moments as follows:

Service Moments	Midspan 1	Interior Support	Midspan 2
M_{DL}	+1.81 ft-k/ft	-2.27 ft-k/ft	+0.57 ft-k/ft
M_{LL}	+0.95 ft-k/ft	-1.51 ft-k/ft	+1.04 or -0.65 ft-k/ft
M_{BAL}	-1.17 ft-k/ft	+1.46 ft-k/ft	-0.36 ft-k/ft

At the time the post-tensioning force is transferred to the concrete, there is typically only dead load and the balancing load acting. Thus, we will algebraically combine M_{DL} and M_{BAL} at the critical sections and compute the concrete stresses assuming an uncracked section, or gross properties. Thus S = 12(5.5)^2/6 = 60.5 in^3/foot.

Recall from Part One that the allowable concrete stresses at transfer per ACI 318 are:

$$Ends\ of\ Simply\ Supported\ Members:\ f_{comp} \leq 0.70\ f'_{ci}$$

$$All\ other\ Cases:\ f_{comp} \leq 0.60\ f'_{ci}$$

$$Ends\ of\ Simply\ Supported\ Members\text{:}\ f_{tens} \leq 6\sqrt{f'_{ci}}$$

$$All\ other\ Cases\text{:}\ f_{tens} \leq 3\sqrt{f'_{ci}}$$

Note that the tensile stresses noted above may be exceeded, but if they are additional bonded reinforcement must be provided in the tensile zone to resist all of the tensile force.

Therefore, we can tabulate the concrete transfer stresses as follows:

Transfer Stresses	Midspan 1	Interior Support	Midspan 2
$M_{DL} + M_{BAL}$	+0.64 ft-k/ft	-0.81 ft-k/ft	+0.21 ft-k/ft
M/S	+/-126 psi	+/-161 psi	+/-42
P/A	-163 psi	-163 psi	-132 psi
f_t	-37 psi	-2 psi	-90 psi
f_c	-289 psi	-324 psi	-174 psi

The allowable concrete stresses at transfer are:

$f_t \leq 3\sqrt{f'_{ci}} = 3\sqrt{3000} = 164$ psi tension

$f_c \leq 0.60 f'_{ci} = 0.6(3000) = 1800$ psi compression

From the stress tabulation, we can see that the concrete is always in compression and that the maximum compressive stress of -324 psi is well below the allowable value of 1800 psi and therefore the stresses at transfer are acceptable.

Now let's check the concrete stresses at service loads. Recall from Part One that this is a check of the concrete tension and compression stresses at sustained service loads (sustained live load, dead load, superimposed dead load, and pre-stress) and a check at total service loads (live load, dead load, superimposed dead load, and pre-stress). These checks are intended to preclude excessive creep deflection and to keep stresses low enough to improve long term behavior. The maximum permissible concrete stresses at the service load state are as follows:

Extreme fiber stress in compression: $f_{comp} \leq 0.45 f'_c = 0.45(4000) = 1800\ psi$

Extreme fiber stress in tension: $f_{tens} \leq 7.5\sqrt{f'_c} = 7.5\sqrt{4000} = 474\ psi$

In this example, we do not have any sustained live load, so we only need to check the load case of total services loads. Thus, we will algebraically combine M_{DL}, M_{LL}, and M_{BAL} at the critical sections and compute the concrete stresses, assuming an uncracked section, or gross properties.

Total Service Stresses	Midspan 1	Interior Support	Midspan 2
$M_{DL} + M_{LL} + M_{BAL}$	+1.59 ft-k/ft	-2.32 ft-k/ft	+1.25 ft-k/ft
M/S	+/-315 psi	+/-460 psi	+/-248
P/A	-163 psi	-163 psi	-132 psi
f_t	+152 psi	+297 psi	+116 psi
f_c	-478 psi	-623 psi	-380 psi

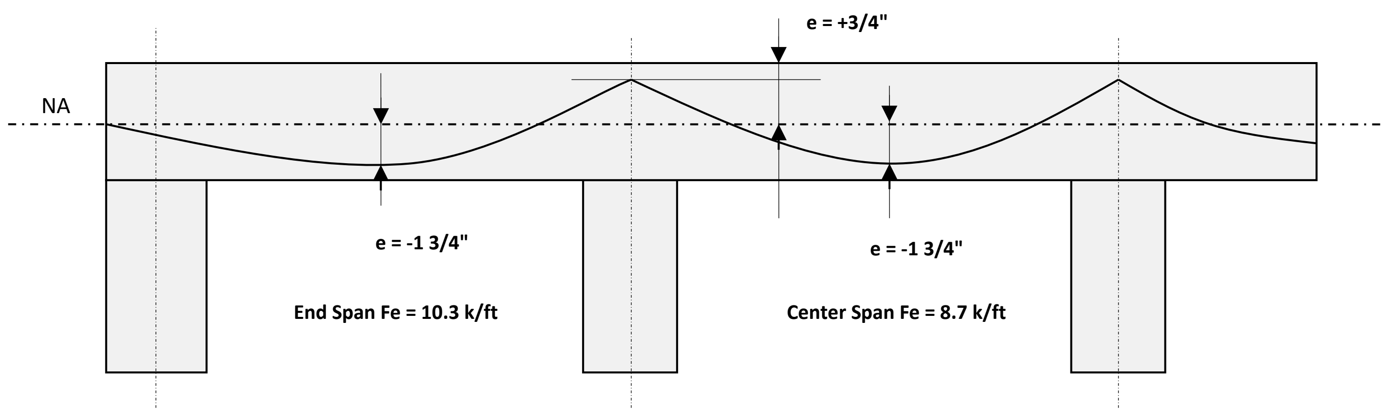
From the above table, we observe the maximum compressive stress of -623 psi and the maximum tensile stress of +297 psi are well below the allowable values and therefore the stresses at service loads are all acceptable.

Now that we have confirmed that the service load stresses are acceptable, we will now determine the hyperstatic moments. Recall from Part One that:

$$M_{HYP} = M_{BAL} - M_1$$

$$M_{HYP} = M_{BAL} - F_e \times e$$

M_1 is called the primary moment, and is simply the effective post-tensioning force times the eccentricity from the neutral axis of the member at critical locations. The eccentricities at critical locations are shown in the following diagram:



Therefore, we can tabulate the hyperstatic moments as follows:

Moment	Midspan 1	Interior Support	Midspan 2
M_{BAL}	-1.17 ft-k/ft	+1.46 ft-k/ft	-0.36 ft-k/ft
M_1 (F_exe)	-1.50 ft-k/ft	+0.64 ft-k/ft	+-1.27 ft-k/ft
M_{HYP}	+0.33 ft-k/ft	+0.82 ft-k/ft	+0.91 ft-k/ft

Now that we have computed all the various bending moment demands, and have confirmed that service load stresses are OK, we can proceed with determining the factored moments. According to ACI 318, the factored moment is calculated using:

$$M_u = 1.2\, M_{DL} + \ 1.6\, M_{LL} + \ 1.0\, M_{HYP}$$

The factored moments are tabulated as follows:

Factored Moments	Midspan 1	Interior Support	Midspan 2
M_{DL}	+2.17 ft-k/ft	-2.72ft-k/ft	+0.68ft-k/ft
M_{LL}	+1.52 ft-k/ft	-2.42ft-k/ft	+1.66 or -1.04ft-k/ft
M_{HYP}	+0.33ft-k/ft	+0.82 ft-k/ft	+0.91 ft-k/ft
M_u	+4.02 ft-k/ft	-4.32 ft-k/ft	+3.25 or +0.55 ft-k/ft

Now that we have calculated the factored moment demand, our next task is to determine the quantity of bonded mild reinforcing steel, if any, required to satisfy the demand. There are two basic approaches to this. The first approach would be to calculate the required area of bonded mild reinforcing steel based on the demand, and then check to make sure that this is not less than the minimum required. The second approach would be to calculate the minimum area of bonded mild reinforcing steel required and use this amount to calculate the ultimate flexural strength of the section, including both mild and post-tensioning steel, and check that the ultimate strength is greater than the demand. For lightly loaded structures, such as the one in this example, the minimum area of bonded mild reinforcing steel is often sufficient to satisfy the flexural demand in conjunction with the post-tensioning steel, and so we will use the second approach in this example. (We will use the first approach later in the continuous beam example).

Recall that the minimum area of bonded mild reinforcing steel for post-tensioned flexural members, regardless of stresses, according to ACI 318 is:

$$Minimum\ A_s = 0.004 A_{ct}$$

Where A_{CT} is defined as the area of that part of the cross section between the flexural tension face and the center of gravity of the cross-section. So, for the slab in this example, we have:

$$Minimum\ A_s = 0.004(12)\left(\frac{5.5}{2}\right) = 0.132\ sq.in./ft$$

USE #3@10

Note that #4@18 would satisfy the area requirement, but would violate the maximum spacing requirement of 3h = 16.5 inches. The area of #3@10" happens to be exactly equal to the minimum requirement of 0.132 in^2.

Now we can check the ultimate flexural capacity using the minimum area of bonded mild reinforcing steel. The span-depth ratio is 18/5.5/12 = 39 which is greater than 35. Therefore, the following equation applies (refer to Part One) in order to determine the stress in the pre-stressing steel at nominal flexural capacity:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f_c'}{300\rho_p}$$

$$\leq f_{py}\ or\ f_{se} + 30{,}000$$

The ratio of prestressed reinforcement, ρ_p, is different for the end spans and the center span.

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{(0.153)}{(12)(4.5)} \times \frac{10.3\ k/ft}{24.5\ k/tendon} = 0.00119\ End\ Span$$

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{(0.153)}{(12)(4.5)} \times \frac{8.7\ k/ft}{24.5\ k/tendon} = 0.00101\ Center\ Span$$

$$f_{ps} = 160{,}000 + 10{,}000 + \frac{4{,}000}{300(0.00119)} = 181\ ksi\ End\ Span$$

$$f_{ps} = 160{,}000 + 10{,}000 + \frac{4{,}000}{300(0.00101)} = 183\ ksi\ Center\ Span$$

These values for f_{PS} are less than both f_{PY} = 270 ksi and f_{SE} + 30,000 = 190 ksi and therefore we will use these values.

Now we can calculate the nominal moment capacity. We will assume that the minimum area of bonded reinforcing steel is at the same depth as the tendons.

$$a = \frac{A_{ps} f_{ps} + A_s f_y}{0.85 f_c' b} = \frac{(0.153)(181)\left(\frac{10.3}{24.5}\right) + (0.132)(60)}{(0.85)(4)(12)} = 0.48 \; inches\ End\ Span$$

$$a = \frac{A_{ps} f_{ps} + A_s f_y}{0.85 f_c' b} = \frac{(0.153)(183)\left(\frac{8.7}{24.5}\right) + (0.132)(60)}{(0.85)(4)(12)} = 0.44 \; inches\ Center\ Span$$

$$\phi M_n = \phi(A_{ps} f_{ps} + A_s f_y)\left(d_p - \frac{a}{2}\right)$$

For the end spans:

$$\phi M_n = 0.90\left[(0.153)\left(\frac{10.3}{24.5}\right)(181) + (0.132)(60)\right]\left(4.5 - \frac{0.48}{2}\right)\left(\frac{1}{12}\right) = 6.25 ft - k/ft$$

For the center spans:

$$\phi M_n = 0.90\left[(0.153)\left(\frac{8.7}{24.5}\right)(183) + (0.132)(60)\right]\left(4.5 - \frac{0.44}{2}\right)\left(\frac{1}{12}\right) = 5.73 ft - k/ft$$

By reviewing the factored moment demands on Page 10 above, we can see that the nominal moment capacities just computed with minimum bonded reinforcing steel are greater. Therefore, $\phi M_n > M_u$ at all critical locations of the spans and is acceptable.

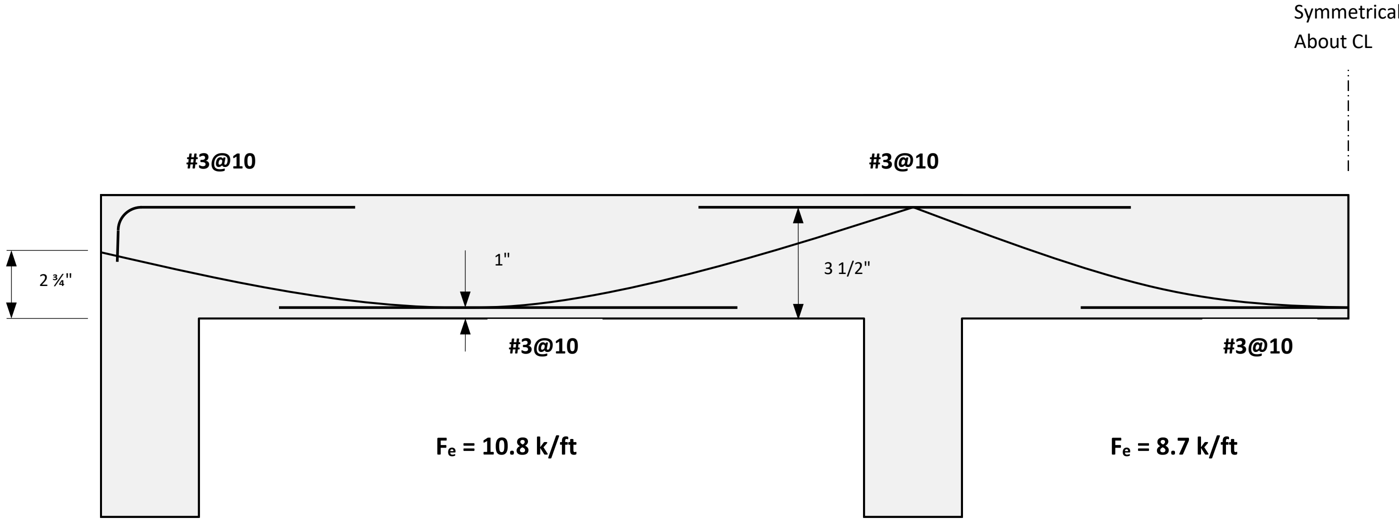
The last item to check is the shear. For pre-stressed members, the nominal shear strength provided by the concrete is:

$$V_c = (0.6\lambda\sqrt{f_c'} + 700\frac{V_u d_p}{M_u})b_w d$$

But V_c need not be less than $2\lambda\sqrt{f_c'}b_w d$ and shall not be greater than $5\lambda\sqrt{f_c'}b_w d$ and $V_u d_p/M_u$ shall not be greater than 1.0. λ is a modification factor for lightweight concrete. Shear strength in one-way slabs is seldom a concern, so let's check the worst case, which will be at the interior face of the exterior support. According to ACI coefficients:

$$V_u = 1.15\frac{w_u l_n}{2} = 1.15\frac{0.148(16.833)}{2} = 1.43\ k$$

$$\phi V_c = \frac{0.85\left(0.6\sqrt{4000} + 700\,\frac{1.43(4.5)}{0}\right)(12\ \times 4.5)}{1000} = \ 1.74\ k > 1.43\ k \qquad \boldsymbol{OK}$$



Final Design Sketch

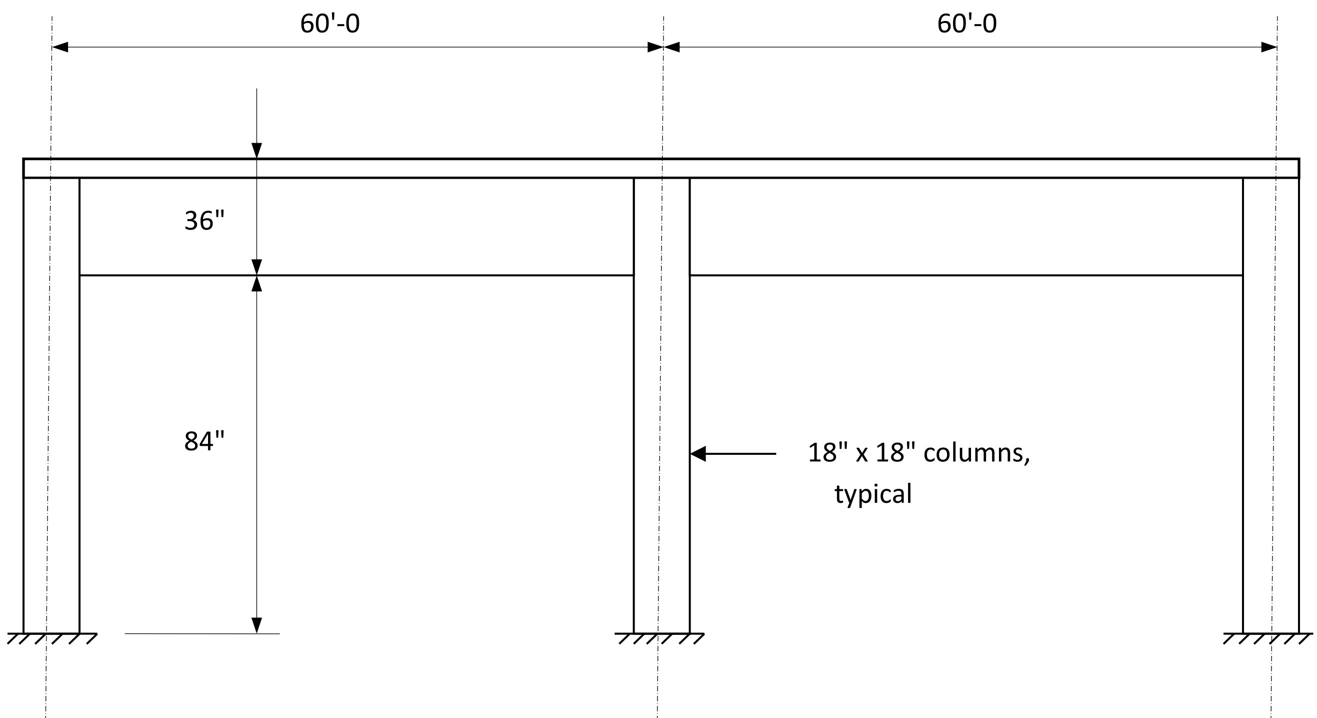
The final lengths of the mild reinforcing may be computed per ACI 318 or left up to the rebar detailer. In reality, the tendon drapes would be reverse parabolas, instead of the single parabolas shown. In this example, the tendons would be spaced at (24.5/8.7) x 12 = 33.8" o.c. with an additional tendon in the end span every 12 x 24.5/(10.8-8.7) = 140 inches. The dead end of the end-span added tendons are normally terminated at the inflection point of the first interior span or at the neutral axis of the member. We have not checked deflection of the slabs as deflection calculations are beyond the scope of this course.

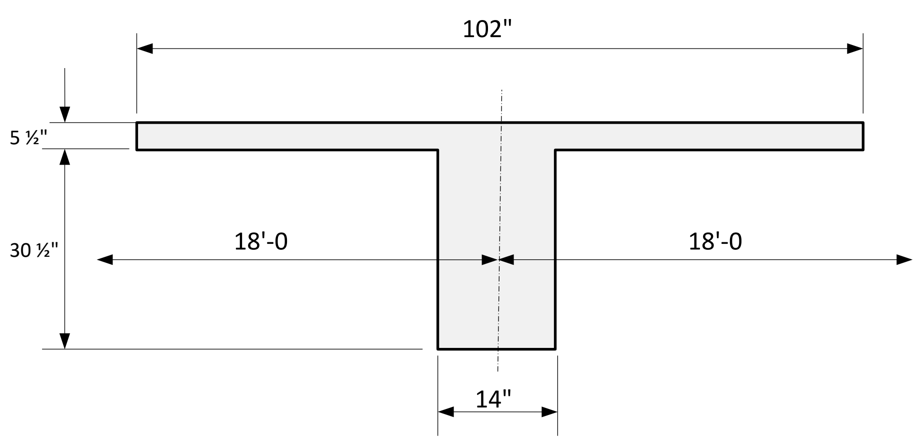
Also be aware that ACI requires a minimum amount of average compressive pre-stress force of 100 psi, for shrinkage and temperature, in the direction perpendicular to the span in one way slab systems. This average pre-stress force is also required in the slab area between effective T-beam flanges, and parallel to their span, in one-way beam systems. Calculations for temperature and shrinkage pre-stress are not shown in this course.

Two-Span Beam Design Example

Find:

Choose an effective post-tensioning force, F_e, and determine the mild reinforcing required for the following two-span beam. The beams are 18'-0 on center.





Given:

- o f_{PU} = 270 ksi, ½"ϕ unbonded tendons, f_{SE} = 160 ksi
- o F_{SE} = 0.153 x 160 = 24.5 kips/tendon
- o f'_c = 4,000 psi, normal weight concrete (150 pcf), f'_{ci} = 0.75 f'_c = 3,000 psi
- o Live Load w_L = 40 psf, Dead Load w_D = 70 psf (slab only)
- o w_{BAL} = 80% (w_D)
- o Use span centerline dimensions (ignore support widths)

Solution:

The gross cross-sectional properties of the beam can be computed to be the following:

$$\mathrm{A} = 988\ sq.in.$$

$$y = 25.5\ in\ from\ the\ bottom$$

$$\mathrm{I} = 113{,}072\ in^4$$

$$\mathrm{S_{top}} = 10{,}769\ in^3$$

$$\mathrm{S_{bot}} = 4{,}434\ in^3$$

The uniform dead load on the beam, including the beam's self-weight, is:

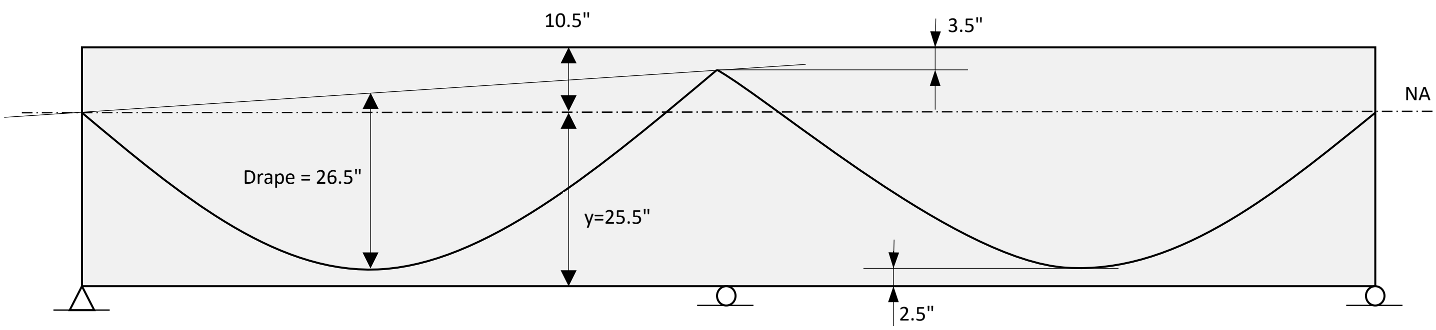
$$\mathrm{W_{DL}} = \left[(18)\frac{5.5}{12} + \frac{14(30.5)}{144}\right]0.15 = 1.68\ kips/foot$$

And the balanced and live loads are:

$$\mathrm{W_{BAL}} = (0.80)1.68 = 1.34\ kips/foot$$

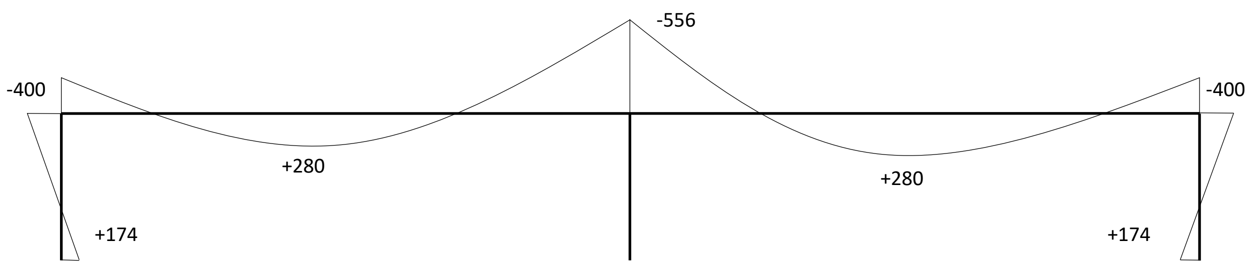
$$\mathrm{W_{LL}} = (18)0.040 = 0.72\ kips/foot$$

Since we have a symmetrical, two-span beam, we can surmise that we can take advantage of the maximum available drape in both spans. Recall from Part One of this course that if the structure is three or more spans, it is in general likely that the end spans will be able to take advantage of the maximum available drape and the interior spans may not, unless additional pre-stressing tendons are placed in the ends spans to account for this. Therefore, the most efficient use of pre-stressing force in this example will be to place the tendons at their maximum available drape in both spans. Assuming that our design will result in a bundle, or several bundles, of tendons, we will use a clear dimension to the centroid of the bundle of 2 1/2" and 3 1/2" at the bottom and top of the beam, respectively. The tendon drape for this example is determined from the following figure:



Drape = (25.5 + 32.5)/2 – 2.5 = 26.5 inches in both spans.

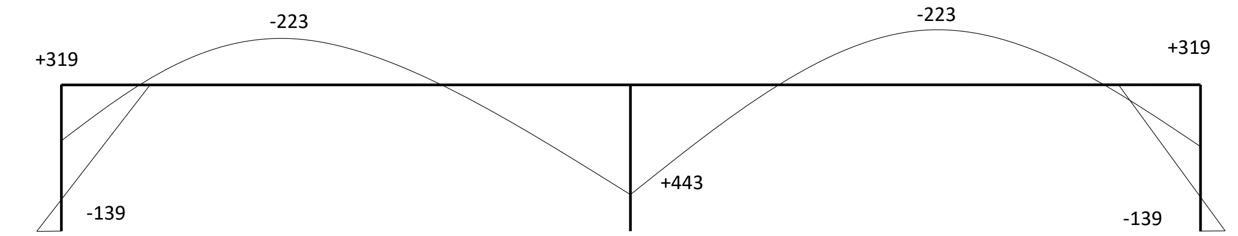
Now let's determine the service load (unfactored) bending moment demands for the two-span beam. The bending moment demands have been computed using two-dimensional software, including the columns, with the assumption that the far ends of the columns below are fixed. This analysis accounts for the column contribution to the moment distribution. For a simplified analysis, assuming pinned supports, one could compute the positive and negative moments using $+0.07wl^2$ and $-0.125wL^2$, respectively, for this two-span beam condition. This would lead to a conservative beam design. Below is the moment diagram for the dead load moments with a 1.68 kips/foot uniformly distributed load applied to both beam spans:



Dead Load Moments (ft-kips)

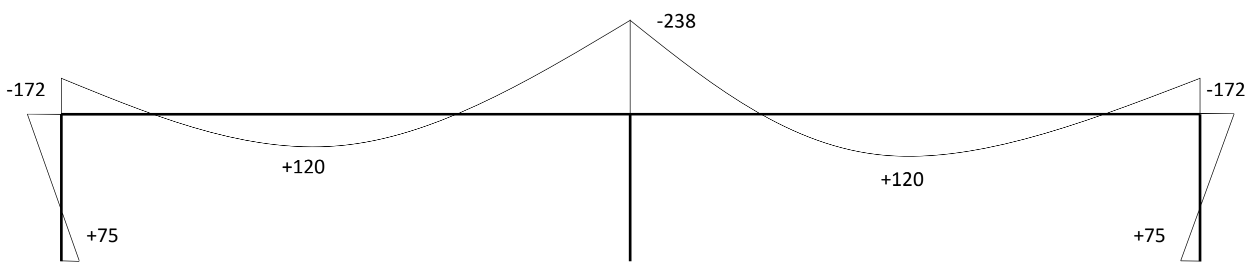
Recall that the sign convention is positive for tension in the <u>bottom</u> of the beam and negative for tension in the <u>top</u> of the beam.

To determine the balanced load moments, we will load the same structure with the balanced load – the upward equivalent load of 1.34 kips/foot due to the tendon force.

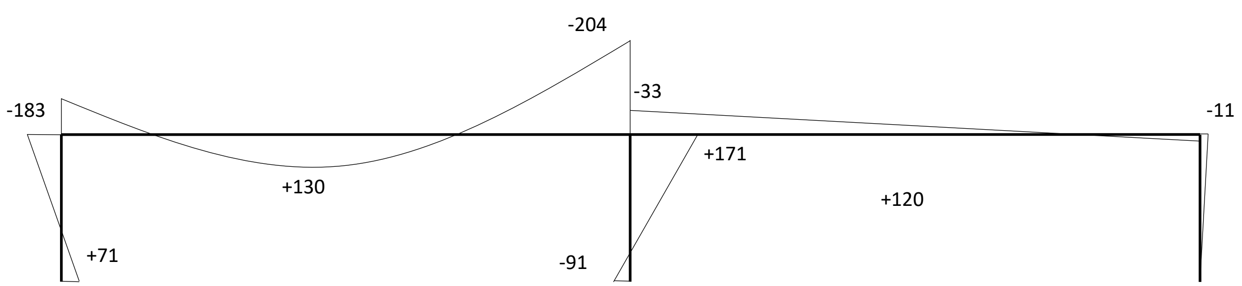


Balanced Load Moments (ft-kips)

Using a similar analysis, the live load moments can be determined for two different load conditions. The first condition is when both spans are loaded with the uniformly distributed live load of 0.72 kips/foot and the second condition is when only one span is loaded with live load. This will give us the maximum and minimum live load moments that we will later use in the design.



Live Load Moments – Both Spans Loaded (ft-kips)



Live Load Moments – Left Span Loaded (ft-kips)

The equivalent tendon force, F_e, required to balance 1.34 kips/foot is:

$$F_e = \frac{w_{BAL}L^2}{8a} = \frac{1.34(60)^2}{8(26.5)} \times 12 = 273\ kips$$

Now let's calculate the concrete stresses at transfer and at service loads. If the concrete stresses exceed the allowable values set by ACI 318, then we will need to revise the balanced load, adjust the drape, or add mild reinforcing steel. First of all, let's summarize the service load moments. Since our two-span beam is symmetrical, we can tabulate the maximum service load moments, in foot-kips, as follows:

Service Moments	Exterior Support	Midspan	Interior Support
M_{DL}	-400 ft-k	+280 ft-k	-556 ft-k
M_{LL}	-183 ft-k	-+130 or -11 ft-k	-238 ft-k
M_{BAL}	+319 ft-k	-223 ft-k	+443 ft-k

At the time the post-tensioning force is transferred to the concrete, there is typically only dead load and the balancing load acting. Thus, we will algebraically combine M_{DL} and M_{BAL} at the critical sections and compute the concrete stresses assuming an uncracked section, or gross properties. Thus, the transfer stresses are:

Transfer Stresses	Exterior Support	Midspan	Interior Support
M_{DL} + M_{BAL}	-81 ft-k	+57 ft-k	-113 ft-k
M/S_{top}	+90 psi	-63 psi	+126 psi
M/S_{bot}	-219 psi	+154 psi	-306 psi
P/A	-276 psi	-276 psi	-276 psi
f_{top}	-186 psi	-339 psi	-150 psi
f_{bot}	-495 psi	-122 psi	-582 psi

The allowable concrete stresses, per ACI 318, at transfer are:

$f_t \leq 3\sqrt{f'_{ci}} = 3\sqrt{3000} = +164$ psi tension

$f_c \leq 0.60\, f'_{ci} = 0.6(3000) = -1800$ psi compression

From the above table, we can see that at transfer, all of the stresses are compressive, and the maximum compression stress is -582 psi, which is well below the allowable value and therefore acceptable.

Now we need to check the service load stresses. Recall from the previous example, the maximum permissible concrete stresses at the service load state are as follows:

Extreme fiber stress in compression: $f_{comp} \leq 0.45f_c' = 0.45(4000) = 1800\,psi$

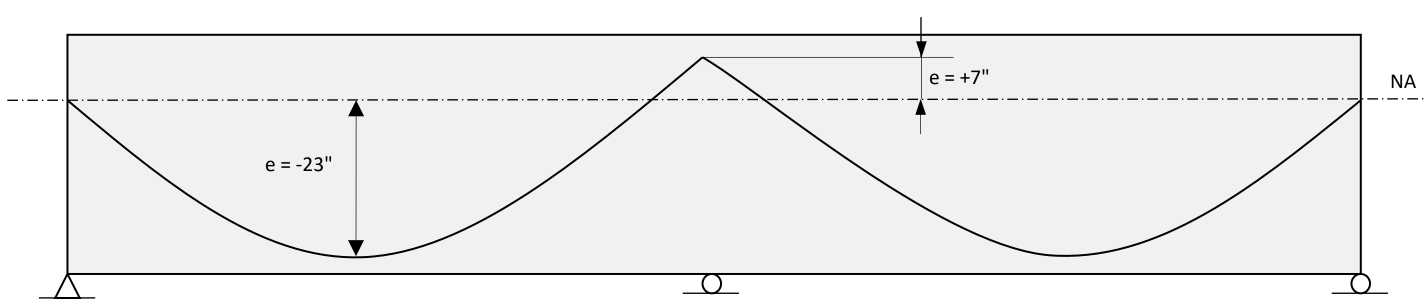
Extreme fiber stress in tension: $f_{tens} \leq 0.75\sqrt{f_c'} = 0.75\sqrt{4000} = 474\,psi$

Service Load Stresses	Exterior Support	Midspan	Interior Support
$M_{DL} + M_{LL} + M_{BAL}$	-264 ft-k	+187 ft-k	-351 ft-k
M/S_{top}	+294 psi	-208 psi	+391 psi
M/S_{bot}	-714 psi	+506 psi	-950 psi
P/A	-276 psi	-276 psi	-276 psi
f_{top}	+18 psi	-484 psi	+115 psi
f_{bot}	-990 psi	-230 psi	-1226 psi

We can see from the above table that at service loads (live load, dead load, and pre-stress load) the maximum tensile stress is +115 psi and the maximum compression stress is -1226 psi, which are both within the allowable limits and acceptable. The intent of the ACI 318 Code in limiting tensile stresses under service load conditions is to guard against premature failure or deterioration of a post-tensioned member due to cracked concrete. Remember that we are using gross section properties and that keeping the stresses within the allowable limits, especially the tensile stresses, assures that the assumption of gross, uncracked section properties is valid.

Now that we have satisfied all of the ACI 318 service load checks, we can move on to analyzing and designing the section using ultimate strength design. Remember that satisfying service load stresses does not ensure adequate structural strength.

The below figure shows the eccentricity of the tendon centroid with respect to the neutral axis at critical points along the span. The primary moment, M_1, is computed as the effective pre-stressing force, F_e, times its eccentricity.

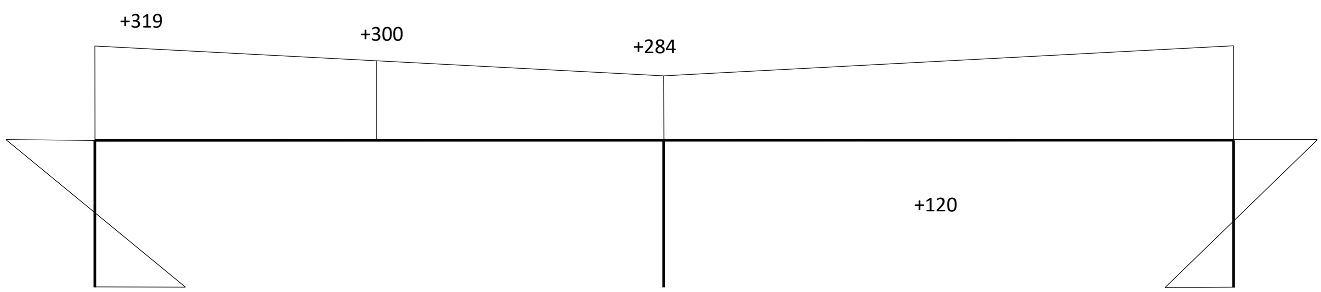


Tendon Eccentricity (F_e = 273 kips)

Referring to the balanced moment diagram above, we can tabulate the hyperstatic moments as follows, remembering that $M_{HYP} = M_{BAL} – M_1$:

Moments	Exterior Support	Mid Span	Interior Support
M_{BAL}	+319 ft-k	-223 ft-k	+443 ft-k
e	0 in	-23 in	+7 in
M_1 (F_exe)	0 ft-k	-523 ft-k	+159 ft-k
M_{HYP}	+319 ft-k	+300 ft-k	+284 ft-k

We can see that hyperstatic moments can be significant. We can also see that when the supports are constructed integrally with the beam, as we have assumed the columns are in this example, the hyperstatic moments in the columns can be significant as well. Hyperstatic moment diagrams are not usually drawn, but we will construct one for this example to create a visual image. Since positive moments cause compression in the top of the beam, the hyperstatic moment diagram looks like this:



Hyperstatic Moments (ft-kips)

It is worth reviewing what hyperstatic actions are. Hyperstatic actions occur in all statically indeterminate post-tensioned members. The restraint forces generated at the supports of indeterminate members when the tendons are stressed are called hyperstatic actions. These hyperstatic actions are due to post-tensioning only. The support reactions due to post-tensioning include vertical reactions (up or down) and moments. Hyperstatic actions must all be in equilibrium when applied to a free-body diagram of the structure. Note that in this course we have only considered hyperstatic moments, but there are also hyperstatic shears that must be considered in the design of post-tensioned members. Hyperstatic actions are more conveniently handled by computer design software. The knowledge gained in this course is intended to enable you to understand the various forces acting on a structure due to post-tensioning, which will aid in understanding computer software output.

Now that we have all of the service load moments at the critical sections, we can determine the factored moments using the following equation:

$$M_u = 1.2M_D + 1.6M_L + 1.0M_{HYP}$$

Factored Moments	Exterior Support	Midspan	Interior Support
M_{DL}	-400 ft-k	+280 ft-k	-556 ft-k
M_{LL}	-183 ft-k	+130 or -11 ft-k	-238 ft-k
M_{HYP}	+319 ft-k	+300 ft-k	+284 ft-k
M_u	-454 ft-k	+844 ft-k	-764 ft-k

Let's first compute the nominal flexural capacity of the section using the tendons alone and then compute what additional mild reinforcing steel, if any, will need to be added to supplement the flexural capacity of the section. Then we can compare this amount of mild steel to the minimum required and use the larger.

From Part One of this course, recall that the flexural capacity of the tendons alone is computed as follows:

$$\phi M_n = \phi A_{ps} f_{ps} (d_p - \frac{a}{2})$$

Where A_{ps} is the area of pre-stressed reinforcement, f_{ps} is the stress in the pre-stressed reinforcement at nominal moment strength, and ϕ is the strength reduction factor (0.90 for flexure). For the span-to-depth ratio in this example (60/3 = 20; < 35), the approximate value for f_{ps} can be calculated as:

$$f_{ps} = f_{se} + 10{,}000 + \frac{f_c'}{100\rho_p}$$

$$\leq f_{py} \ or \ f_{se} + 60{,}000$$

Where f_{se} is the effective stress in the pre-stressing steel after all losses.

ρ_p is the ratio of pre-stressing steel to the cross sectional area, or:

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{273k/160ksi}{(14)(32.5)} = 0.00375$$

$$f_{ps} = 160{,}000 + 10{,}000 + \frac{4000\ psi}{100(0.00375)}$$

$$f_{ps} = 180.7\ ksi \quad \Longrightarrow \quad \boxed{\text{Use } f_{ps} = 181 \text{ ksi}}$$

$$\leq\ f_{py} = 270\ ksi$$

$$\leq 160{,}000 +\ 60{,}000 = 220\ ksi$$

Next, determine depth of compression block, a, for a positive moment:

$$a = \frac{A_{ps}\,f_{ps}}{0.85 f_c' b} = \frac{1.71\ sq.in.(181\ ksi)}{0.85\ (\ 4\ ksi)(102\ in)} = 0.89\ inches < 5.5\ inches -\ \boldsymbol{OK}$$

Now we can compute the nominal positive flexural capacity, using the tendons alone, at the midspan of this beam:

$$\phi M_n\ =\ \phi A_{ps}\,f_{ps}\left(d_p - \frac{a}{2}\right)$$

$$\phi M_n\ =\ 0.90(1.71\ sq.in.)(181\ ksi)\left(32.5\ in. - \frac{0.89\ in.}{2}\right)\frac{1}{12}$$

$$\phi M_n\ = 744\ ft - kips < 844\ ft - kips$$

The positive flexural capacity of the beam at midspan is less than the maximum factored demand of 844 ft-kips. Therefore, we will have to supplement the flexural capacity of the beam section with bonded mild reinforcing in the bottom. An approximation of the additional amount of bonded flexural reinforcing steel required is:

$$Additional\ A_s\ = \frac{M_u - \phi M_n}{\phi f_y (d - \frac{a}{2})} = \frac{(844 - 744)12}{0.9(60)(32.5 - \frac{0.89}{2})} =\ 0.69\ in^2$$

This is less than the minimum area of steel in the bottom of = 14(25.5)(0.004) = 1.42 in^2, and so we will use the minimum amount. We will use 2#8 in the bottom. Just as a check, let's compute the nominal moment capacity with the tendons and 2#8, conservatively assuming the 2#8s are at the same depth as the tendon bundle:

$$a = \frac{A_{ps}\,f_{ps} + A_s\,f_y}{0.85 f_c' b} = \frac{1.71(181) +\ 1.58(60)}{0.85(4)(102)} =\ 1.166\ inches < 5.5\ inches -\ \boldsymbol{OK}$$

$$\phi M_n\ =\ \phi(A_{ps}\,f_{ps} + A_s\,f_y)(d_p - \frac{a}{2})$$

$$\phi M_n = 968\, ft - kips > 844\, ft - kips - \boldsymbol{OK}$$

Without showing the calculations for the negative moment region at the interior support, we find that the negative flexural capacity using the tendons alone is:

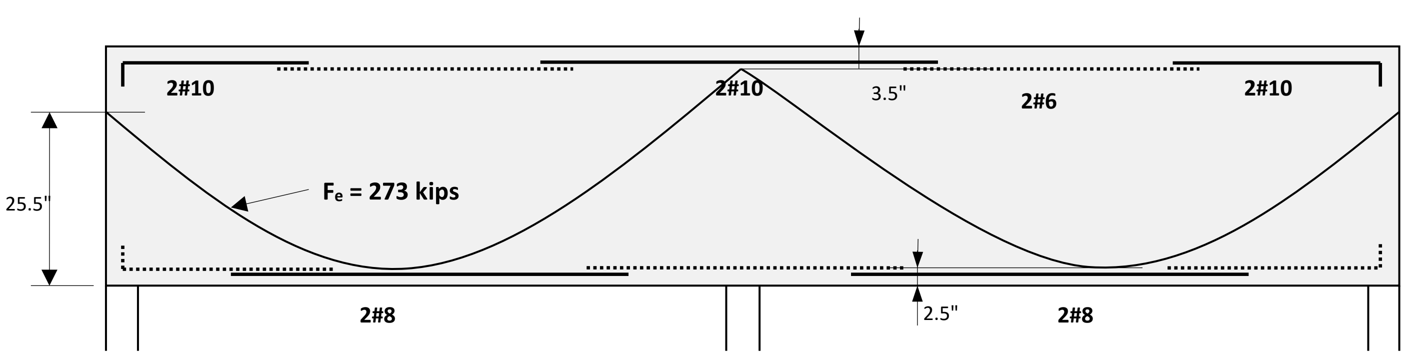
$$\phi M_n = 682\, ft - kips < 764\, ft - kips - \boldsymbol{NO\ GOOD}$$

The minimum area of mild bonded reinforcement in the top is = [14(5) + 102(5.5)](0.004) = 2.52 in^2, or 2#10. Using 2#10s in the top, the negative flexural capacity at the support, including minimum bonded reinforcing, is:

$$\phi M_n = 958\, ft - kips > 764\, ft - kips - \boldsymbol{OK}$$

We can see that the negative flexural capacity with the minimum area of bonded reinforcing included is greater that the negative moment demand of 764 ft-kips and therefore is acceptable. The negative flexural capacity at the exterior support is similarly calculated.

The shear analysis and stirrup design for as beam like this has been covered in Part One of this course and is not repeated here. A difference is that the structure in this example is a statically indeterminate structure, and the maximum shear in the beam will be influenced by hyperstatic shears, which are not covered in this course. Also note that in this example, the gravity loads on the structure and hyperstatic actions induce moments and shears into the columns that need to be taken accounted for when designing the columns. Fortunately, most structural analysis and design software available accounts for these forces. The knowledge gained in this course is intended to enable you to at least understand the various forces acting on a structure due to post-tensioning, and to perform quick and basic checks of computer output.



Final Design Sketch

Notes on the mild reinforcing detailing:

- Shear stirrups are not shown for clarity.
- Since the width of this beam is rather narrow (14"), it is important to make sure the shear and flexural reinforcing all fits in the beam. Therefore, we have called for only two bottom bars to minimize congestion.
- The top bars are allowed to be placed outside the beam stirrups if they don't otherwise fit, but this should be avoided if possible.
- Bar cut off lengths should meet the ACI minimums, as shown in Part One of this course, and should be greater if required by analysis.
- The reinforcing that is dashed is not strictly needed by analysis, but it is usually provided for stirrup support. These stirrup support bars need to be minimally lapped with the main steel, so be sure there is enough room inside the stirrups for these lap splices. These bars also tend to reduce the overall shortening of the beam due to the pre-stress force.
- The bottom end bars are usually hooked (or otherwise developed) into the support in case the hyperstatic moment exceeds the dead load moments, thereby creating unintended tension in the bottom of the beam.
- It is good practice to provide stirrups the entire length of the beam, at some maximum spacing such as 48" o.c., even where stirrups are not required by analysis. These stirrups are used to tie tendon support bars to at the appropriate height.

Conclusion

Part Two of this three-part course covers some of the more typical design topics related to post-tensioned concrete for building structures using unbonded tendons. It should be abundantly obvious by now that that the detailed analysis and design of post-tensioned systems can be extremely tedious and difficult to accomplish by hand and is best handled by computer software specifically intended for this purpose. However, with a good understanding of the material in Part Two of this course, you should:

- Be able to quickly check a computer generated design for typical one-way systems such as a continuous one-way slab and a continuous beam.
- Understand how to compute an approximate effective pre-stress force using an assumed balance load.
- Understand allowable transfer and service load stresses according to ACI 318-08.
- Determine the amount of bonded flexural reinforcing and when it is required.
- Calculate the nominal moment capacity ϕM_n of a one-way slab or beam system.

References

Building Code Requirements for Structural Concrete, ACI 318-08, American Concrete Institute, 2008.

Notes on ACI 318-08, Portland Cement Association, 2008.

Design of Prestressed Concrete Structures, T.Y. Lin and Ned H. Burns, Third Edition, John Wiley & Sons, 1981

Design Fundamentals of Post-Tensioned Concrete Floors, Bijan O. Aalami & Allan Bommer, Post-Tensioning Institute, 1999

Design of Post-Tensioned Slabs Using Unbonded Tendons, Post-Tensioning Institute, Third Edition, 2004

Post-Tensioning Manual, Post-Tensioning Institute, Fourth Edition, 1985

Precast and Prestressed Concrete, PCI Design Handbook, Precast Concrete Institute, Third Edition, 1985

Control of Deflections in Concrete Structures, ACI Committee 435, ACI Manual of Concrete Practice, 2002

