



A SunCam online continuing education course

What Every Engineer Should Know About Structures

Part B – Statics Applications

by

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This is the second course of a series in the area of study of engineering mechanics called Statics and Strength of Materials. The series of courses provides the tools for solving some of the most common statically determinate structural problems.

What Every Engineer Should Know About Structures, Part B – Statics Applications is an applied statics course focusing on presenting simplified methods of solving some common statics problems. Anyone who has taken the first course of this series – **What Every Engineer Should Know About Structures, Part A – Statics Fundamentals** – can complete this course.

Part One – REACTIONS presents a method for solving for the reactions of statically determinate beams and trusses.

Part Two – TRUSSES presents two methods for solving for the forces in truss members.

Part Three – CABLES presents a simple method for solving for the tensile force in cables supporting single, multiple, and uniformly distributed loads.

Part Four – FRICTION introduces the concept of static friction.

Part Five – SHEAVES and PULLEYS introduces terminology and procedures for raising and lowering loads and the effects of friction.

Basic Trigonometry, Significant Figures, and Rounding – A Quick Review is a zero credit course intended for those who might find themselves a bit rusty and would like a quick refresher. The information in the course is useful for application to the solution of structural problems especially in the fields of statics and strength of materials.

This course is free and can be downloaded by “Ctrl+Click” the link below.

<http://www.suncam.com/authors/123Glon/TRIG.pdf>

The trigonometry review includes the use of the basic trigonometric functions including: the sine, cosine and tangent and their inverse; the Pythagorean Theorem; the Sum of the Angles; the Law of Sines; and the Law of Cosines. The significant figures and rounding review includes a discussion of the precision and validity of an answer, along with rules and guidelines for using the appropriate number of significant figures, and for rounding answers appropriately.



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WHAT IS STATICS

Statics is the study of bodies that are stationary. It is part of the division of physics known as mechanics. Statics is concerned with determining the forces acting on a body. Beginning with Newton's Three Laws of Motion and eliminating the parts that pertain to acceleration and motion, we are left with two laws. The study of statics is based on these two revised portions of Newton's First Law and his Third Law.

- **First Law** – a body at rest will stay at rest if the forces acting on it are in equilibrium.
- **Third Law** – for every action there is an equal and opposite reaction.

INTRODUCTION

The methods and techniques presented in the first course of this series can be applied to a great number of real-life applications. Except for cutting edge, highly theoretical, and very precise work we can now make some simplifying assumptions and approximations and solve most of the statics problems encountered in everyday life.

As a practical matter, determining design loads on structural members involves several approximations. Seldom is a live load known exactly – either in precise location or precise magnitude. For example, wind loads used for design are based on past weather conditions and atmospheric records. Knowing that information is no guarantee that the future winds on the face of a building or bill-board will be within those parameters. Building codes specify minimum design wind loads. The specified minimum design loads must be a reasonable estimate of the actual maximum wind force that may occur – taking into account natural or man-made extraneous conditions such as nearby buildings, trees, mountains, etc., and other natural, atmospheric anomalies. In the end, the building code minimum design wind load is a best guess, albeit a thoroughly thought out and reasoned guess, and should be used as a minimum load.

Support conditions for structural members are another condition that is not precisely known in the majority of situations. Dealing with exact support conditions usually requires very complicated math to properly analyze. Approximating the conditions at the support eliminates the complex math, allows for simple computations, and produces an answer reasonably close to



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the actual answer. Applying the “worst case” support condition – the condition that will produce the larger load on the member – is standard practice in determining the design loads on a structural member. The lack of precision in loads and support conditions also works well with the practice of rounding answers to only three significant figures.

A story Solving practical, everyday statics problems involves an ability to imagine the “real-life” uses of the member being analyzed. Now, I realize that this is a problem solving course, not a story telling exercise, but I would like to share one short story with you that clearly demonstrates the limitations of code dictated design loads. I was designing the structural system for a new addition to the Tau Kappa Epsilon fraternity chapter house on the Purdue campus in West Lafayette, IN. The house sat on a side street that the college marching band used every day during football season to march to their practice field. The band often stopped in the street, turned to face the fraternity house, and play the Purdue fight song for the group of men standing on the front porch. The men of the house waited for them every day. It was great fun.

The new house addition was designed with a second story, open air deck, approximately 30 feet by 30 feet. It was a large deck for house activities and it faced the side street that the band used. It was intended to, among other things, allow the men of the house to be able to view and appreciate the marching band as they stopped and played for them.

The building code in effect at the time dictated a minimum live load for the design of all decks of this sort. In anticipation of a large group of men – and maybe women from the neighboring sororities – enjoying life, watching the band on a beautiful, bright, cloudless autumn afternoon, the sun shining, and perhaps bouncing in rhythm to the music, I decided that the minimum design load might not be enough for a truly safe design. So, for the design of those deck floor joists I used a design live load of nearly half again as much as the required code minimum to determine the load each joist was to be able to support. At this point in the design, it was a simple statics problem. Plus, the additional cost for the larger joists was minimal.

The house was built, the chapter members moved in, and the music of the marching band was enjoyed. Then one afternoon I received a call from the student chapter president, who was also a friend of mine. “Professor Glon”, he began. “Is our deck strong enough to have a Hawaiian themed party on it?” The men of the house wanted to seal the perimeter drains, place visqueen to hold water, and haul 8 tons of sand on the deck to create a beach around a pool of water about two feet deep! I did a few quick calculations to check the actual live loads and found that the joists were indeed strong enough to support that kind of load. They did not have to cancel or



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materially change their party. Although this wasn't the "anticipated" live load, it was indeed an imaginative live load, and it was certainly greater than the code required minimum.

There are two morals to this story. The first is that when calculating loads on members, try to anticipate what the "worst case" scenario might be and be sure that the code required loads are adequate for that use; and, second, that determining an anticipated live load to a high degree of precision during the design phase is often nearly impossible. Again, that's another reason the precision used in statics calculations is only three significant figures. [End of story](#).

Part One -REACTIONS

Background In structural mechanics there are a number of terms that are used to classify and describe a problem. **Structural members** are the pieces in a structure that carry a load. There are many names for these members – **beam** and **truss** are probably the most common. It turns out that trusses and beams behave very similarly when loaded. Because they do, the word truss can be substituted for beam in any of the following discussion on reactions.

Supports are the name of the things that hold structural members up. There are several different kinds of supports and each one behaves differently.

Reactions refer to the forces that occur at the supports of structural members.

Classification Of Beam Supports The supports for beams, trusses, and other structural members come in a wide variety of shapes and types, and are often complex. Structural members are attached to their supports with nails, bolts, or welds and some members are simply set on the support. The exact manners in which the forces are transferred from the beam to the support are varied and can be difficult to determine. A system of support classifications has been developed to allow relatively simple solutions to complex statics problems – all within the reasonable limits of the accuracy of the loads.

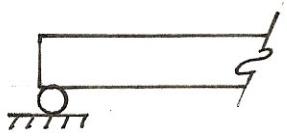
Idealized supports To simplify the solution of statics problems, four idealized supports have been created. These four idealized supports approximate how the actual support conditions interact with the structural member. It is an ideal condition – not actually how it works. For example, the roller support assumes absolutely no friction. This is not true in real life. There is actually some friction at a roller support. And the fixed support assumes perfect rigidity. This is



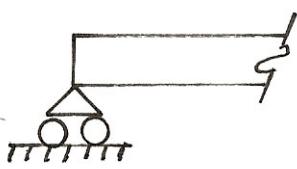
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not true either. There is a certain, slight, rotation in a fixed support. However, the error introduced by using an idealized support instead of the actual support is usually negligible – certainly beyond the three-significant figure range of accuracy for the most common problems.

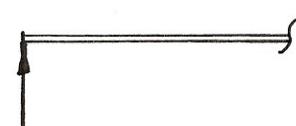
Roller Support A roller support is a support that permits the end of the beam to move parallel to the support. It is assumed to be frictionless. If the beam wants to shorten because of a drop in temperature, a roller will permit it to do so. There is no friction force to keep the beam from sliding on the support. A roller support also permits the end of the beam to rotate and does not permit the transfer of a moment reaction. As a beam deflects in the center, the ends of the beam want to rotate a bit. A roller support is assumed to allow the rotation to happen with no resistance whatsoever. A roller support can have only a single reaction, one that is normal (perpendicular) to the supporting surface. It does not permit the beam to move either up or down, normal to the supporting surface. The support can “push up” and “pull down” on the end of the beam. Notice in the free-body diagram that the beam is shown as a double line. This is common, though not mandatory.



Pictorial Views

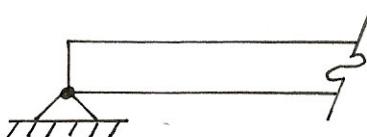


Roller Support Representations



Free-Body Diagram

Pinned Support A pinned support is a support that does not permit the end of the beam to move in any direction (up or down or left or right) but it does allow the beam to rotate about the pin. Therefore, this type of support can provide a reaction in any direction, but it cannot have a moment reaction. Instead of showing the reaction at an odd angle, the reaction is divided into its horizontal and vertical components.



Pictorial View



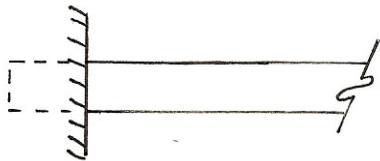
Free-Body Diagram

Pinned Support Representations



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Fixed Support A fixed support holds the end of the beam rigidly, permitting no translation – up, down, left, or right – or rotation. Again, this is an idealized condition. It is nearly impossible to construct a reasonably priced support that allows absolutely no rotation. The effect of any actual minimal rotation at a fixed joint is usually beyond the third significant figure. This type of support provides a vertical reaction, a horizontal reaction, and a moment reaction.



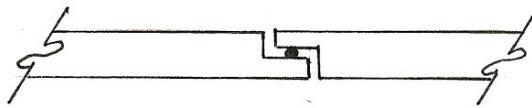
Pictorial View



Free-Body Diagram

Fixed Support Representations

Hinge Support A hinge support is similar to a pinned support except that a pinned support is assumed to be attached to some immovable object. A hinged support is attached to a moveable object – a beam. It is used to connect two beams and to allow some vertical displacement of the joint. A hinged support only has a vertical reaction.



Pictorial Views



Free-Body Diagram

Hinge Support Representations

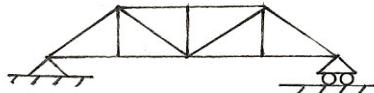
Classification Of Structural Members Beams, trusses, and frames are similar structural members with different names. All of them are used to carry loads across horizontal distances. A **beam** is a straight, solid member. A **truss** is composed of many small members that are connected in a triangular pattern. And a **frame** is a solid section that is not straight. Usual practice is to show beams and frames as two parallel lines (which represent the top and bottom of the member), and each member of a truss as a single line. Notice that a single beam or truss is supported with a roller support and a pinned support – not two rollers or two pins. This limits the number of unknown forces in the problem to a maximum of three – equal to the three equilibrium equations in 2-dimensional problems.



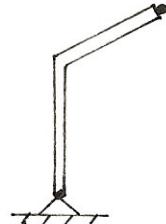
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Beam



Truss
Structural Members

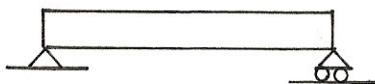


Frame

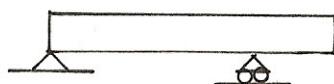
Types of Beams and Trusses Beams and trusses are classified according to the type and location of their supports. Although the term “beam” is used here, the same classification is used to describe both beams and trusses. There are four types of beams (trusses) that can be solved by the laws of statics alone. There is another category of beams, indeterminate beams, which cannot be solved by the laws of statics alone.

Simple beams are beams with two supports, one at each end. One support must be a pinned support and the other support must be a roller support.

Overhang beams are beams with two supports. At least one of the supports is not located at the end of the beam. One support must be a pinned support and the other support must be a roller support.



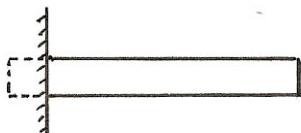
Simple Beam



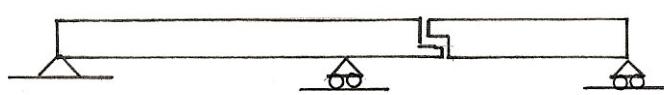
Overhang Beam

Cantilever beams are beams with a fixed support at one end and no other support.

Hinged beams are segmented (2 or more) beams that are connected to each other with hinges. The number of supports must be equal to the number of hinges plus two. One of the supports must be pinned and the others must be roller supports.



Cantilever Beam

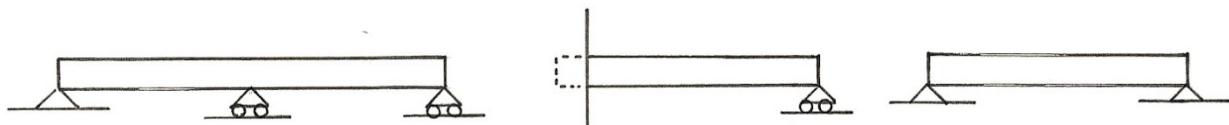


Hinged Beam



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Indeterminate beams are beams that cannot be solved by using the laws of statics alone. Beams with more than two supports are indeterminate. Beams with a fixed support at one end and another support of any type are indeterminate. Beams with two pinned supports are indeterminate, although they are frequently treated as simple beams.



Indeterminate Beams

Procedure For Computing Reactions The general procedure for computing reactions is straightforward. It is described below. Examples using the procedure follow.

1. Draw a free-body diagram of the problem. Show all the loads and the necessary dimensions. Label reference points and reactions using any symbols you choose.
2. If there are any distributed loads, determine the resultant and show it on the diagram as a dashed line (or show the load as a dashed line and the resultant as a solid line).
3. Convert diagonal forces to their horizontal and vertical components. Show the diagonal force as a solid line and the components on the diagram as dashed lines (or the diagonal force as a dashed line and the components as solid lines).
4. Determine the left reaction by summing moments at the right reaction. Note that the moment arm for the right reaction is zero, and thus the right reaction does not appear in the equation.
5. Repeat Step 4 at the other reaction.
6. Check your solution for the vertical reactions by summing forces vertically. They must add to zero – up must equal down.
7. If there are horizontal forces, find the horizontal reaction by summing forces horizontally.
8. Show the reactions on the diagram. Be sure to show the units.

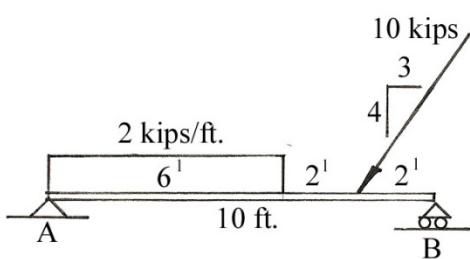
Computing Reactions For Different Types Of Structural Members The procedure for solving for the reactions for a **simple beam** is as described above. Note that the procedure calls for the computing of each reaction by means of a moment equation, then checking the answers



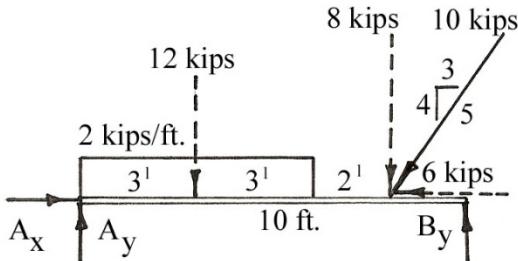
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by summing the forces vertically. This is a better procedure than computing one reaction, then finding the other by subtracting from the total load.

Example: Find the reactions for a simple beam with a uniform and diagonal load.



Problem



Free-Body Diagram

Begin by redrawing the problem as a free-body diagram. Show the reactions on the free-body according to the type of support given in the problem—i.e., replace the pinned support with a horizontal and vertical reaction; and replace the roller support with a vertical reaction. Label them appropriately. In this case use A_x , A_y , and B_y . Convert the uniform load into its resultant and note its location; and convert the diagonal force into its x- and y- components. Show these forces as dashed lines to distinguish them from the given loads.

Now sum moments about A equals zero to find B_y . Then sum moments about B equals zero to find A_y

$$\begin{array}{c} \sum M @ A = 0 \\ \curvearrowleft \quad \curvearrowright \\ \hline \begin{array}{|c|c|} \hline B_y(10) & 12(3) \\ \hline & 8(8) \\ \hline \end{array} \\ 10 B_y = 100 \\ \underline{\underline{B_y = 10 \text{ kips} \uparrow}} \end{array}$$

$$\begin{array}{c} \sum M @ B = 0 \\ \curvearrowleft \quad \curvearrowright \\ \hline \begin{array}{|c|c|} \hline 8(2) & A_y(10) \\ \hline 12(7) & \\ \hline \end{array} \\ 100 = 10 A_y \\ \underline{\underline{A_y = 10 \text{ kips} \uparrow}} \end{array}$$

Now check the work by summing forces vertically equals zero. And finally, sum forces horizontally equals zero to find A_x .

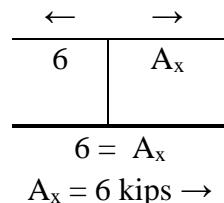
$$\begin{array}{c} \sum F_y = 0 \\ \uparrow \quad \downarrow \\ \hline \begin{array}{|c|c|} \hline 10 & 12 \\ \hline 10 & 8 \\ \hline \end{array} \\ 10 = 10 \end{array}$$



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Check✓

$$\sum F_x = 0$$



$$A_x = 6 \text{ kips} \rightarrow$$

Overhang Beam A special problem associated with overhang beams is the possibility of a negative or tie-down reaction. If the loads on the overhang portion of the beam are large enough, the “other” end of the beam may want to lift up. The pinned, roller, and fixed supports defined earlier all prevent displacement in the vertical direction – both up and down. There is no problem with the supports being able to handle an upward lift on the connection point.

There are two ways of handling the tie-down condition. The first procedure is to assume each reaction is acting upward. If the reaction is actually acting downward, the computed value will be negative. The answer can be shown on the diagram with an upward arrow and a negative value. The second procedure is to determine the correct direction of the reaction with an educated guess before starting the calculations. In this case, the answer is shown on the diagram with a downward arrow and a positive value.

A second problem associated with overhang beams deals with the calculations for a distributed load that extends onto the overhang portion of the beam. The distributed load could be divided at the support(s) into separate segments at Step 2, and the reactions computed for each of the supports. However, the preferred procedure is to leave the distributed load as one piece and show the total (single) reaction through the center of gravity of the distributed load diagram.

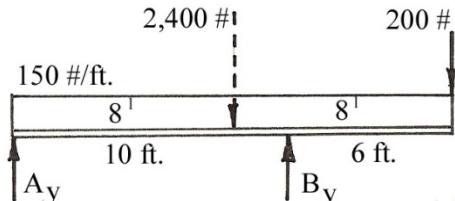
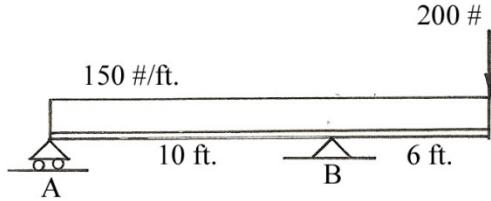
Example: Find the reactions for a beam with an overhang using the preferred procedure.

This loading is used to design floor joists for a second floor apartment deck. The interior floor joists of the apartment extend through the exterior wall and overhang outside the building to form the floor of the deck. Often there is a door opening in the exterior wall below. The reaction B_y is then used as the loading on the header above that door opening below.



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The procedure includes assuming both reactions act up and leaving the uniformly distributed load as one piece.



Problem

Free-Body Diagram

Begin by redrawing the problem as a free-body diagram. Show the reactions on the free-body according to the type of support given in the problem. In this case, there are no horizontal loads on the beam; therefore no horizontal force component is shown on the free-body diagram. Remember, with two pinned supports (and, therefore four unknowns) the beam is indeterminate and can't be solved with just three equilibrium equations; however, just the two vertical unknowns can be solved with only two equilibrium equations. It would be meaningless to show a horizontal force at each support because it's obvious that their values would be zero. Again, both vertical reactions on the free-body are assumed to act in the up direction. Label them appropriately. In this case use A_y , and B_y . Convert the whole uniform load into one resultant and show as a dashed line. Note its location as 8 feet from both ends.

Now sum moments about A equals zero to find B_y . Then sum moments about B equals zero to find A_y

$$\begin{array}{c} \sum M @ A = 0 \\ \curvearrowleft \quad \curvearrowright \\ \hline B_y(10) & | & 2400(8) \\ & | & 200(16) \\ \hline 10 B_y & = & 22,400 \\ \underline{B_y = 2,240 \# \uparrow} \end{array}$$

$$\begin{array}{c} \sum M @ B = 0 \\ \curvearrowleft \quad \curvearrowright \\ \hline 2,400(2) & | & A_y(10) \\ & | & 200(6) \\ \hline 4,800 & = & 10 A_y + 1,200 \\ 10 A_y & = & 3,600 \\ \underline{A_y = 360 \# \uparrow} \end{array}$$

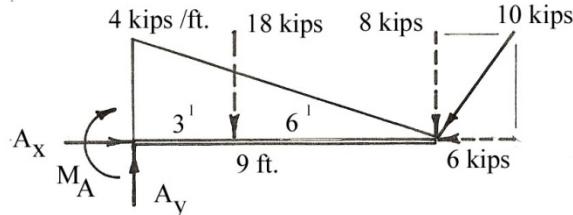
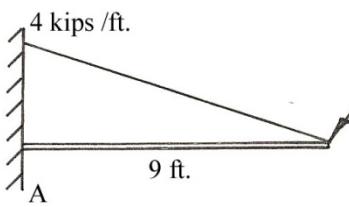
Now check the work by summing forces vertically equals zero.

$$\begin{array}{c} \sum F_y = 0 \\ \uparrow \quad \downarrow \\ \hline 2,240 & | & 2,400 \\ 360 & | & 200 \\ \hline \end{array}$$

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$$\overline{2,600} = \overline{2,600}$$

Check ✓



Cantilever Beams A cantilever beam has only one support, and all of the reactions must be at that support. The three possible reactions are: (1) vertical reaction, (2) horizontal reaction, and (3) end moment. The vertical and horizontal reactions are determined by summing forces in the vertical and horizontal directions. The end moment is determined by summing moments at the face of the support.

Example: Find the reactions for a cantilever beam supporting a uniformly varying load and a diagonal load.

Problem

Free-Body Diagram

To solve the problem, draw the free-body diagram; show the three reactions (all the reactions are at the left end of the beam – labeled Point A); assume a direction for each reaction (often simply use up, right, and clockwise – if any are the wrong direction, the answer will simply be a negative number); label the reactions – in this case A_y , A_x , and M_A ; calculate and show the resultant of the uniformly varying distributed load as a dashed line and note its location; and resolve the diagonal force into its x- and y- components and show as dashed lines.

The resultant of the uniformly varying distributed load is the area under the load diagram, $(1/2)bh = (1/2)(9 \text{ ft.})(4 \text{ kips/ft.}) = 18 \text{ kips}$. Its location is at the $1/3 - 2/3$ point.

Now sum forces vertically equal zero to find A_y . Then sum forces horizontally equals zero to find A_x . Finally sum moments at A equals zero to find M_A . Notice that M_A is a negative number

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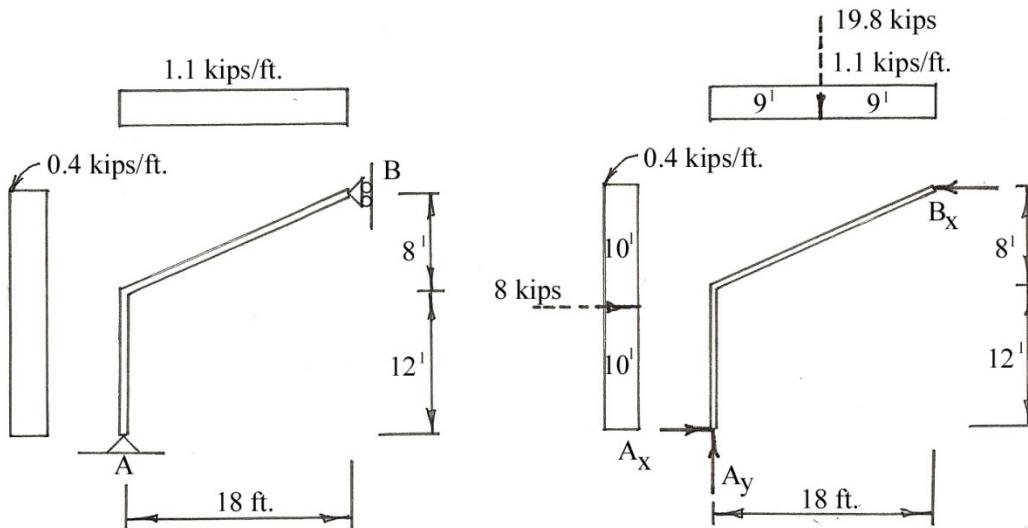
which means that the assumed direction is wrong. The direction of the moment is actually counterclockwise.

$\sum F_y = 0$ $\begin{array}{c} \uparrow \\ A_y \end{array} \quad \begin{array}{c} \downarrow \\ 18 \\ 8 \end{array}$ <hr/> $A_y = 26 \text{ kips } \uparrow$	$\sum F_x = 0$ $\begin{array}{c} \leftarrow \\ 6 \end{array} \quad \begin{array}{c} \rightarrow \\ A_x \end{array}$ <hr/> $6 = A_x$ $A_x = 6 \text{ kips } \leftarrow$	$\sum M @ A = 0$ $\begin{array}{c} \curvearrowleft \\ 0 \end{array} \quad \begin{array}{c} \curvearrowright \\ M_A \\ 18(3) \\ 8(9) \end{array}$ <hr/> $0 = M_A + 126$ $M_A = -126 \text{ kip-ft. } \curvearrowright$ $M_A = 126 \text{ kip-ft. } \curvearrowleft$
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Frames Frames come in many shapes and arrangements. They frequently have hinges. The procedure for determining the reactions must be developed for each problem. Frequently, simultaneous equations must be solved.

Example: Find the reactions for the frame supporting horizontal and vertical uniformly distributed loads.

This problem is similar to the loading on a steel frame building. The frame represents the structural frames of a metal building which are typically placed 20 to 30 feet apart. The





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uniformly distributed downward load represents a snow load and the uniformly distributed horizontal load represents a wind load on the frame.

Problem

Free-Body Diagram

Although this problem may look complicated, it is not. Simply attack the solution one step at a time beginning with drawing the free-body diagram. Show all the loads, dimensions, and reactions. The pinned support includes a horizontal and a vertical reaction and the roller support has only one reaction acting perpendicular to the support surface (in this case the reaction is horizontal which is perpendicular to the wall or other half of the frame). Determine the resultant of the uniformly distributed loads, show as dashed lines, and note their locations.

To solve, sum forces vertical equals zero yields A_y . Then sum moments at A equals zero yields B_x . Finally, sum forces horizontally equals zero yields A_x .

$$\begin{array}{c} \sum F_y = 0 \\ \uparrow \quad \downarrow \\ \hline A_y & | & 19.8 \\ \hline \end{array}$$

$A_y = 19.8 \text{ kips } \uparrow$

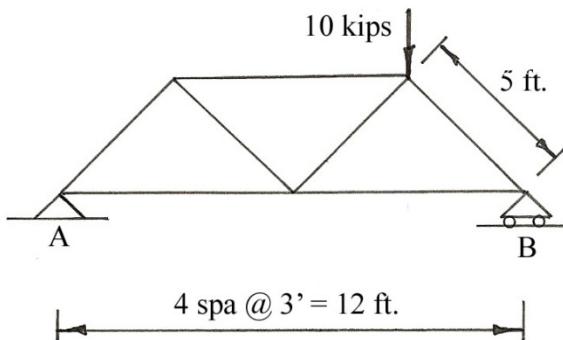
$$\begin{array}{c} \sum M @ A = 0 \\ \curvearrowleft \quad \curvearrowright \\ \hline B_x (20) & | & 19.8 (9) \\ & & 8 (10) \\ \hline 20 B_x = & 258.2 \\ \hline B_x = 12.9 \text{ kips } \leftarrow \end{array}$$

$$\begin{array}{c} \sum F_x = 0 \\ \leftarrow \quad \rightarrow \\ \hline B_x & | & A_x \\ & & 8 \\ \hline 12.9 & = & A_x + 8 \\ \hline A_x = 4.9 \text{ kips } \rightarrow \end{array}$$

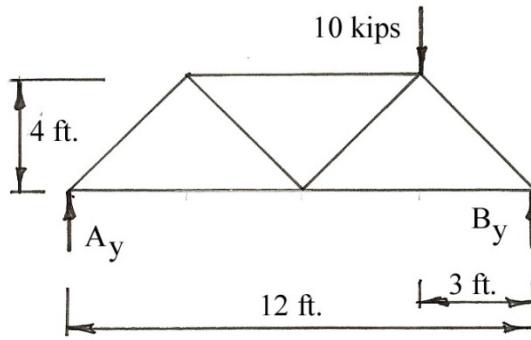
Trusses The procedure for determining reactions for trusses is exactly the same as for beams. As the problem is solved, don't forget that the moment created by a load applied at a different level than the supports is equal to the force times the perpendicular distance to the line of action of the force (not the direct line distance to the point of application).

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Example: Find the reactions for the truss shown below.



Problem



Free-Body Diagram

Draw the free-body diagram of the truss exactly as for a beam. Use a single line for the members of the truss. Replace the supports with vertical reactions (there is no horizontal load on the truss shown therefore there is no horizontal reaction). Then sum moments at A equals zero to find B_y ; sum moments at B equals zero to find A_y ; and then sum forces vertical to check the arithmetic.

$$\begin{array}{c} \sum M @ A = 0 \\ \curvearrowleft \quad \curvearrowleft \\ B_y(12) \quad 10(9) \\ \hline 12B_y = 90 \\ B_y = 7.5 \text{ kips } \uparrow \end{array}$$

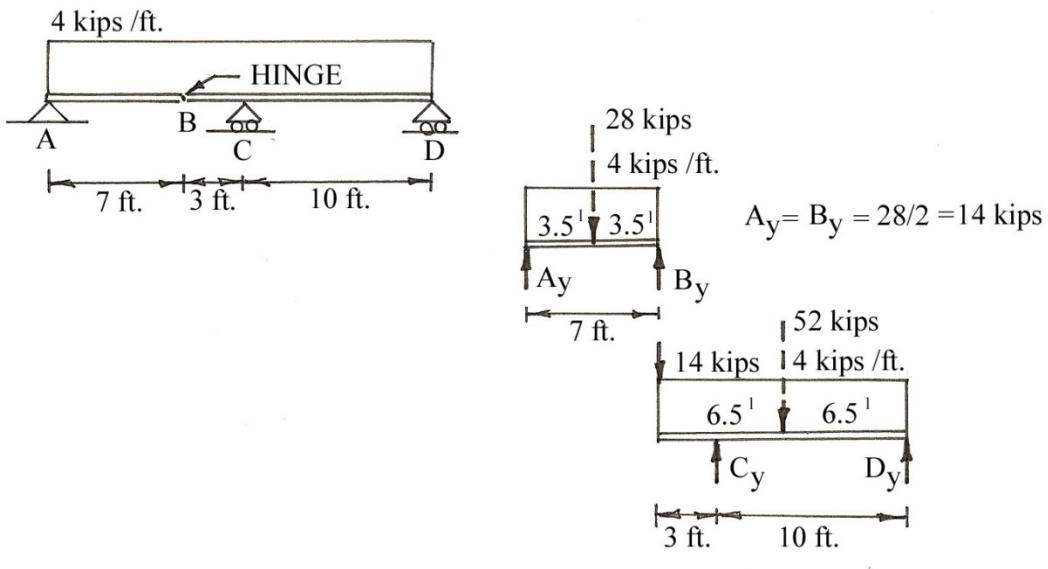
$$\begin{array}{c} \sum M @ B = 0 \\ \curvearrowright \quad \curvearrowright \\ 10(3) \quad A_y(12) \\ \hline 30 = 12A_y \\ A_y = 2.5 \text{ kips } \uparrow \end{array}$$

$$\begin{array}{c} \sum F_y = 0 \\ \uparrow \quad \downarrow \\ 2.5 \quad 10 \\ \hline 10 = 10 \\ \text{Check } \checkmark \end{array}$$

Hinged Beams A hinged beam is actually two or more beams connected in series with a hinge. At each hinge, one beam is supporting the other. The beam being supported is called a suspended beam. The suspended beam is analyzed first. The reaction of a suspended beam then becomes a load on the supporting beam.

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Example: Find the reactions for a beam with a hinge.



Problem

Free-Body Diagram

Draw the free-body diagrams of the beam with a hinge. Show two sections of beam – one being supported, the other doing the supporting. It helps to visualize the interaction of the two beams by drawing one above the other. Replace all supports with reaction forces. Replace the hinge with a reaction on the suspended beam and a load on the supporting beam.

Solve for the “reactions” of the suspended beam first. Then use the reaction at the hinge as a load on the supporting beam and solve for the reactions of the second beam.

Note that in the drawing above, the calculations for the reactions for the suspended beam, A_y and B_y , have been simplified. This is a symmetrical problem – both the load and the supports are symmetrical about the vertical centerline. Since the problem is symmetric, the reactions must be symmetric (equal). When an answer can be determined by intuition and simple arithmetic, it is unnecessary to use a more complex analysis. $A_y = 14 \text{ kips } \uparrow$. Use the reaction $B_y = 14 \text{ kips}$ of the suspended beam as a 14 kip load at the end of the supporting beam. Then solve for the reactions of the supporting beam.

$$\sum F_y = 0$$



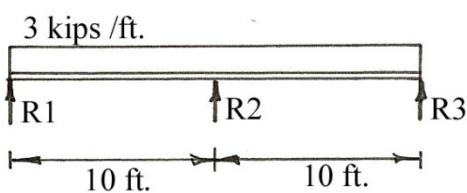
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$$\begin{array}{c} \sum M @ C = 0 \\ \curvearrowleft \quad \curvearrowright \\ \hline 14(3) & 52(3.5) \\ D_y(10) & \\ \hline 10D_y + 42 = & 182 \\ D_y = 14 \text{ kips } \uparrow & \end{array}$$

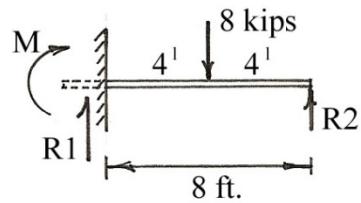
$$\begin{array}{c} \sum M @ D = 0 \\ \curvearrowleft \quad \curvearrowright \\ \hline 52(6.5) & C_y(10) \\ 14(13) & \\ \hline 520 = & 10C_y \\ C_y = 52 \text{ kips } \uparrow & \end{array}$$

↑	↓
14	28
52	52
14	
<hr/> 80 = 80	
Check ✓	

Indeterminate Beams The solution of an indeterminate beam is beyond the scope of a course in statics – it requires knowledge of the behavior of the material. However, it is important to be able to recognize problems that you can't solve. Two indeterminate beams are shown below. In both cases, there are three unknowns, but only two equations can be written.



3 unknowns: R_1, R_2, R_3
 2 equations $\sum F_y = 0, \sum M = 0$



3 unknowns: R_1, R_2, M
 2 equations: $\sum F_y = 0, \sum M = 0$

Indeterminate Beams
Part Two - TRUSSES

Introduction A truss is special kind of beam normally used to span horizontal distances. Trusses are composed of structural members that are joined together in a lattice framework. Each section of the lattice is in the shape of a triangle. Each point of the triangles is called a joint – where several truss members are connected. A single truss member goes no further than joint to joint.

A truss is an idealized structural system. The joints are assumed to be pinned with a single bolt and to have no friction. No friction means the members of the truss – the lattice framework – are free to rotate. Each structural member of the lattice network only spans from joint to joint where they are connected to other members. Another idealization is that loads to the truss can only be

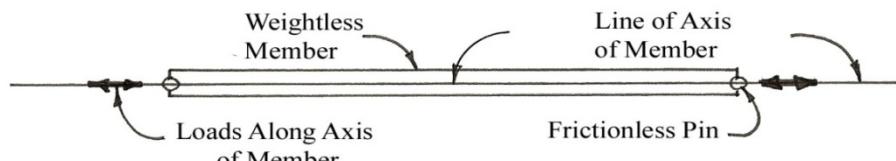
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applied at the joints (which require the members to be weightless!). Since the members of the truss are weightless, the truss itself must be weightless – another idealization.

No truss meets these idealized conditions completely, but all trusses are analyzed as if they did.

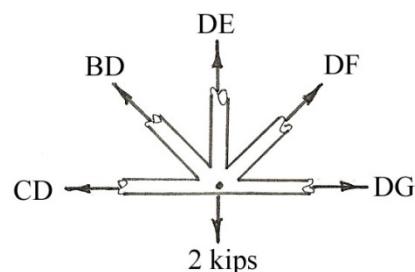
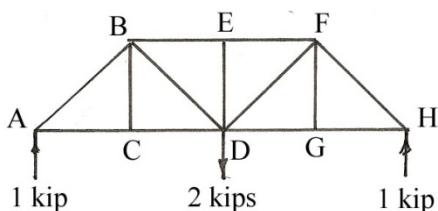
Making these assumptions about trusses – idealizing – allows the analysis of a truss to be straightforward. Using an idealized truss makes the answers slightly less accurate but the trade-off is that the analysis can be made using only the basic principles of statics. The error in accuracy is very small compared to the relatively large loads trusses usually carry. Also, the negligible error is acceptable because we only calculate answers to three significant figures.

Pinned, frictionless joints with weightless members limits the forces in each truss member to only tension or compression – either pulling on the member, or pushing on the member. These tensile or compressive forces must be aligned with the axis of the member. Since there can only be two external forces on the member, the members are known as **two-force members**. Each member in an ideal truss is a two-force member and is in either tension or compression – or has a zero load – and all of the forces at a joint are concurrent and coplanar (two-dimensional).



Two-Force Member

When sketching a truss, the members are drawn as a single line. The standard way of labeling a truss is to label each joint. The simplest way is to label them with A, B, C, etc. Then each member of the truss can be defined by the joint at each end. For example, the member between joints D and E is called DE. The force in the member is labeled DE or F_{DE} .



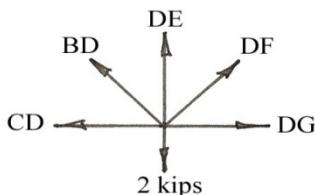


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Truss

Diagram of Joint D

Every joint in a truss must be in equilibrium. Therefore, the equilibrium equations can be used to compute the unknown forces. Since the forces are concurrent, there is no moment and only two equilibrium equations can be used: $\sum F_x = 0$, and $\sum F_y = 0$. With two equations, there can only be two unknowns. Therefore, if there are three or more unknowns at a joint, it cannot be solved.

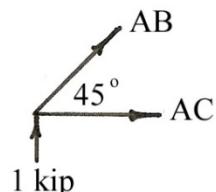
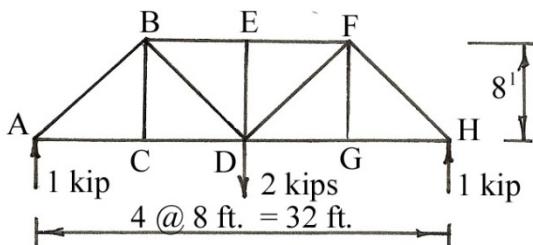


Free-Body Diagram of Joint D

The sign convention for truss analysis is not universal – different areas of the country and different engineering firms use different sign conventions. One method is to use plus for tension and minus for compression; another method is to use plus for compression and minus for tension; and yet another method to indicate the sense of the force is to use symbols, most commonly T for tension, and C for compression. Use one or the other – sign or symbol – **but not both**. In this course we will use symbols: T for tension and C for compression.

<u>Force</u>	<u>Sign</u>	<u>Sign</u>	<u>Symbol</u>	<u>Examples</u>
Tension	+	-	T	+8 kips or -8 kips or 8 kips T
Compression	-	+	C	-8 kips or +8 kips or 8 kips C

Example: Determine the forces acting on members AB and AC in the truss below.





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Truss

Free-Body Diagram of Joint A

In this problem, there are two unknowns acting on joint A – AB and AC. Therefore, the two equilibrium equations, $\sum F_x = 0$, and $\sum F_y = 0$, are enough to solve the problem. In the free-body diagram, notice that both forces are shown acting in tension. If, during the solution, one (or both) of them turns out to be negative, we will know that we chose the wrong direction. Also notice that we have a choice of which equilibrium equation to use first. If we choose $\sum F_x = 0$ first, we will have two unknowns in the equation. We would then need to solve forces vertically equal zero to create another equation – and then solve two simultaneous equations. If, however, we choose to sum forces vertically equal zero first, the equilibrium equation will only have one unknown, AB, which we can solve for directly.

In the example above, when summing the forces vertically equals zero, the force AB turned out to be negative. That simply means that we chose the wrong direction for AB in the free-body diagram of joint A. AB is actually a compressive force – not a tensile force. When the second equilibrium equation was used (sum forces horizontally equals zero), we placed the horizontal component of AB in the proper column – i.e., acting to the left, and used a positive number indicating that the direction was correct. NOTE: The positive number did not mean tension or compression. It only meant that we were using the correct direction for the force.

Calculating Forces In Truss Members There are two common ways to solve for the forces in truss members – the Method of Joints and the Method of Sections.

The **Method of Joints** for solving for the magnitude of the tensile or compressive forces in each

$$\begin{array}{c} \sum F_y = 0 \\ \uparrow \quad \downarrow \\ \hline \begin{array}{|c|c|} \hline AB \sin 45 & 0 \\ \hline \end{array} \\ \hline \begin{array}{l} AB (.707) + 1 = 0 \\ (.707) AB = -1 \\ AB = -1.41 \text{ kips} \\ \hline AB = 1.41 \text{ kips C} \end{array} \end{array}$$

$$\begin{array}{c} \sum F_x = 0 \\ \leftarrow \quad \rightarrow \\ \hline \begin{array}{|c|c|} \hline AB \cos 45 & AC \\ \hline \end{array} \\ \hline \begin{array}{l} AB (.707) = AC \\ (1.41) (.707) = AC \\ 1 = AC \\ \hline AC = 1 \text{ kip T} \end{array} \end{array}$$

member of a truss involves sequentially analyzing the forces at each joint of the truss. The rules for the procedure are as follows:

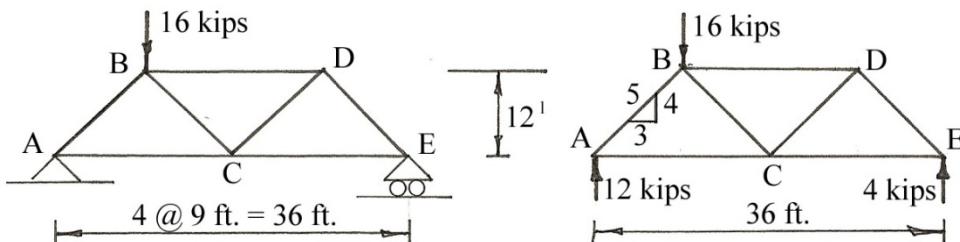
1. Compute the reactions of the truss.



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2. Select a joint (normally at the end of the truss) as the starting point. The joint must have no more than two unknowns because the forces are concurrent – there are only two equilibrium equations. There is no moment equilibrium equation for concurrent forces.
3. Draw a free-body diagram showing the forces at the joint. Show both known and unknown forces. The known forces are shown as acting in the appropriate direction. The unknown forces are shown as being in tension.
4. Compute the unknown forces at the joint.
5. Indicate the newly computed values on the truss diagram. Use C or T to indicate whether the force is compression or tension.
6. Select an adjacent joint for analysis. The joint must have no more than two unknowns.
7. Repeat Steps 3 through 6 until all joints have been analyzed.

Be sure to complete the analysis for all of the joints. The final two joints will not have two unknowns, but sum the forces horizontally and vertically anyway. These summations will serve as an arithmetic check. If the numbers do not check, you must go back to the beginning and work through all your calculations again to find the errors, and then correct subsequent values!



Example: Use the **Method of Joints** to compute the force in each member of the truss.

Truss

Free-Body Diagram

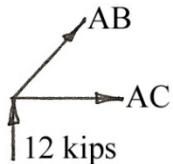
The problem is shown on the left above and the free-body diagram of the problem is on the right. The reactions have been computed and are shown on the free-body diagram. Notice that the sloped members of the truss are defined by a 3-4-5 triangle.



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To begin a solution using the Method of Joints, first select a joint to begin. The only two joints in the truss with only two unknowns are Joint A and Joint E. All other joints have more than two unknowns. Use Joint A, the left support, as the starting point – the first joint to be analyzed.

Since all the triangles of the truss are 3-4-5 triangles, the vertical components of the forces are $4/5 = 0.8$ times the force, and the horizontal components are $3/5 = 0.6$ times the force.



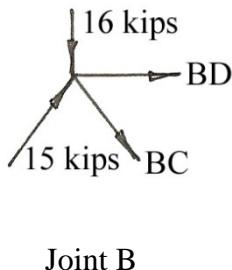
Joint A

From Joint A we must go to Joint B because there are three unknowns at Joint C.

$$\begin{array}{c} \sum F_y = 0 \\ \hline \begin{array}{c|c} \uparrow & \downarrow \\ AB (.8) & 0 \\ \hline 12 & \end{array} \\ \hline (.8) AB + 12 = 0 \\ (.8) AB = -12 \\ AB = -15 \text{ kips} \\ \hline \underline{AB = 15 \text{ kips}} \end{array}$$

$$\begin{array}{c} \sum F_x = 0 \\ \hline \begin{array}{c|c} \leftarrow & \rightarrow \\ AB(.6) & AC \\ \hline \end{array} \\ \hline (.6) AB = AC \\ (.6) (15) = AC \\ 9 = AC \\ \hline \underline{\frac{AC = 9 \text{ kips}}{\sum F_x = 0 \text{ kips T}}} \\ \hline \begin{array}{c|c} \leftarrow & \rightarrow \\ (.6) BC & BD \\ \hline (.6) (15) & \end{array} \\ \hline (.6) BC = BD + 9 \\ (.6) (5) - 9 = BD \\ BD = -6 \text{ kips} \end{array}$$

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$$\sum F_y = 0$$

↑	↓
(.8)(15)	16

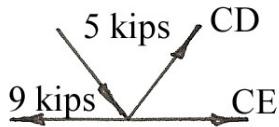
12 =	(.8) BC + 16

$$(.8) BC = -4$$

$$BC = -5 \text{ kips}$$

$BC = 5 \text{ kips C}$

From here, we could go to Joint C or Joint D. It is better to follow a regular pattern, so go to Joint C.



$$\sum F_y = 0$$

↑	↓
(.8) CD	(.8)(5)

(.8) CD =	4

$$CD = 5$$

$$CD = 5 \text{ kips}$$

$CD = 5 \text{ kips T}$

$$\sum F_x = 0$$

←	→
9	(.6)(5)

(.6)(5)	CE

9 =	CE + 6

$$CE = 3$$

$CE = 3 \text{ kips T}$

Joint C

$$\sum F_y = 0$$

↑	↓
0	(.8)(5)

(.8) DE	

0 =	(.8) DE + 4

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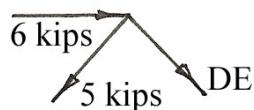
From here we go to Joint D.

Joint D. The sum of forces
 in DE directly. The sum of forces
 on our work.

$$(.8) DE = -4$$

$$DE = -5 \text{ kips}$$

$$\underline{DE = 5 \text{ kips}}$$



Joint D

Only Joint E is left. There are no unknowns at this joint therefore the calculations are a check on the work.

There is only one unknown at vertical equals zero gets the force horizontal equals zero is a check

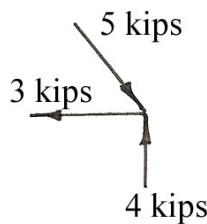
$$\sum F_x = 0$$

$$\begin{array}{c|c} \leftarrow & \rightarrow \\ \hline (.6)(5) & 6 \\ (.6)DE & \\ \hline (.6)DE + 3 & = 6 \end{array}$$

$$(.6)(5) + 3 = 6$$

$$3 + 3 = 6$$

Check ✓



$$\begin{array}{c} \sum F_y = 0 \\ \uparrow \downarrow \\ \hline 4 & | (.8)(5) \\ \hline 4 = 4 \\ \text{Check } \checkmark \end{array}$$

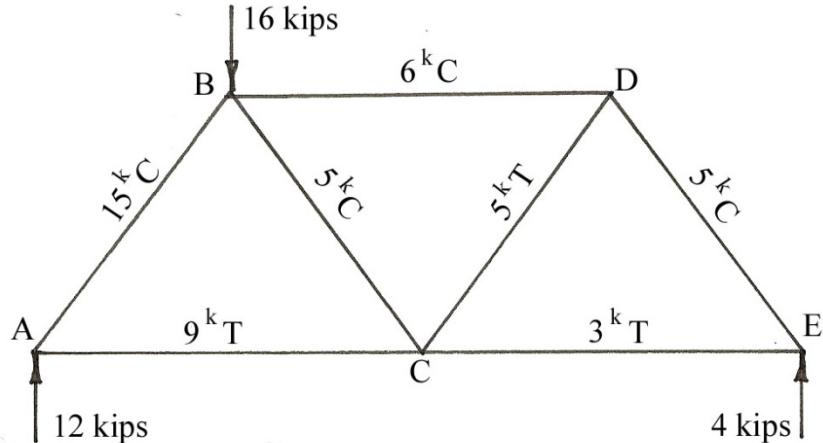
$$\begin{array}{c} \sum F_x = 0 \\ \leftarrow \rightarrow \\ \hline 3 & | (.6)(5) \\ \hline 3 = 3 \\ \text{Check } \checkmark \end{array}$$

Joint E



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The solution is shown below. Normally the load diagram and the solution diagram are combined into only one diagram. The solution diagram is shown below only for clarity.



Solution Diagram

The **Method of Sections** for solving for the magnitude of the tensile or compressive forces in each truss member is a method used for individual members in any order. A serious flaw in the **Method of Joints** is that all the calculations must be made without error. If an error is made somewhere along the way, you must go back to the beginning and start over to find it. Another major detraction from the Method of Joints is that if you want to find the force in a particular member, you must start at the beginning and it may require a great many calculations to get to the member of interest.

The Method of Sections does not have these weaknesses. The force in each member is computed independently.

In the Method of Sections, a section is cut through the truss to create a free-body. The free-body diagram can include any portion of the truss. In general, the forces in such a free-body will not be concurrent, but they will be coplanar. This means that there are three equilibrium equations: $\Sigma F_v = 0$, $\Sigma F_h = 0$ and $\Sigma M = 0$. With three equations, there can be three unknowns, and the free-body must be selected accordingly.



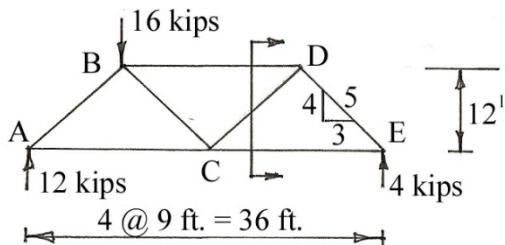
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Note that it is never necessary to solve the equations simultaneously. For every member, an equation can be written that is independent of all other variables. When moments are computed, they are taken about a point where the other two members intersect.

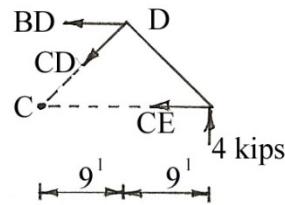
If the top and bottom chords (flanges) are parallel, as in the following example, the forces in the top and bottom members are determined by taking moments at a point on the opposite flange. The forces in the web members are determined by summing forces vertically.

If the top and bottom chords are not parallel, each member is determined by taking moments at the point of intersection of the other two members.

Example: Use the Method of Sections to compute the force in truss members BD, CD, and CE.



Load Diagram



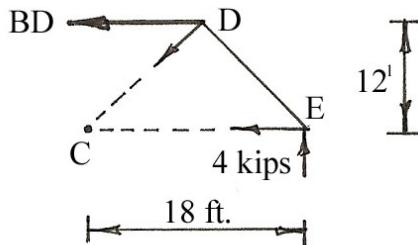
Free-Body Diagram

Begin by cutting the truss as shown on the load diagram. The three members of interest are cut. The portion of the truss to the right of the cut becomes the free-body diagram with only three unknowns. All three unknown forces can be determined from this free-body.

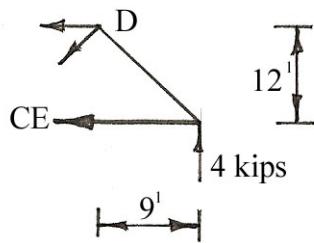
First, solve for the force in member BD. The other two members, CD and CE, intersect at Joint C. If moments are taken at Joint C, both CD and CE will have a moment arm of zero and will not appear in the equation.

$$\begin{array}{c} \sum M @ C = 0 \\ \curvearrowright \quad \curvearrowright \\ \hline \begin{array}{c|c} BD (12) & 0 \\ (4) (18) & \\ \hline 12 BD + 72 & 0 \end{array} \\ 12 BD + 72 = 0 \\ BD = -6 \text{ kips} \\ \underline{BD = 6 \text{ kips}} \end{array}$$

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Next solve for the force in member CE. The other two members, BD and CD, intersect at Joint D. If moments are taken at Joint D, both BD and CD will have a moment arm of zero and will not appear in the equation.

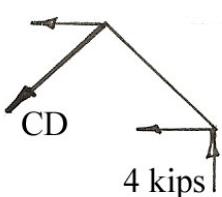


$$\begin{array}{c}
 \sum M @ D = 0 \\
 \curvearrowleft \qquad \curvearrowright \\
 \hline
 (4)(9) \qquad CE(12) \\
 \hline
 36 = 12 CE \\
 CE = 3 \text{ kips} \\
 \underline{CE = 3 \text{ kips T}}
 \end{array}$$



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Finally, solve for the force in member CD. The other two members, BD and CE, are parallel and do not intersect. By summing forces perpendicular to BD and CE, they are eliminated from the equation. Sum forces vertically equals zero to find CD.



$$\begin{array}{c} \sum F_y = 0 \\ \uparrow \quad \downarrow \\ 4 \quad | \quad CD (.8) \\ \hline 4 = (.8) CD \\ CD = 5 \text{ kips} \\ \underline{CD = 5 \text{ kips T}} \end{array}$$

NOTE: There is no check on your arithmetic with the Method of Sections. You can check your work by summing forces at one or more joints, or you can visually check to see that the values are reasonable.

The **Qualitative Method** for “solving” trusses does not yield a magnitude of force in a member. It is a method of determining – by inspection – whether there is a tensile or compressive force in a member (or zero force). The method involves the following steps:

1. Mentally create a free-body section that cuts through the member to be analyzed (or draw a sketch if you cannot visualize the problem).
2. If the member is a top or bottom chord member, mentally take moments about a point on the opposite chord. It should be obvious whether the unbalanced moment is clockwise or counterclockwise. Hence it should be obvious whether a tensile or compressive force is needed to assure equilibrium for the section.
3. If the member is a web member, sum forces vertically. It should be obvious whether the net sum of the external forces on the free-body is upward or downward, and whether a tensile or compressive force is needed to assure equilibrium for the section.



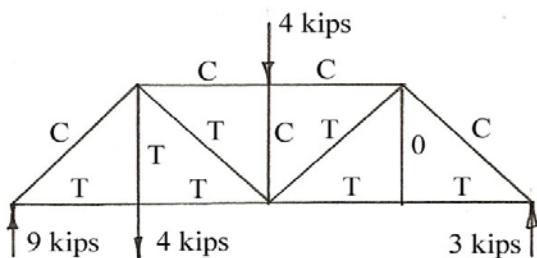
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4. If the member is a web member that intersects the top or bottom chord at a point where there is no other web member, then it serves as a hanger or column for the load applied at that point. If there is no load at that point, then there is no force in the member.

In addition to the above rules, it is useful to remember the following generalities:

5. The top chord members of an ordinary truss are in compression.
6. The bottom chord members of an ordinary truss are in tension.
7. The web members of an ordinary truss alternate between tension and compression.

See if you can verify the sense of the force in each member of the truss below.



Qualitative Truss Analysis
Part Three – CABLES

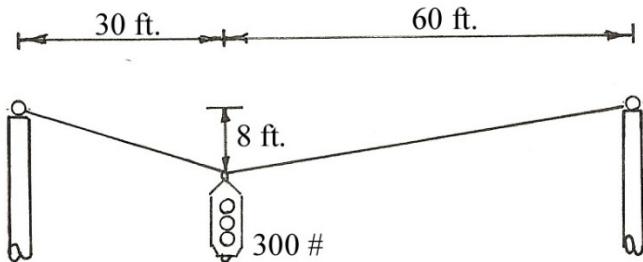
Introduction A **Cable** is used for suspending objects that are intended to stay in place. For example, a suspension bridge is built in one place and is suspended from cables. A hanging traffic signal is suspended from a cable across an intersection. The force in a cable is always tension – there cannot be any compression or bending. You can't push on a rope.

Solution of cable problems is usually straightforward when the line of action is known. The line of action of the tension in a cable is the centerline of the cable. To solve a cable problem, simply draw the free-body diagram and sum forces.

Suspension Cables With A Single Load One common cable problem that we see all the time is a cable that supports a single traffic signal over an intersection. If the cable supports a single concentrated load, the tension in the cable can easily be determined from the geometry of the problem if the vertical offset of the load is known.

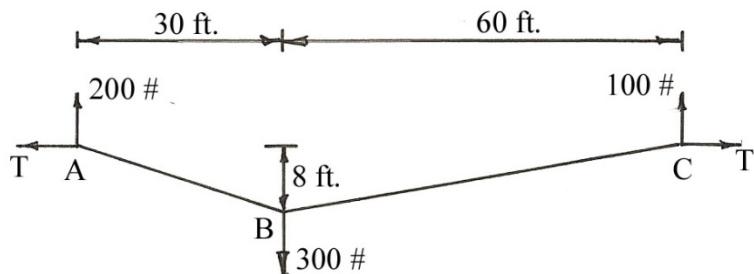
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Example: What is the tension force in each segment of the cable supporting a single concentrated load – in this case a single traffic light?



Load Diagram

The first step in the solution is to construct the free-body diagram. Show all dimensions and loads. Remove the supports at each end of the cable and replace with the vertical and horizontal components of the reactions. Identify the three critical points on the diagram as A, B, and C. Compute the magnitude of the vertical components of the reactions as 200 # and 100 # by summing the moments at C equals zero and summing the moments at A equals zero (the same way reactions are computed for beams and trusses). Because the entire structural system is in equilibrium, the horizontal component of the tensile force at both supports must be equal and opposite – in this case labeled T.



Free-Body Diagram

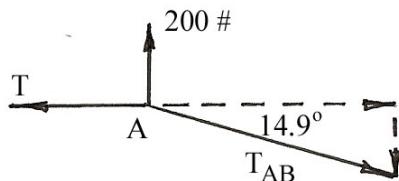
From this point on, there are two ways to solve the problem. The two procedures are analogous to the Method of Joints and the Method of Sections used for truss analysis. Both methods will be demonstrated by using the “Method of Joints” to solve for the AB section of the cable and the “Method of Sections” to solve for the BC section of cable.



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The “Method of Joints” procedure involves the summation of forces at each critical point along the cable by using the geometry of the point. Select joint A as the point and draw the free-body diagram of the joint. Notice that it is simply a concurrent, coplanar problem with three forces and two unknowns. Begin by determining the angle the cable makes with the horizontal as follows: $\tan \Theta = (8 \text{ ft.})/(30 \text{ ft.})$; $\Theta = \tan^{-1} (8/30)$; $\underline{\Theta = 14.9^\circ}$. Then sum forces vertically equals zero to find the tension in the cable (T_{AB}), and sum forces horizontally equals zero to find T.

$$\begin{array}{c} \sum F_y = 0 \\ \uparrow \quad \downarrow \\ \hline 200 & | T_{AB} (\sin 14.9^\circ) \\ \hline 200 = T_{AB} (0.2577) \\ \underline{T_{AB} = 776 \#} \end{array} \qquad \begin{array}{c} \sum F_x = 0 \\ \leftarrow \quad \rightarrow \\ \hline T & | T_{AB} (\cos 14.9^\circ) \\ \hline T = (776) (0.96623) \\ \underline{T = 750 \#} \end{array}$$



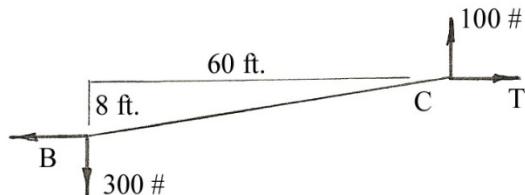
Point A

The “Method of Sections” procedure involves the summation of moments at each critical point along the cable. Use a free-body diagram that cuts the cable at a point, and then take moments at that point. In this case cut the cable at Point B; then sum moments equals zero at B to find T. Now the vertical and horizontal components of the tension in the cable BC are known and the cable tension (T_{BC}) can be computed. Use the Pythagorean Theorem to find T_{BC} .

$$\begin{array}{c} \sum M @ B = 0 \\ \curvearrowright \quad \curvearrowright \\ \hline 100 (60) & | T (8) \\ \hline 6,000 = 8 T \\ \underline{T = 750 \#} \end{array}$$



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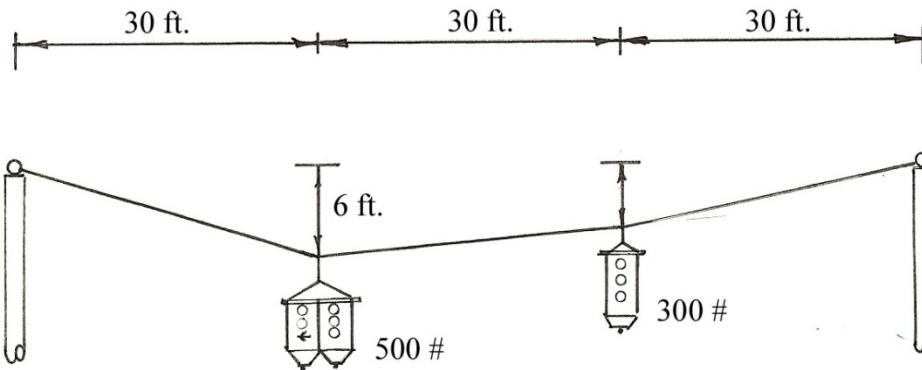
$$\begin{aligned}T_{BC} &= \sqrt{100^2 + 750^2} \\T_{BC} &= 756.64 \\T_{BC} &= 757 \#\end{aligned}$$

Point A

Notice that the tension is different in each section of the cable. Had the load been centered on the cable, both cable tensions would have been the same. Also notice that the horizontal component of both reactions equals 750 #. This makes sense because the complete static system is in equilibrium – which means that the sum of the forces horizontally equals zero.

Suspension Cables With Multiple Loads Another common cable problem is a cable that supports two traffic signals over an intersection – perhaps a double light on the left with a turn only arrow plus the regular three-light signal on the right. For cables that support multiple loads, the vertical offset at one (but not more than one) point along the cable must be known or assumed.

Example: What is the tensile force in each segment of the cable supporting two concentrated loads – in this case a pair of traffic signals? The vertical offset at the 500 # load is 6 feet.

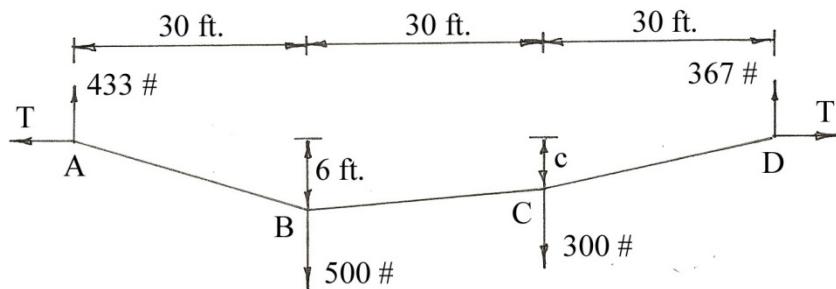


Load Diagram



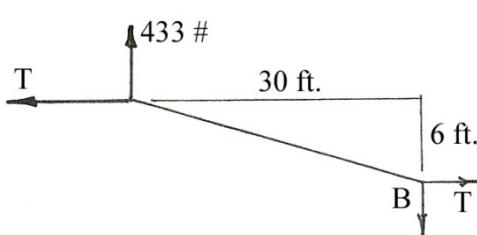
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Begin this problem, as we have all the others, with a free-body diagram. Show all dimensions and loads; replace the supports with the vertical and horizontal components; identify the critical points A through D; compute the magnitude of the vertical components of the reactions as 433 # and 367 #; label the unknown tensile force, T; and the unknown vertical distance at C as “c”.



Free-Body Diagram

Before we can compute the tensile forces in each section of the cable, we must first determine the two unknowns in the free-body diagram, T and c. To find T, sum the moments at the connection the offset is known. Sum moments at B equals zero to find T. Note that this horizontal force T is the same horizontal component for all points along the cable, i.e., the horizontal force component is the same from one end of the cable to the other.



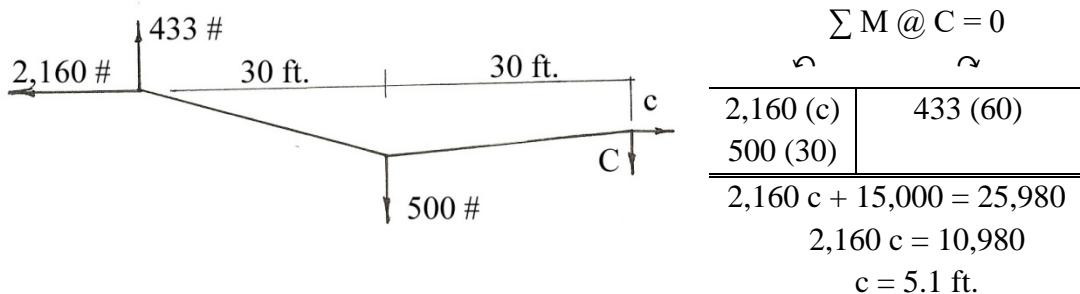
$$\begin{array}{c} \sum M @ B = 0 \\ \curvearrowleft \quad \curvearrowright \\ \hline \text{Table:} & \\ \hline T(6) & 433(30) \\ \hline \end{array}$$
$$6T = 12,990$$
$$T = 2,165 \#$$
$$\underline{T = 2,160 \#}$$

Free-Body Diagram



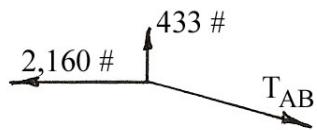
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Now find the vertical offset c by summing moments at that point – sum moments at C equals zero to find c .

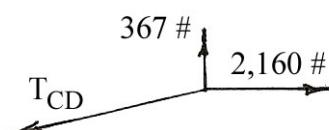


Free-Body Diagram

Now the force in each cable segment can be determined by the geometry of the cable. Use the “Method of Joints” at each end to find T_{AB} and T_{CD} . Use the Pythagorean Theorem.



$$T_{AB} = \sqrt{2160^2 + 433^2}$$
$$T_{AB} = 2,200 \text{ #}$$

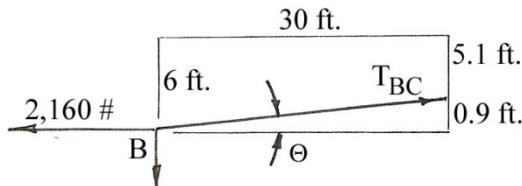


$$T_{CD} = \sqrt{367^2 + 2160^2}$$
$$T_{CD} = 2,190 \text{ #}$$



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To find T_{BC} , determine the angle the cable between B and C makes with the horizontal and use the cosine function, i.e., divide the horizontal force by the cosine of the angle that the cable makes with the horizontal. B is offset 6 ft.; C is offset 5.1 ft.; therefore line BE is offset 0.9 ft.



$$\sin \Theta = 0.9 / 30$$

$$\Theta = 1.72^\circ$$

$$\cos \Theta = 2160 / T_{BC}$$

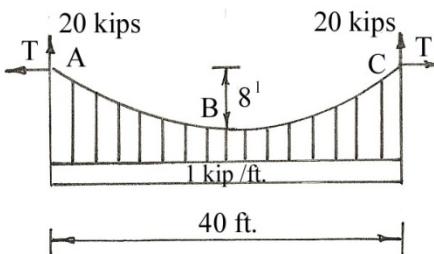
$$T_{BC} = 2160 / \cos 1.72^\circ$$

$$T_{BC} = 2160 \text{ #}$$

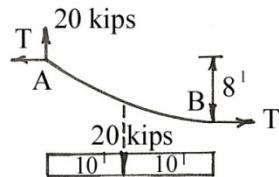
Suspension Cables with a Uniformly Distributed Load There are two types of cable problems that are analyzed as carrying a uniformly distributed load. Neither type exactly fits that load description. The first, the suspension bridge, is actually a series of concentrated loads, each hanging from a vertical cable. The suspension cable is a series of straight lines between the hanging loads. The second type, the high tension wire such as a power line, carries its own weight which is considered a uniformly distributed load. It's not precisely a uniformly distributed load because the wire has sag. It would be a uniform load if the wire had no sag.

Both types of problems are solved in the same manner. To solve the problem, the vertical loading must be known and either the vertical offset at mid-span or the horizontal force in the cable must be known.

Example: Compute the force in the cable at mid-span if the mid-span offset is 8 feet. Also compute the maximum force in the cable.



Load Diagram



Free-Body Diagram

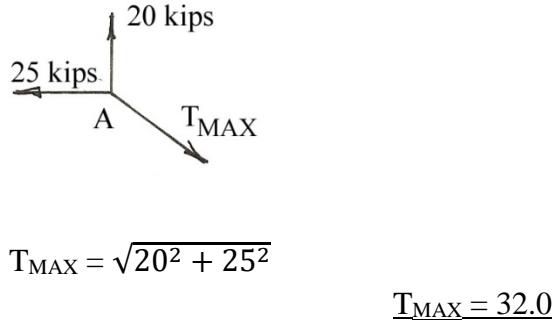
The first step is to compute the vertical component of the reactions as 20 kips each. Then draw a free-body diagram that cuts the cable at midpoint. Solve for the horizontal component of the



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reaction by summing moments at the mid-point. Sum moments at B equals zero to find T. The tension in the cable at mid-span is equal to the horizontal component of the reaction, T. The maximum tension in the cable (at either support) can then be found by drawing the free-body of the left reaction and using the Pythagorean Theorem to find T_{MAX} .

$$\begin{array}{c} \sum M @ B = 0 \\ \hline 20(10) & 20(20) \\ T(8) & \\ \hline 8T + 200 = 400 \\ 8T = 200 \\ \underline{T = 25 \text{ kips}} \end{array}$$



kips

As the vertical offset at the center of the span gets smaller, the horizontal component of the reaction gets larger, and the tensile force in the cable gets larger. For example, if the vertical offset in the problem above were cut in half to 4 feet, the tensile force at mid-span would be 50 kips, twice that for an 8 foot offset. If the off-set were cut in half again to 2 feet, the tensile force at mid-span would be twice again, or 100 kips. Halving the offset again, to 1 foot, would double the force again too; to 200 kips. At some point the cable could no longer resist the tensile force and would break.

When supporting its own weight, the more sag in a power line, the less the tensile force in the cable. Power company workers know to string power lines with a minimum amount of sag. After a big storm where lines are down, notice that the worker making the final hook-up to a home will sight down the line to make sure the proper amount of sag is in the line. This is how they make sure to keep the tensile force in the cable below a safe magnitude. We know this by applying the fundamentals of statics to a real life problem.

Part Four – FRICTION

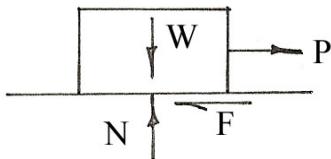


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Introduction Friction is the resistance to movement between two bodies that are in contact with each other. **Static friction** refers to problems in which there is not movement between the two bodies.

Static Friction Static friction is when there are frictional forces present on a body but the body remains at rest. Consider the block shown below. It is not moving and therefore is in static equilibrium. The weight of the block, W , is forcing the block against the supporting surface, and the resisting normal force, N , must be equal to W . A horizontal force, P , is applied to try to move the block, and a friction force, F , resists the force P . By summing forces horizontally, it is apparent the F is equal to P while the block is at rest.

If P has a value of zero, then there is no friction force. If the force P is gradually increased, it will reach a maximum value that can be resisted by friction. The status at which motion is about to begin is referred to as **motion impending**. Most static friction problems are based on the motion impending status. If P is increased further, the block will slide and the problem becomes a dynamics problem rather than a statics problem. In all static friction problems, the friction



force, F , is parallel to the surface, and the normal force, N , is perpendicular to the surface.

Static Friction Forces

Coefficient of Static Friction The coefficient of static friction is the ratio of the friction force, F , to the normal force, N , when motion is impending.

$$\mu = F/N$$

The coefficient of static friction is a function of a number of factors, including the type of materials, the smoothness of the surfaces, and the lubrication of the surfaces. These values are determined experimentally. Also notice that the coefficient of static friction is a function of N , the force, and not the area over which the weight is spread. A 100 # box that has a footprint of 1 sq. ft. will develop the same frictional force as a 100 # box that has a footprint of 20 sq. ft.



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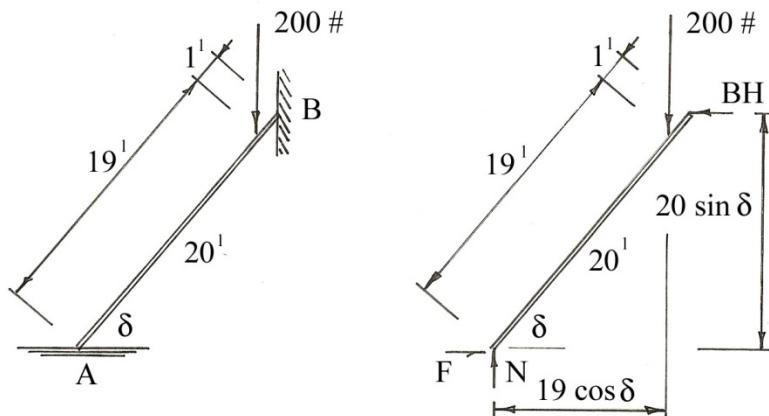
The coefficients of friction for several materials are listed below. A range of values is shown because of the range of differences in the smoothness of the surfaces. Coefficients of friction are always dimensionless, and thus they have the same value in the American and SI systems.

Coefficients of Static Friction for Dry Surfaces

Steel on Steel	0.6 – 0.8
Wood on Wood	0.3 – 0.6
Wood on Concrete	0.7 – 1.2
Steel on Concrete	0.4 – 0.6
Rubber on Concrete	0.7 – 0.9

Ladder safety is a big concern. OSHA regulates the maximum lean an extension ladder can have on a construction site. Statics plays a big part in determining a reasonable safe lean for a ladder. By making some reasonable simplifying assumptions, the safe lean angle of an extension ladder can be calculated. This calculated angle is then adjusted to set a reasonable minimum lean angle for use in regulations.

Example: Calculate the minimum lean angle, δ , for the ladder if the coefficient of static friction is 0.80.



Load Diagram

Free-Body Diagram

To solve this problem of determining the minimum lean angle, δ , first make the assumption that the top of the ladder contact at the wall is frictionless. This is a reasonable assumption with motion pending since the frictional force at the bottom of the ladder, the floor, will be the



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dominant force. Plus, there can only be three unknowns to solve the ladder problem using the principals of statics. Once this assumption is made, draw the free-body diagram. Use F and N as the friction force and the normal force at the bottom of the ladder. Since δ is not yet known, indicate the horizontal and vertical distances as a function of δ .

From this point, it is simply applying the rules of statics and trig to arrive at the solution – the minimum lean angle δ – as follows:

Sum forces vertical equals zero to find N. $N = 200 \# \uparrow$. From this, compute the maximum friction force using $\mu = F / N$. $F_{MAX} = \mu N = (0.80)(200)$; $F_{MAX} = 160 \# \rightarrow$.

Since the maximum friction force is the maximum horizontal force allowed on the system without the ladder slipping on the floor and falling, the maximum horizontal force at the top of the ladder must be the same. $BH = 160 \# \leftarrow$. Now the only unknown in the problem is the lean angle δ . Sum moments at A equals zero to find δ .

$$\begin{array}{c} \sum M @ A = 0 \\ \curvearrowleft \quad \curvearrowright \\ \hline BH(20 \sin \delta) \quad 200(19 \cos \delta) \\ \hline 20BH \sin \delta = (200)(19) \cos \delta \\ (20)(160) \sin \delta = (200)(19) \cos \delta \\ 3,200 \sin \delta = 3,800 \cos \delta \end{array}$$

And then solve for δ :

$$\frac{\sin \delta}{\cos \delta} = \tan \delta = \frac{3,800}{3,200} = 1.1875$$
$$\delta = \tan^{-1}(1.1875)$$
$$\delta = 49.9^\circ$$

Knowing the calculated minimum lean angle, the safety experts can apply practical knowledge to the issue and come up with a reasonable angle to allow for safe ladder use.

Part Five – SHEAVES and PULLEYS

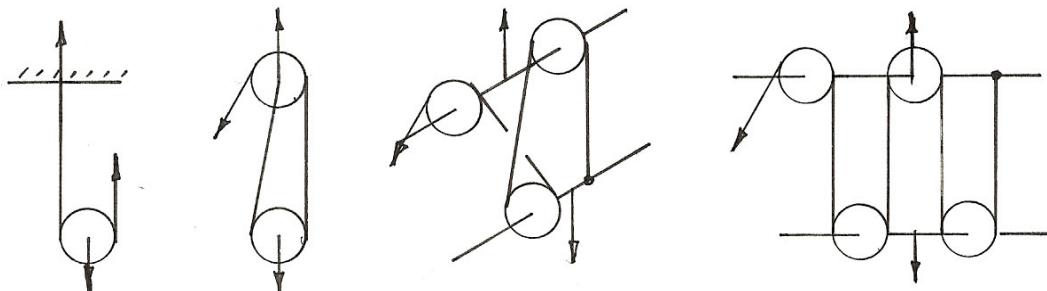
Terminology **Sheave** (pronounced shiv) and **pulley** are two names for the same thing. What a layman calls a pulley, the engineer calls a sheave. Generally speaking, sheaves lift larger loads than pulleys. A group of sheaves (or pulleys) that are ganged together is called a **block**. **Rope** is the term that is used for the hoisting. Rope can be fiber or wire, and is made by twisting together



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the fibers or wires. The **line**, as in hoisting line, refers to the entire hoisting system including the upper block, the lower block, and the rope. The line a home builder uses to hoist a wooden beam (a relatively light load) is called a pulley and uses rope made of fibers. The line a bridge builder has on the crane used to lift a pre-cast concrete beam in place is called a sheave and uses a rope made of wire. **Reeving** refers to the pattern of rope as it is tread from the winding drum of the crane to the hook used to lift the beam. **Parts of line** refer to the number of ropes running between the blocks, i.e. a four-part line. As in cables, all forces in a rope are tensile. There is no compression or bending. As the saying goes, “You can’t push on a rope.” Again, generally speaking, fiber rope is used in pulleys and wire rope is used in sheaves.

Multiple Part Lines The parts of line reeved between the upper and lower blocks provide a mechanical advantage. A sketch of the reeving pattern is helpful when you are first learning to



compute the forces in the rope. Shown below are a simple pulley and three different reeving patterns, and three different methods by which the reeving pattern can be drawn. Choose the method that is the easiest or clearest to you. After you learn to make the calculations, you will not need a sketch except when you are trying to describe the reeving pattern to others.

Simple Pulley

Two-part Line

Three-part Line

Four-part Line

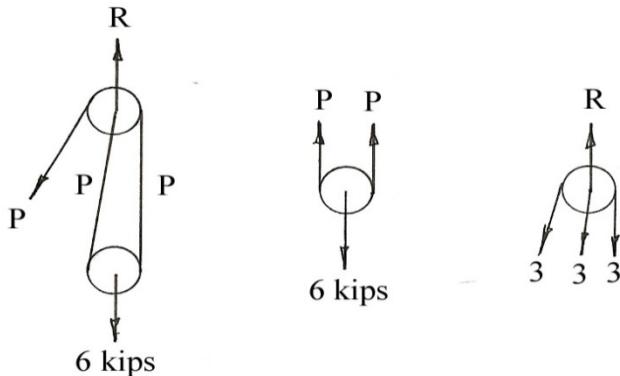
Reeving Sketches

Frictionless Sheaves When the load is static, neither being raised nor lowered, there is no friction. Hence, the force in the rope on each side of a sheave is equal, and the force in the rope is the same from one end to the other. To determine that force, draw a free-body that cuts the line between the two blocks. The tension in the line must be divided equally between the ropes running between the two blocks.



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Example: Ignore friction in the line. What is the maximum force in a 2-part line that supports a 6,000 # load?



Two-Part line

Lower Block

$$P = 3 \text{ kips}$$

Upper Block

$$R = 9 \text{ kips}$$

Sheaves with Friction – General Sheave friction is the resistance of a rope passing over a sheave or pulley. It includes both the friction of the bearings and the losses of the rope itself (the rubbing of the individual strands as the rope is bent). Coefficients of sheave friction (μ) are determined experimentally. The range of coefficients of sheave friction varies from $\mu = 0.01$ for ball bearing sheaves to a high of about $\mu = 0.045$ for less efficient sheaves. The coefficients are based on a rope making a 180° turn, but no adjustment to the coefficient is made for lesser angles.

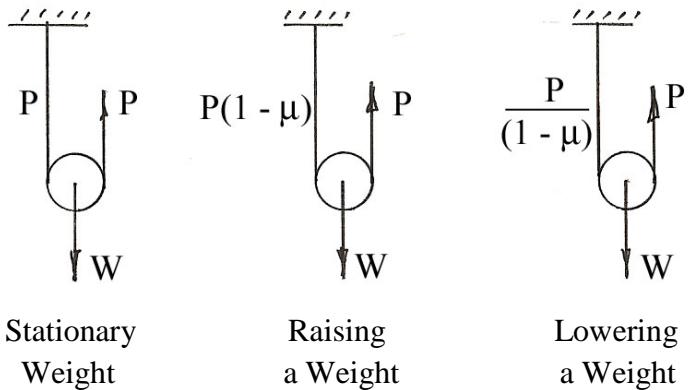
If a weight W is hanging from a simple pulley, and is not moving either up or down, the force in each rope is one-half the weight ($0.5 \times W$). If the weight W is being raised, friction must be overcome and, therefore slightly more than $0.5W$ will be required on the pulling rope. If the weight is being lowered, friction is helping and a slightly less force than $0.5W$ is required on the rope doing the pulling. The differences between the static force and the forces required to raise or lower the load is due to overcoming friction losses in the pulley and the rope. The net effect of the friction on two ropes supporting a sheave is for the force in the stationary rope to be more or less than the force in the other by a factor of $(1 - \mu)$. The force in the rope on one side of the sheave is $(1 - \mu)$ times the force on the other.



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Example: What are the forces in the pulling ropes for a stationary hanging weight (static load); raising a hanging weight; and lowering a hanging weight if the coefficient of sheave friction is 0.04?

For the stationary weight, the sum of forces vertical equals zero yields $P = 0.5 W$ as seen below.



To raise a weight and to lower a weight, the maximum force P is found by summing the forces vertical equals zero as follows:

Raising

$$\begin{array}{c} \uparrow \sum F_y = 0 \downarrow \\ \hline P & | & W \\ P(1-\mu) & | & \\ \hline P (1 + (1-\mu)) = W \end{array}$$

$$P = W / 1 + (1-\mu)$$

$$P = W / 1 + (1 - .04)$$

$$P = W / (1 + .96)$$

$$\underline{P = 0.51 W}$$

Lowering

$$\begin{array}{c} \uparrow \sum F_y = 0 \downarrow \\ \hline P & | & W \\ P / (1-\mu) & | & \\ \hline P (1 + (1 / (1-\mu))) = W \end{array}$$

$$P = W / 1 + (1 / (1-\mu))$$

$$P = W / 1 + (1 / (1 - .04))$$

$$P = W / 1 + (1 / .96)$$

$$P = W / 1 + 1.042$$

$$\underline{P = 0.49 W}$$

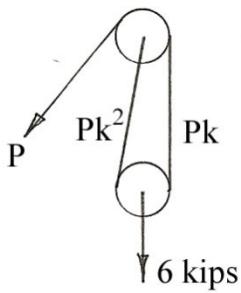
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The pulling rope needs just a bit more than half the load to raise it and the pulling rope needs to reduce to just a bit less than half the weight to allow it to lower. This makes sense.

Sheaves with Friction – Raising a Load For a multiple part line, sheave friction must be overcome at each sheave. At each sheave the effective force on the pulling rope is reduced by the coefficient of sheave friction factor, μ , i.e., the force in the rope on one side of each sheave is $(1 - \mu)$ times the force on the other. For simplicity of notation, let $k = 1 - \mu$.

Example: What is the maximum force in the rope of a 2-part line that is lifting a 6,000 pound load if the coefficient of sheave friction is 0.04?

For this problem, the pulling force P is the maximum force in the line. In each segment of the line, the force in the line is reduced by a factor of k . To draw the free-body diagram, cut the ropes between the sheaves and label properly. $\mu = 0.04$; $k = 1 - \mu$; $k = 1 - .04$; $k = 0.96$



$$\begin{aligned} P(k + k^2) &= 6 \text{ kips} \\ P &= \frac{6}{k + k^2} \\ P &= \frac{6}{.96 + .96^2} \\ P &= 3.19 \text{ kips} \end{aligned}$$

Problem Diagram

Free-Body Diagram

Solution

The maximum force in the line, i.e., the maximum force required to lift the 6,000 pound load, is 3.19 kips. This is, as expected, more than the 3 kips to keep the load static.

The general solution for the maximum force in a rope with an n -part line is:

$$P = \frac{W}{k + k^2 + \dots k^n}$$

Sheaves with Friction – Lowering a Load When the load is being lowered friction is acting for you and the maximum force in the rope is less than in the static condition. When designing for the maximum load in a rope, this is not the maximum condition and, therefore this calculation can be ignored.



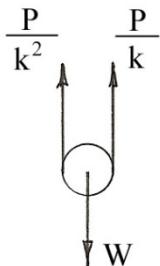
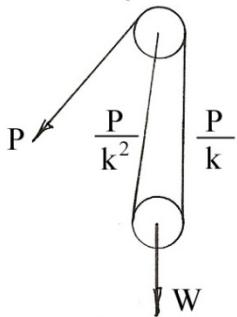
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What cannot be ignored is the lowering of the hook of a crane when there is no load. The weight of the hook and lower block of a crane hoisting system may not be enough to overcome sheave friction, rope friction, weight of the rope back to the winding drum of the crane, and the friction in the drum. In this case, an **overhauling weight** may have to be added. An overhauling weight is the minimum weight required to overcome the friction in the system and to counterbalance the weight of the rope between the drum and the upper block. In practice, the actual overhauling weight must be greater than the computed force so that the hook will be lowered quickly.

Example: What amount of additional overhauling weight needs to be hung from the lower block of a sheave hanging from a crane if the hoisting system is a two-part line with a coefficient of sheave friction of 0.04? The weight of the hook attached to the lower block is 60 pounds. The make and model of the crane is such that 100 pounds of overhauling weight is necessary just to overcome the weight of the lifting rope and the drum friction.

The solution to this problem is the answer to the real-life question of why you often see a generator or air compressor hanging from the hook of a crane overnight and on weekends.

To solve this problem we must first recognize that the total overhauling weight must be at least 100 pounds plus the force required to overcome friction in the sheave (neglect the weight of the sheave). Also recognize that the force required to overcome the sheave friction is dependent on the amount of force in the line – in this case a minimum of 100 pounds. The calculation is one of lowering a weight. $P_{min} = 100 \text{ #}$; $\mu = 0.04$; $k = 1 - \mu$; $k = 1 - .04$; $k = 0.96$



$$\begin{aligned}P_{min} \left(\frac{1}{k^2} + \frac{1}{k} \right) &= W_{min} \\W_{min} &= P_{min} \left(\frac{1}{.96^2} + \frac{1}{.96} \right) \\W_{min} &= P_{min} (2.22) \\W_{min} &= (100 \text{ #}) (2.22) \\W_{min} &= 222 \text{ #} \\ \underline{\text{Overhaul Weight}} &= 222 - 60 = 162 \text{ #}\end{aligned}$$

Problem Diagram

Free-Body Diagram

Solution



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The answer $W_{\min} = 222 \text{ #}$ is the minimum weight required to just overcome friction and very slowly move the hook down. The hook itself weighs 60 pounds, so an additional 162 pounds must be added to the load to move the hook. Overnight and on weekends, when the hook is raised out of harm's way, a generator or air compressor will be hung from it to allow the hook to be quickly lowered in the morning. A heavy steel ball is also used as an overhauling weight during the day when the hook must be lowered without a load.

Importance of Friction Obviously it is easier to determine the rope forces by assuming that the sheaves are frictionless. The table below is a comparison of frictionless vs. friction calculations for several friction factors, μ , and several parts of line. With low friction or few parts of line, there is little difference between friction and frictionless calculations. For high friction and high parts of line, friction plays a significant role.

Parts of Line	Coefficient of Sheave Friction, μ			
	0.01	0.02	0.03	0.045
2	1.02	1.03	1.05	1.07
4	1.03	1.05	1.08	1.12
6	1.04	1.07	1.11	1.17
8	1.05	1.09	1.14	1.22
10	1.06	1.12	1.18	1.28
12	1.07	1.14	1.21	1.33

Ratio Between Friction and Frictionless Calculations