Fundamentals of Foundation Design

By

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I. Introduction to Elements of Foundation Design

The various types of loads produced from buildings, bridges, or any other structure must be transmitted to the soil through foundations. Because soil bearing pressures are significantly lower than the compressive stresses of steel, concrete or masonry columns or walls foundations must be used to reduce the pressure applied directly to the soil by spreading the column or wall load over an area large enough such that the soil bearing pressure is not exceeded.

A comprehensive foundation design involves both a geotechnical study of the soil conditions to determine the most suitable type of foundation and a structural design to determine the proportions of the foundation elements. There are many factors which come into play when evaluating a specific soils capacity to support a load.

These factors include but are not limited to:

1.) Strength and compressibility of the various soil strata at the site
2.) The depth of the water table at the time of the construction of the foundation and is the water table likely to fluctuate such that the bearing pressure of the various soil strata could be reduced
3.) Does the soil experience significant expansive properties when saturated
4.) Is the top layer of the soil strata a “fill” soil and if so was it adequately compacted

The type of soil present at the site greatly influences the type of foundation design required as soil strength and compressibility are dependent on soil type.

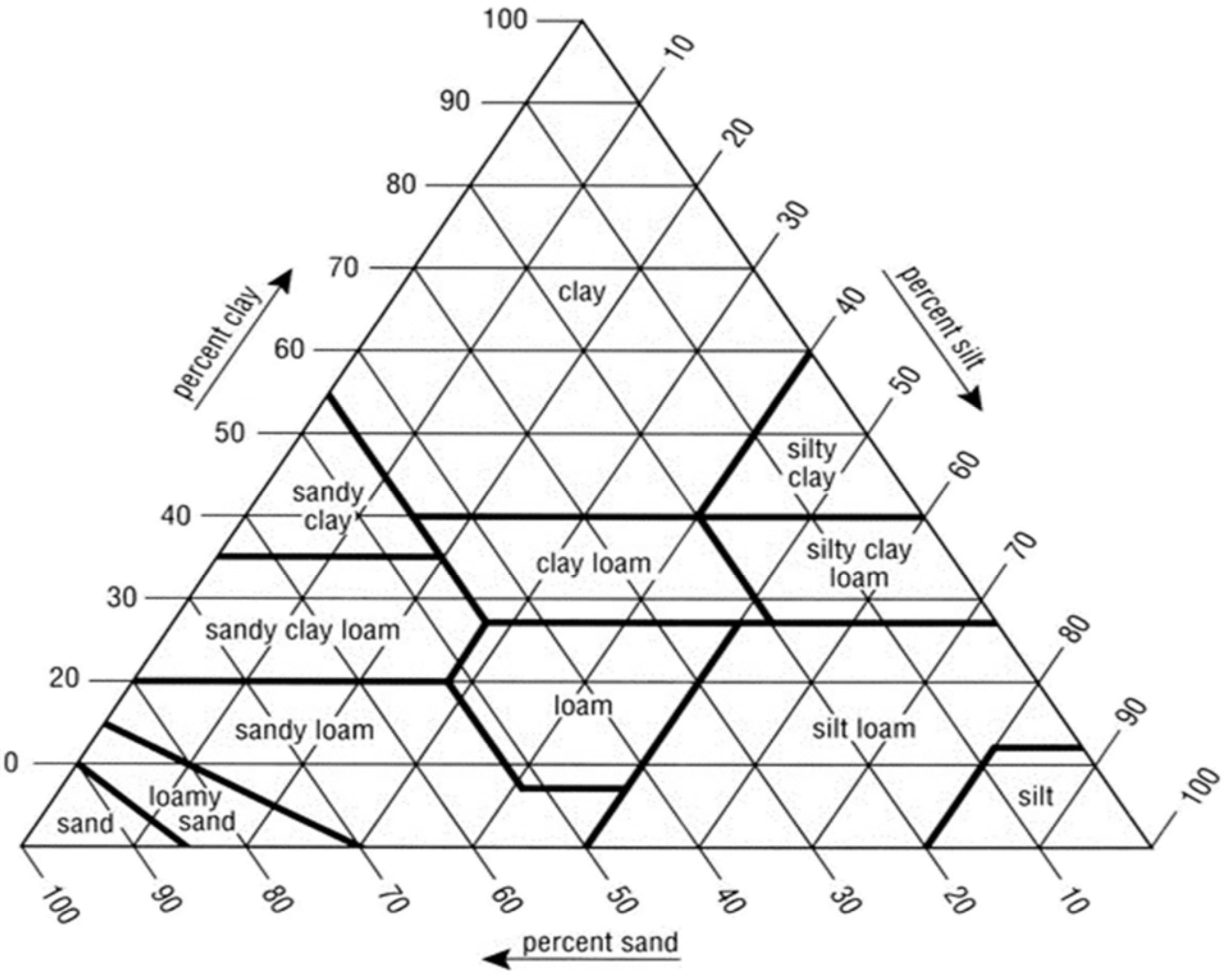
II. Soil Mechanics

Most soils are comprised of a variety of sediments or particles in addition to air, water and sometimes organic matter. Soils are typically non-homogeneous with particle sizes varying greatly within a given sample. The soil particle sizes and distribution of soil particle sizes influence soil properties and performance. The chart below shows the classification of particle sizes used by the ASTM Unified Soil Classification System.

Soil Particle Sizes			
Type	**Fraction**	**Sieve Size**	**Diameter**
Boulders	**-**	**12″ Plus**	**300 mm Plus**
Cobbles	**-**	**3″ – 12″**	**75 – 300 mm**
Gravels	**Coarse**	**0.75″ – 3″**	**19 – 75 mm**
	Fine	**No. 4 – 0.75″**	**4.76 – 19 mm**
Sand	**Coarse**	**No. 10 – No. 4**	**2 – 4.76 mm**
	Medium	**No. 40 – No. 10**	**0.42 – 2 mm**
	Fine	**No. 200 – No. 40**	**0.074 – 0.412 mm**
Fines(silts & clays)	**-**	**Passing No. 200**	**0.074 mm**

The two basic soil types that are defined by particle size are coarse-grained soils and fine-grained soils. Coarse-grained soils consist of particles that are too large to pass through a #200 sieve (0.074 mm). A #200 sieve has 200 openings per inch. Cobbles, gravels and sands are coarse-grained soils and are commonly referred to as non-cohesive soils. The particles in a non-cohesive soil typically do not stick together unless sufficient moisture is present, which is caused by the surface tension of the water molecules. Fine-grained soils consist of particles that are small enough to pass through a #200 sieve. Silt particles typically range from 0.074 to 0.002 mm while clays are typically smaller than 0.002 mm. Silts and clays are fine-grained soils and are commonly referred to as cohesive soils. Molecular attraction causes the particles of cohesive soils to stick together.

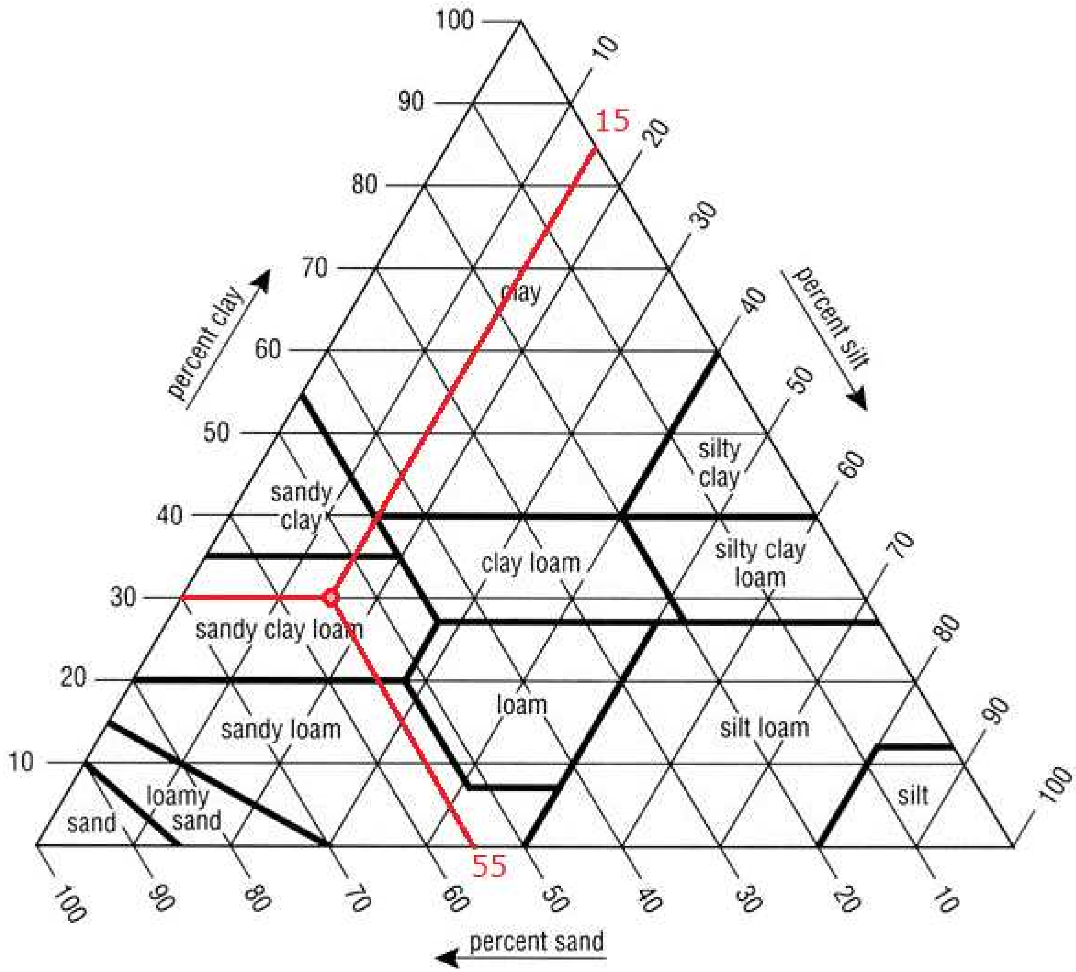
The USDA classifies soil types according to a soil texture triangle chart which gives names to twelve combinations of clay, sand, and silt. The chart can be a little confusing at first glance, however, it makes sense after seeing a few examples.


100
90
80
70
60
50
40
30
20
10
10
20
30
40
50
60
70
80
90
100
percent clay
percent silt
percent sand
clay
silty clay
sandy clay
silty clay loam
clay loam
sandy clay loam
loam
sandy loam
silt loam
silt
loamy sand
sand

First, look at the orientation of the percentages on the sides of the triangle. The numbers are arranged symmetrically around the perimeter. On the left the numbers correspond to the percentage of clay, and on the right the numbers correspond to the percentage of silt. At the bottom of the triangle chart are the percentages of sand.

To classify a soil sample, you find the intersection of the three lines that correspond to the three proportions. On the chart, all of the percent's will add up to 100%.

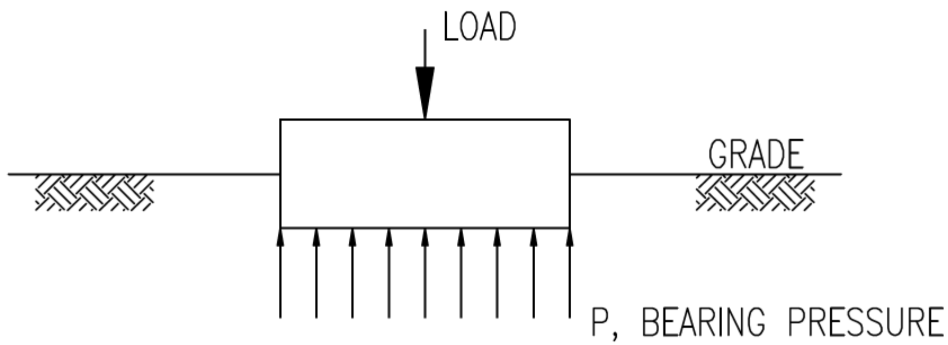
Example: Classify a soil sample that is 30% clay, 15% silt, and 55% sand. First locate 30% on the clay axis, and draw a line horizontally from left to right. Next, locate 15% on the silt axis, and draw a line going down diagonally to the left. Finally, locate 55% on the sand axis, and draw a line going up diagonally to the left. The intersection is in a region called Sandy Clay Loam. See figure below. (Actually, you only need to make two lines.)



100
90
80
70
60
50
40
30
20
10
percent clay
percent silt
percent sand
15
55
clay
silty clay
sandy clay
clay loam
silty clay loam
sandy clay loam
loam
sandy loam
silt loam
loamy sand
sand
silt

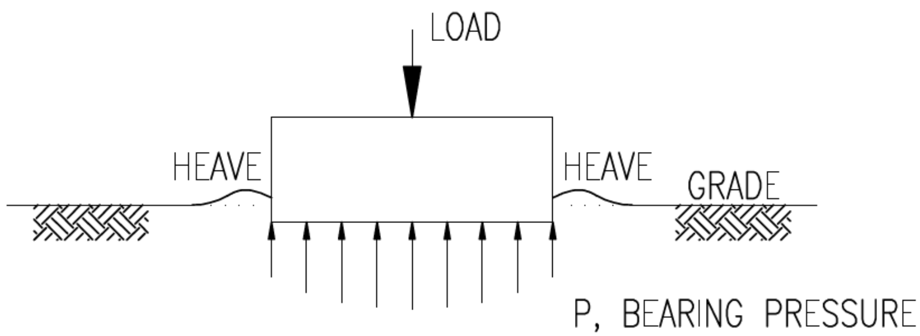
III. Bearing Pressures on Axially Loaded Footings

The treatment of bearing pressures below the base of a footing is determined by analyzing the footing as a rigid element and the soil directly below the footing as a homogeneous elastic material which is isolated from the surrounding soil. By analyzing the soil pressure using the model described above for an axially loaded footing with the load located at the centroid of the footing the soil pressure would be found to be directly proportional to the deformation of the soil and a uniform pressure would act under the base of the axially loaded footing. The diagram below shows the assumptions made in this model.



ASSUMED PRESSURE DISTRIBUTION

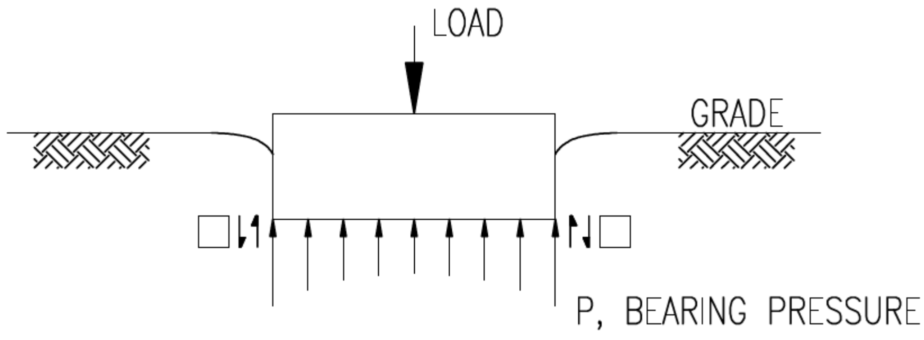
The pressure distribution below the base of an axially loaded footing is in actually not uniform but varies. The primary factors which contribute to the variation in soil pressure are the flexibility of the footing, the depth of the footing below grade and the type of soil, e.g., cohesive (clay) or non-cohesive (sand). Consider an axially loaded footing on a non-cohesive soil with the base of the footing a small distance below grade. The downward displacement of the loaded footing on the non-cohesive soil located below the perimeter of the footing produces a lateral movement of the soil from under the edge of the footing. This lateral movement of the non-cohesive soil at the footings perimeter results in the soil heaving upward and from the removal of this soil produces a parabolic pressure distribution as shown in the diagram below.



DEFORMATIONS AND APPROXIMATE PRESSURE UNDER
A RIGID SURFACE FOOTING ON A COHESIONLESS SOIL

If the base of the footing on the non-cohesive soil in the above diagram been placed well below grade the pressure distribution would have been more nearly uniform because the weight of the larger depth of soil at the perimeter of the footing would be more restrictive to the lateral movement of the soil under the edges of the footing.

Consider an axially loaded footing on a cohesive soil with the base of the footing a small distance below grade. The downward displacement of the axially loaded footing on the cohesive soil located below the perimeter of the footing produces shear stresses in the soil surrounding the base of the footing which provides additional vertical support at the edge of the footing. The resulting pressure distribution produces higher stresses at the perimeter than at the center of the footing as shown in the diagram below.



DEFORMATIONS AND APPROXIMATE PRESSURE UNDER
A RIGID SURFACE FOOTING ON A COHESIVE SOIL

Regardless of the soil type, the magnitude of the soil pressures below the base of an axially loaded footing are not uniform, however, footing design is usually based on the assumption of a uniform pressure distribution.

IV. Allowable Soil Pressures

Values of allowable soil pressures for various types of soils are usually specified in building codes. The table below is from the Florida 2010 Building Code.

TABLE R401.4.1

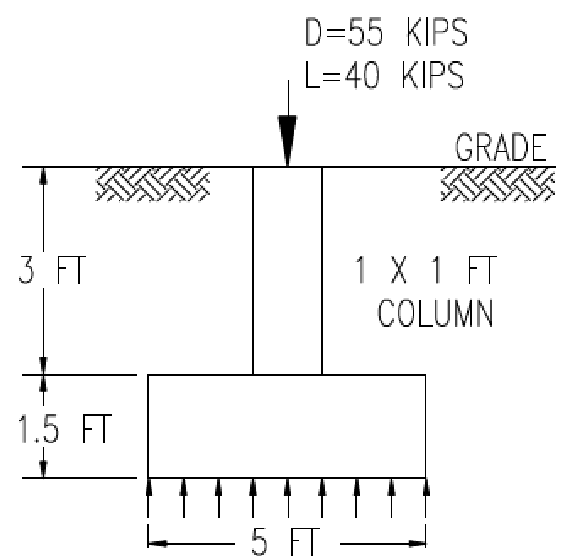
PRESUMPTIVE LOAD-BEARING VALUES OF FOUNDATION MATERIALS

CLASS OF MATERIAL	LOAD-BEARING PRESSURE (pounds per square foot)
Crystalline bedrock	12,000
Sedimentary and foliated rock	4,000
Sandy gravel and/or gravel (GW and GP)	3,000
Sand, silty sand, clayey sand, silty gravel and clayey gravel (SW, SP, SM, SC, GM and GC)	2,000
Clay, sandy clay, silty clay, clayey silt, silt and sandy silt (CL, ML, MH and CH)	1,500

If the foundation design engineer is familiar with the allowable soil pressures at the location of the proposed foundation a geotechnical engineer may not be required, however, if the foundation design engineer is not certain of the soils at a specific location soil boring(s) and a geotechnical engineering report would be required.

In the design of shallow foundations there are actually two pressures that should be investigated, namely *gross* pressure and *net* pressure. The gross pressure is the sum of all loads that the base of the footing is supporting which include service loads, weight of footing and soil between top of footing and grade. The net pressure is computed by subtracting from the gross pressure the weight of a 1 ft^2 column of soil from grade to the base of the footing. The following example illustrates the calculations for the gross and net pressures below the base of the 5ft X 5ft footing.

Example: A geotechnical report is provided for the bearing pressure for this site location and the net allowable soil pressure chart in the report for the depth of 3 to 6 ft. states that the net allowable soil pressure is 4 kips/ft^2 and a unit wt. of 130 pcf, (density of concrete is typically 150 pcf).



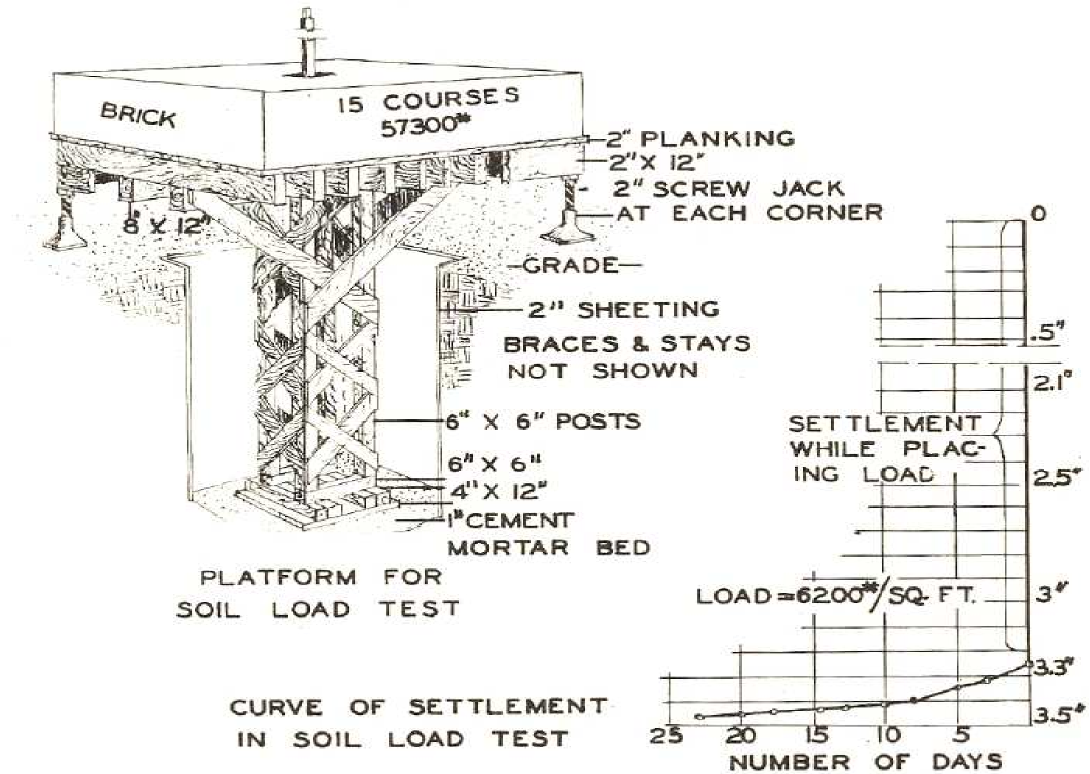
Calculate the total load at the base of the square footing:
Weight of footing = (5)(5)(1.5)(0.15) = 5.63 kips
Weight of column= (1)(1)(3)(0.15) = 0.45 kips
Weight of soil= (3)(25-1)(0.13) = 9.36 kips
Service loads= 55+40 = 95 kips
Total load at the base of the footing = 110.4 kips
The gross soil pressure is:

$$Q_{gr} = \frac{P}{A} = \frac{110.4 \text{ KIPS}}{25 \text{ ft}^2} = 4.4 \text{ kips/ft}^2$$

The net soil pressure is:
$Q_n = Q_{gr}$ – wt. of 4.5 ft. col. of soil = 4.4 kips/ft^2 – (4.5)(0.13) = 3.8 kips/ft^2

The gross pressure exceeds the geotechnical reports allowable soil pressure value, however, the net pressure is within the allowable soil pressure value.
Note that load factors are not used when calculating soil pressures.
What is being shown in this example is that the plane of soil located 4.5 ft. below grade (bottom of footing) was supporting a 4.5 foot column of soil weighing 130 pcf or 0.6 Kips/ ft^2 prior to the excavation for the placement of the foundation.

Testing allowable soil pressures and settlement is in most cases one of the analysis which is performed by a geotechnical engineer. Early methods of testing allowable soil pressures and settlement are illustrated in the figure below.

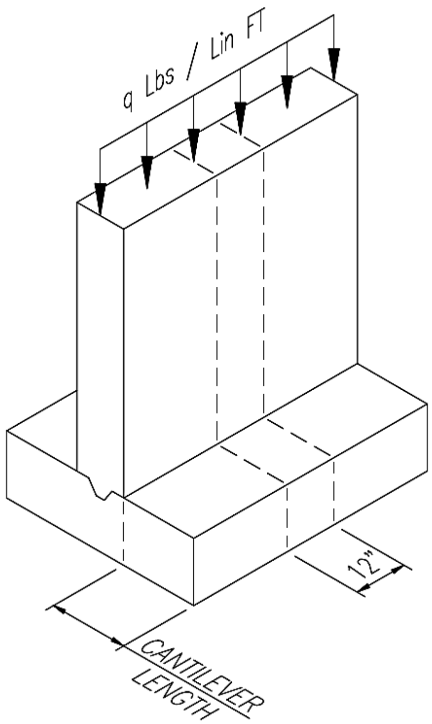


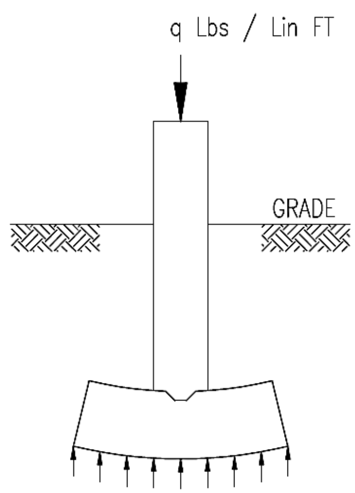
The ASTM D 1194, Standard Test Method for Bearing Capacity of Soil for Static Load and Spread Footings is the standard method used to determine allowable soil pressures and settlement and is basically very similar to the testing method shown in the diagram above.

V. Design of Wall Footings for Vertical Uniform Load

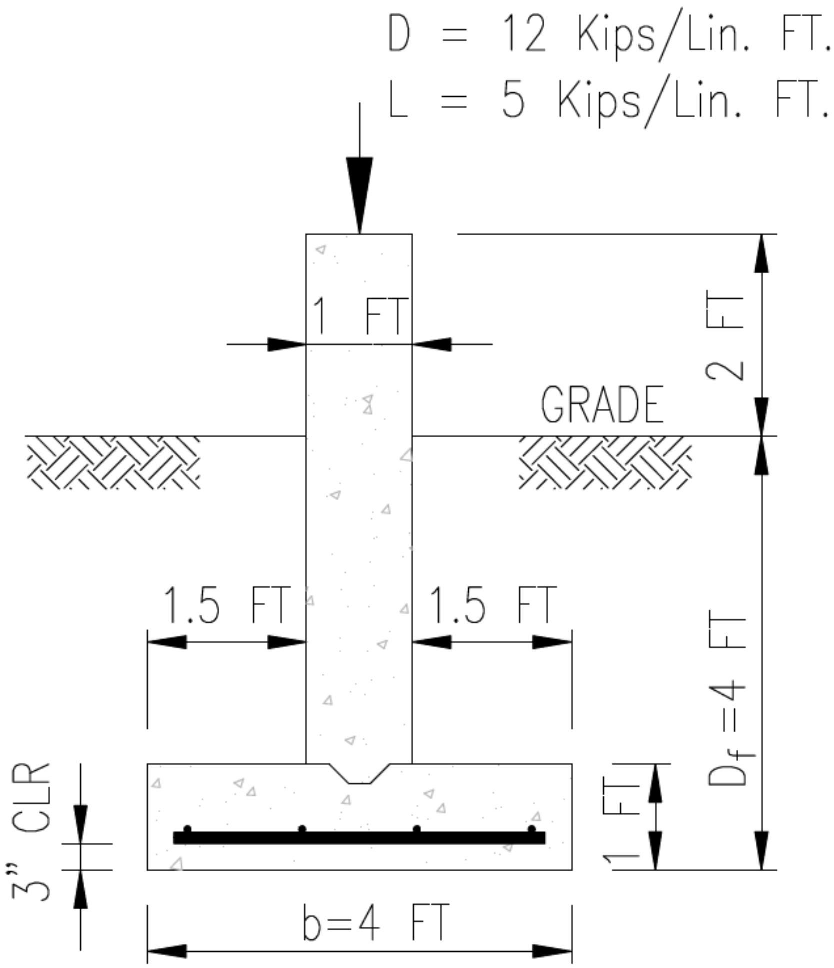
The soil pressure on the segment of the wall footing which extends beyond the face of the wall acts like a cantilever beam and all sections behave the same, therefore, the design of a wall footing can be based on the analysis of a 12" or 1 foot slice cut by traverse planes normal to the longitudinal axis of the wall. See adjacent figure. The steps required to design a wall footing are outlined below in the example.

Note: All Tables included in this course are from ACI 318-14, for reference to notes in Tables see Building Code Requirements for Structural Concrete (ACI 318-14)





Example: Design a reinforced concrete footing shown in the adjacent figure. The maximum allowable pressure on the soil under the foundation is 5 Kips/ft^{2}.
Compressive strength of concrete (f'_c) = 3 Kips/in^2
Tensile strength of reinforcing steel (f_y) = 60 Kips/in^2
The unit weight of the soil (γ) = 130 pcf



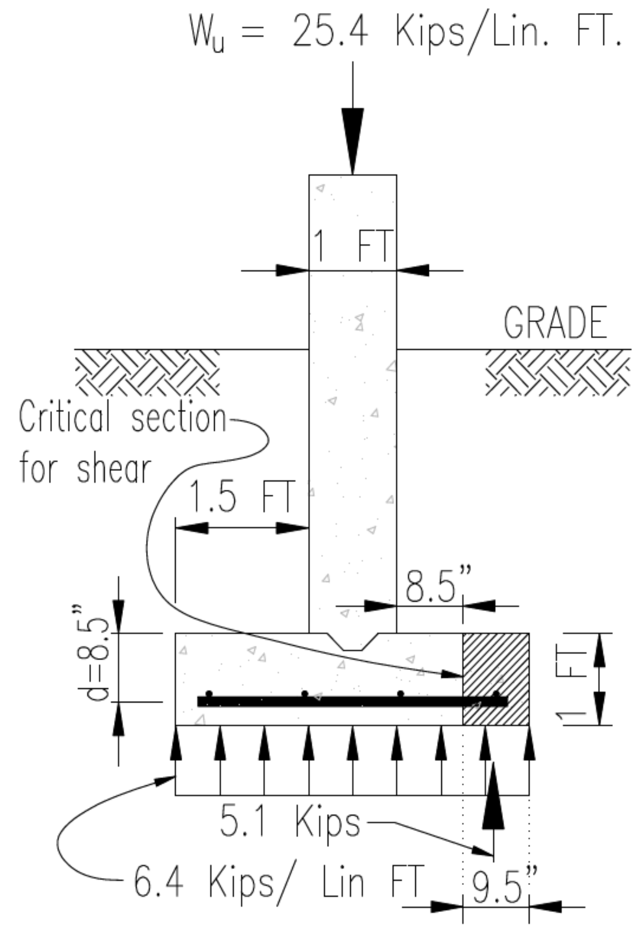
Solution:
Calculate the total load at the base of the footing:
Weight of footing = (4)(1)(1)(0.15) = 0.60 kips/LF
Weight of column= (5)(1)(1)(0.15) = 0.75 kips/LF
Weight of soil= (3)(4-1)(0.13) = 1.17 kips/LF
Service loads= 12+5 = 17 kips
Total load at the base of the footing = 19.5 kips/LF

The gross soil pressure is:

$Q_{gr} = \frac{P}{A} = \frac{19.5 \text{ Kips/}}{4 \text{ ft}^2} = 4.88 \text{ Kips/ft}^2$

The net soil pressure is:
$Q_n = Q_{gr}$ – weight of 4 ft. column of soil =
$4.88 \text{kips/ft}^2 – (4)(0.13) = 4.36 \text{ kips/ft}^2 < 5 \text{ Kips/ft}^2$, ***OK***
Calculate soil pressure produced by factored loads:
U = 1.2(D) + 1.6(L)

$$P_u = \frac{1.2(12+0.6+0.75+1.17 \text{ Kips})+1.6(5 \text{ Kips})}{4 \text{ ft}^2} = \frac{25.4 \text{ Kips}}{4 \text{ ft}^2} = 6.4 \text{ Kips/ft}^2$$



Check shear at ***d*** inches (critical section) from face of wall,
d =footing thickness – (3 inches clear space + ½ rebar diameter).
Therefore, in this example ***d*** = 8.5 inches ***assuming*** a #8 rebar.

$$v_u = \frac{9.5 \text{ in.}}{12 \text{ in}} (6.4 \text{ Kips}) = 5.1 \text{ Kips}$$

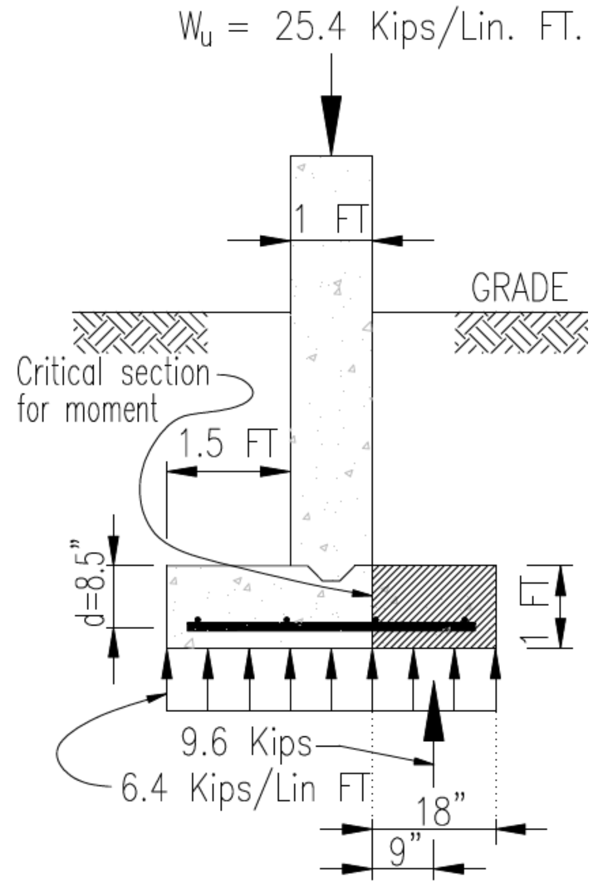
The shear capacity at the critical section is equal to
$v_c = \phi 2\sqrt{f'c}\ b_w d$, from Table 21.2.1, ϕ for shear = 0.75 and ***d*** = 8.5 in.

$v_c = 0.75(2)\left(\frac{\sqrt{3000}}{1000}\right)(12)(8.5) = 8.4$ Kips, therefore, $v_u = 5.1$ Kips $< v_c = 8.4$ Kips, OK

Table 21.2.1-Strength Reduction Factors ϕ

	Action or structural element	ϕ	Exceptions
(*a*)	Moment, axial force, or combined moment and axial force	0.65 to 0.90 in accordance with 21.2.2	Near ends of pretensioned members where strands are not fully developed, ϕ shall be in accordance with 21.2.3.
(b)	Shear	0.75	Additional requirements are given in 21.2.4 for structures designed to resist earthquakes effects.
(c)	Torsion	0.75	—
(d)	Bearing	0.65	—
(e)	Post –tension anchorage zones	0.85	—
(f)	Brackets and corbels	0.75	—
(g)	Struts, ties, nodal zones, and bearing areas designed in accordance with strut-and-tie method in Chapter 23	0.75	—
(h)	Components of connections of precast members controlled by yielding of steel elements in tension	0.90	—
(i)	Plain concrete elements	0.60	—
(j)	Anchors in concrete elements	0.45 to 0.75 in accordance with Chapter 17	—

Determine the area of reinforcement steel required to resist the moment in the footing.
The critical section of the footing is taken at the face of the wall.
M_u = (6.4 Kips/Lin. FT)X(1.5 FT)X(0.75 FT)
= (1.6 Kips/FT)X(1.5 FT)X(0.75 FT)
=7.2 FT-Kips



To determine the area of reinforcement steel,
five quantities are required in addition to the value of the moment:
From Table 21.2.2 Φ = 0.90, f'_c = 3 Kips/in^2,
f_y = 60 Kips/in^2 , b = 12 inches and ***d*** = 8.5 inches
the solution of the quadratic equation shown below
will provide the solution for ρ**,**

where ρ represents the ratio of the $\frac{\text{area of steel}}{\text{area of concrete}}$

$$R_u = \frac{M_u}{\emptyset\, b\, d^2} = \frac{7.2 \text{ X } 12{,}000}{0.9\, X\, 12\, X\, 8.5^2} = 110.7 \text{ psi}$$

$$\rho = \left(\frac{0.85\, f'_c}{f_y}\right)\left(1 - \sqrt{1 - \frac{2\, R_u}{0.85\, f'_c}}\right)$$

$$\rho = \left(\frac{0.85 \text{ X } 3{,}000}{60{,}000}\right)\left(1 - \sqrt{1 - \frac{2 \text{ X } 110.7}{0.85 \text{ X } 3{,}000}}\right) = 0.00189$$

$A_s = \rho bd$ = 0.00189 X 12 X 8.5 = 0.19 in^2 per linear foot of footing.

Table 21.2.2 Strength Reduction Factors φ for moment, axial force or combined moment and axial force

Net tensile strain ε_t	Classification	φ			
		Type of traverse reinforcement Classification			
		Spirals conforming to 25.7.3		Other	
$\varepsilon_t \leq \varepsilon_{ty}$	Compression-controlled	0.75	(a)	0.65	(b)
$\varepsilon_{ty} < \varepsilon_t < 0.005$	Transition[1]	$0.75+0.15\frac{(\varepsilon_t-\varepsilon_{ty})}{0.005-\varepsilon_{ty}}$	(c)	$0.65+0.25\frac{(\varepsilon_t-\varepsilon_{ty})}{0.005-\varepsilon_{ty}}$	(d)
$\varepsilon_t \geq 0.005$	Tension-controlled	0.90	(c)	0.90	(f)

[1]For sections classified as transition, it shall be permitted to use φ corresponding to compression controlled sections.

The minimum allowable value for ρ is $\rho_{\min}$ which is equal to the greater of

(a) $\frac{3\sqrt{f'c}}{f_y} = 0.0027$ or **(b)** $\frac{200}{f_y} = 0.0033$

$A_{s,min} = \rho_{min}bd = 0.0033 \text{ X } 12 \text{ in. X } 8.5 \text{ in.} = 0.337 \text{ in}^2$ per linear foot of footing.

$A_{s,min}$ is greater than the calculated A_s, therefore, the required area of reinforcement steel linear per foot of footing is 0.337 in^2.

Checking Tempature and Shrinkage reinforcement area for gross area of footing from ACI 318-14 Table 24.4.3.2 shown below results in

A_{smin} = 0.0018 X 12 in. X 12 in. = 0.259 in^2 per linear foot of footing.

Table 24.4.3.2 Minimum ratios of deformed shrinkage and reinforcement area to gross concrete area

Reinforcement Type	f_y, psi	Minimum reinforcement ratio	
Deformed bars	< 60,000	0.0020	
Deformed bars or welded wire reinforcement	$\geq$ 60,000	Greater of:	$\frac{0.0018\ X\ 60{,}000}{f_y}$
			0.0014

Use the $A_{s,min}$ value of 0.337 in^2 of reinforcing steel per linear foot of footing.

The area of a #6 rebar is 0.44 in^2, use #6's @12" OC.

Checking development length l_d , using ACI 318-14 Tables 25.4.2.2 and 25.4.2.4 shown below:

Table 25.4.2.2-Development length for deformed bars and deformed wires in tension

Spacing and cover	No. 6 and smaller bars and deformed wire	No. 7 and larger bars
Clear spacing of bars or wires being developed or lap spliced not less that d_b, clear cover at least d_b, and stirrups throughout l_d but not less than the code minimum or Clear spacing of bars or wires being developed or lap spliced at least $2d_b$ and clear cover at least d_b	$\frac{f_y \Psi_t \Psi_e d_b}{25\lambda \sqrt{f'c}}$	$\frac{f_y \Psi_t \Psi_e d_b}{20\lambda \sqrt{f'c}}$
Other cases	$\frac{3f_y \Psi_t \Psi_e d_b}{50\lambda \sqrt{f'c}}$	$\frac{3f_y \Psi_t \Psi_e d_b}{40\lambda \sqrt{f'c}}$

Table 25.4.2.4-Modification factors for development of deformed bars or deformed wires in tension

Modification factor	Condition	Value of factor
Lightweight $\boldsymbol{\lambda}$	Lightweight Concrete	0.75
	Lightweight Concrete, where f_{ct} is specified	In accordance with 19.2.4.3
	Normalweight concrete	1.0
Epoxy[1] $\boldsymbol{\Psi}_e$	Epoxy-coated or zinc and epoxy dual-coated reinforcement with clear cover less than $3d_b$ or clear spacing less than $6\ d_b$	1.5
	Epoxy-coated or zinc and epoxy dual-coated reinforcement for all other conditions	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Size $\boldsymbol{\Psi}_s$	No. 7 and larger bars	1.0
	No. 6 and smaller bars and deformed wire	0.8
Casting position[1] $\boldsymbol{\Psi}_t$	More than 12 in. of fresh concrete placed below horizontal reinforcement	1.3
	Other	1.0

[1] The product of $\boldsymbol{\Psi}_t\ \boldsymbol{\Psi}_e$ need not exceed 1.7.

Using the formula from Table 25.4.2.2 under "No. 6 bars and smaller" and to the right of "Other cases", the development length available is 18 in. less the clear distance of 3 in. which leaves 15 in.

$$l_d = \frac{3 f_y \Psi_t \Psi_e d_b}{50\lambda\sqrt{f'c}} = \frac{(3)(60{,}000)(1.0)(1.0)(0.75)}{50(1.0)\sqrt{3{,}000}} = 49.3 \text{ in.} > 15 \text{ in.}$$

The development length is inadequate for #6's with 15 in. of available development and an alternate design needs to be considered. There are 3 alternatives or combinations thereof,

1.) Increase the width of the footing
2.) Use smaller rebar spaced more closely
3.) Use hooks to provide the required development length

The formula for standard hooks in tension per ACI 318-14 Section 25.4.3.1 is

$$l_{dh} = \frac{f_y \Psi_e \Psi_c \Psi_r d_b}{50\lambda\sqrt{f'c}} = \frac{(60{,}000)(1.0)(0.7)(1.0)(0.75)}{50(1.0)\sqrt{3{,}000}} = 11.5 \text{ in.} < 15 \text{ in.}, \textbf{\textit{OK}}$$

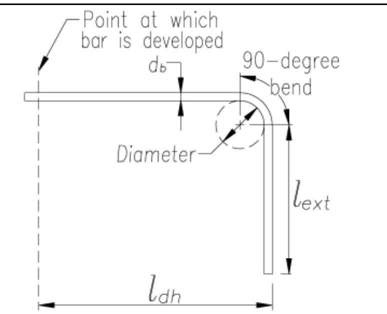
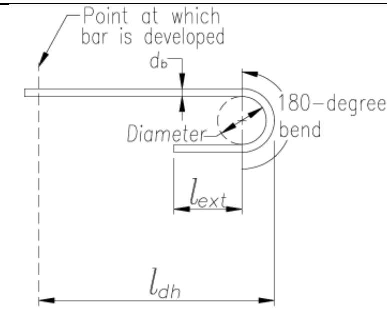
Table 25.4.3.2-Modification factors for development of hooked bars or in tension

Modification factor	Condition	Value of factor
Lightweight **λ**	Lightweight Concrete	0.75
	Normalweight concrete	1.0
Epoxy[1] $\boldsymbol{\Psi_e}$	Epoxy-coated or zinc and epoxy dual-coated reinforcement.	1.2
	Uncoated or zinc-coated (galvanized) reinforcement	1.0
Cover $\boldsymbol{\Psi_c}$	For No. 11 bars and smaller hooks with side cover (normal to plane of hook) $\geq$ 2½ in. and for 90° hook with cover on bar extension beyond hook $\geq$ 2 in.	0.7
	Other	1.0
Confining reinforcement $\boldsymbol{\Psi_r}$ [2]	For 90° hooks of No. 11 and smaller bars (1) enclosed along $\boldsymbol{l}_{dh}$ within ties or stirrups[1] perpendicular to $\boldsymbol{l}_{dh}$ at s $\leq 3\boldsymbol{d}_b$, or (2) enclosed along the bar extension beyond hook including the bend within ties or stirrups[1] perpendicular to $\boldsymbol{l}_{dh}$ at s $\leq 3\boldsymbol{d}_b$	0.8
	Other	1.0

[1] The first tie or stirrup shall enclose the bent portion of the hook within $2d_b$ of the outside of the bent.

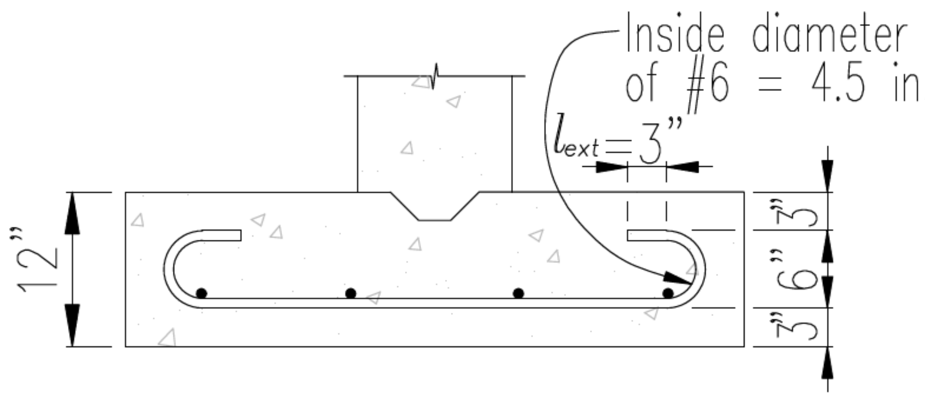
[2] d_b is the nominal diameter of the hooked bar

Table 25.3.1-Minimum hook geometry for development of deformed bars in tension

Type of standard hook	Bar size	Minimum inside bend diameter, in.	Straight extension[1] l_{ext}, in.	Type of standard hook
90-degree hook	No. 3 through No. 8	$6d_b$	$12d_b$	
	No. 9 through No. 11	$8d_b$		
	No. 14 and No. 18	$10d_b$		
180-degree hook	No. 3 through No. 8	$6d_b$	Greater of $4d_b$ and 2.5 in.	
	No. 9 through No. 11	$8d_b$		
	No. 14 and No. 18	$10d_b$		

[1] The standard hook for deformed bars in tension includes the specific inside bend diameter and straight extension length. It shall be permitted to use a longer straight extension at the end of a hook. A longer extension shall not be considered to increase the anchorage capacity of the hook.

A 90° hook has a l_{ext} length per Table 25.3.1 of $12d_b$ or 12 X 0.75 in. = 9 in. This 9 in. extension would intrude into the required clear space at the top of the footing. Therefore, a 180° hook is considered, the minimum inside diameter for a #6 rebar per the above Table is $6d_b$ or 6 x 0.75 in. = 4.5 in. and the required straight extension length is the greater of $4d_b$ or 2.5 in., $4d_b$ = 4 X 0.75 in. = 3.0 in. See figure below for hook details.



VI. Design of an Axially Loaded Two-Way Footing

The design for an axially loaded rectangular footing that supports a single column requires an analysis of bending, beam shear (one-way shear) and punching shear (two-way shear). In the following example space is limited to a maximum footing width of 4 feet, footing length is not limited and the top of the footing is at grade. The steps required to design a two-way footing are outlined below in the example.

Example: Design a reinforced concrete footing shown in the figure below.

The maximum allowable pressure on the soil under the foundation is 2.5 Kips/ft$^{2.}$

Compressive strength of concrete (f'_c) = 3 Kips/in^2

Tensile strength of reinforcing steel (f_y) = 60 Kips/in^2

Solution:

A dimension for the footing length must first be determined by equating the total weight of the dead & live loads and footing weight to the 2.5 kips/ft^2 for the area beneath the footing. (Ignore weight of pedestal)

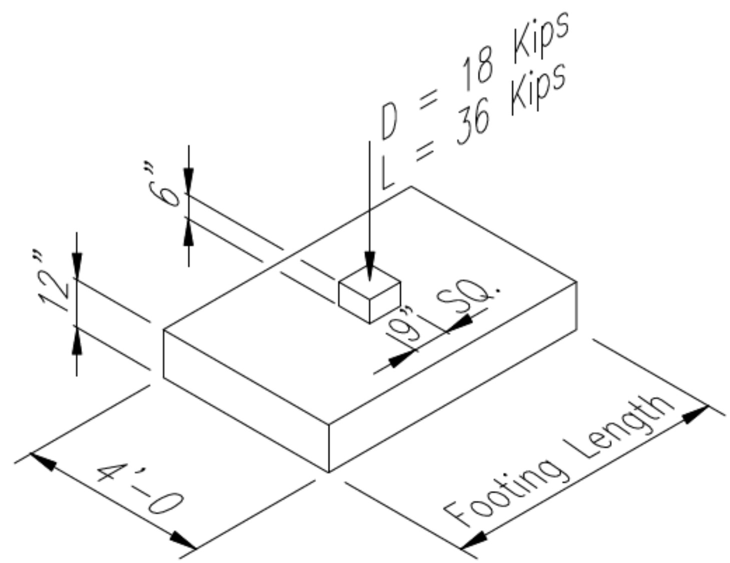
Length X 4 FT. X 2.5 kips/ft^2 =

Length X 4FT. X 1 FT. X 0.15 kips/ft^3+36 kips + 18 kips

or,

Length X 10 = Length X 0.6 + 54 kips

therefore, Length = 5.75 FT.



Calculate soil pressure produced by factored loads:

U = 1.2(D) + 1.6(L)

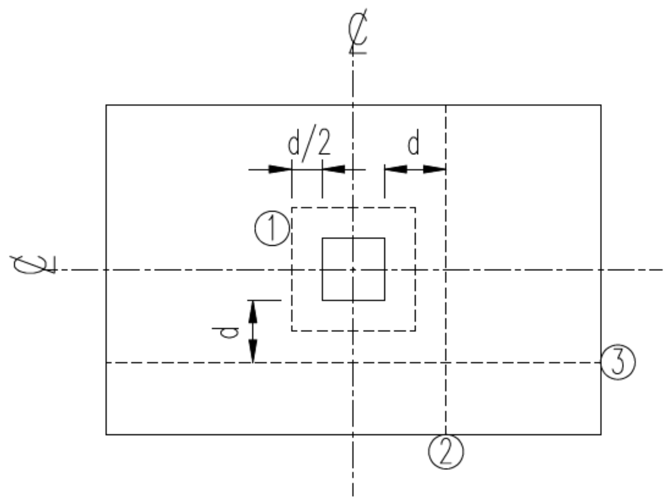
$$P_u = \frac{1.2(18\text{ Kips}+(4\text{ X }5.75\text{ X }0.15)\text{Kips})+1.6(36\text{ Kips})}{(4\text{ X }5.75)\text{ ft}^2} = \frac{83.3\text{ Kips}}{23\text{ ft}^2} = 3.6\text{ Kips/ft}^2$$

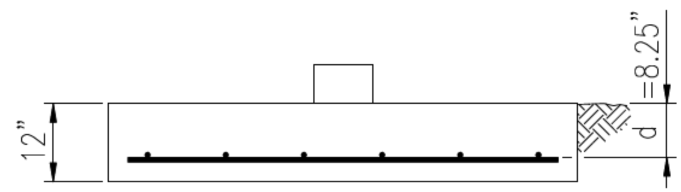
Check Shear:
The punching shear is calculated based on the length of the perimeter at a distance of d/2 from the column as indicated by ①.

The short direction beam shear is calculated based on the length and location of the line located at a distance of d from the column as indicated by ②.

The long direction beam shear is calculated based on the length and location of the line located at a distance of d from the column as indicated by ③.

Bending of the footing in both directions requires having two rows of rebars, one on top of the other. Assuming ¾ inch diameter rebars (# 6's) and a clear distance of 3 inches yields the average ***d*** value of 8.25 inches.





Check Two-way or Punching Shear:
The perimeter distance of the punching shear distance b_o = 4 X (8.25" + 9") = 69 inches
and β = 1.0 & normal weight concrete λ = 1.
The formula for the calculation of v_c for two-way shear per ACI 318-14 is shown below in Table 22.6.5.2:

Table 22.6.5.2- Calculation of V_c for two-way shear

V_c		
Least of (a), (b), and (c)	$4\lambda\sqrt{f'c}$	(a)
	$\left(2+\dfrac{4}{\beta}\right)\lambda\sqrt{f'c}$	(b)
	$\left(2+\dfrac{\alpha_s d}{b_o}\right)\lambda\sqrt{f'c}$	(c)

Note: β is the ratio of long side to short side of the column, concentrated load, or reaction area and α_s is given in 22.6.5.3.

Section 22.6.5.3 states that α_s is equal to 40 for interior columns, 30 for edge columns and 20 for corner columns.

Equation (a) from Table 22.6.5.2 yields 4 (1) $\sqrt{3,000}$ = 219 psi

Equation (b) from Table 22.6.5.2 yields (2+ $\frac{4}{1}$)(1) $\sqrt{3,000}$ = 329 psi

Equation (c) from Table 22.6.5.2 yields (2+ $\frac{(40)(8.25)}{69}$)(1) $\sqrt{3,000}$ = 371 psi

Equation (a) controls, use = 219 psi

v_c = (219 psi)(69 in.)(8.25 in.) =124,666 pounds or 125 Kips

From Table 21.2.1 Shear reduction factor ϕ = 0.75

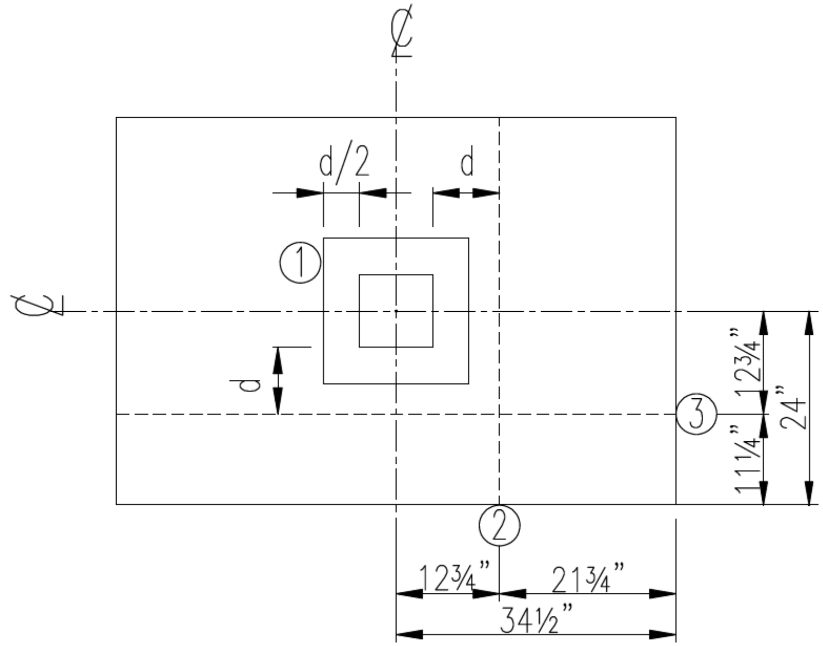
$\phi\ v_c$ = (0.75)(125) = 94 kips

The value v_u equals the force pushing upward on the total area of the bottom of the footing minus the area inside the perimeter of the punching shear.

$v_u = \left((4)(5.75) - \frac{(9"+8.25")^2}{144}\right)$ 3.6 Kips/ft^2 = 75.4 Kips

Check to see if design strength exceeds required strength, $\phi\ v_c \geq v_u$?

$\phi\ v_c$ = 94 kips $\geq v_u$ = 75.4 Kips; Therefore, Two-way shear or punching shear strength is adequate.



Check short & long direction One-way shear:

For short direction One-way shear, b_w = 4 feet

$\phi\, v_c = \phi\,(2)\sqrt{f'c}\, b_w d = (0.75)(2)\frac{\sqrt{3,000}}{1000}\ (48)(8.25) = 32.5$ Kips

$v_u = (21.75/12)(4)\ 3.6\ \text{Kips/ft}^2 = 26.1$ Kips

Check to see if design strength exceeds required strength,

$\phi\, v_c \geq v_u$?

$\phi\, v_c = 32.5$ kips $\geq v_u = 26.1$ Kips; Therefore, One-way short direction shear strength is adequate.

For long direction One-way shear, b_w = 5.75 feet

$\phi\, v_c = \phi\,(2)\sqrt{f'c}\, b_w d = (0.75)(2)\frac{\sqrt{3,000}}{1000}\ (69)(8.25) = 46.8$ Kips

$v_u = (11.25/12)(5.75)\ 3.6\ \text{Kips/ft}^2 = 19.4$ Kips

Check to see if design strength exceeds required strength,

$\phi\, v_c \geq v_u$?

$\phi\, v_c = 46.8$ kips $\geq v_u = 19.4$ Kips; Therefore, One-way long direction shear strength is adequate.

Check short & long direction flexural bending:

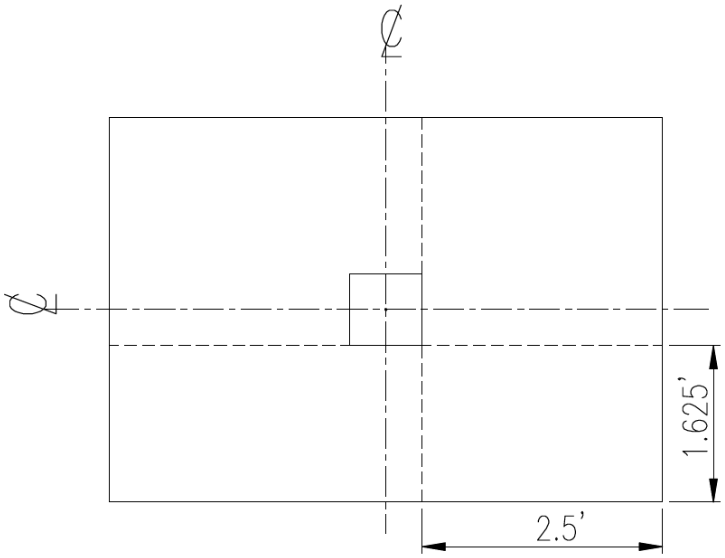
The critical section is located at the face of the column.

For short direction flexural bending the moment is equal to

$M_{u\text{-short}} = (3.6\ \text{Kips/ft}^2)\ X\ \left(\frac{1.625^2}{2}\right)(5.75) = 27.3$ FT-Kips

For long direction flexural bending the moment is equal to

$M_{u\text{-long}} = (3.6\ \text{Kips/ft}^2)\ X\ \left(\frac{2.5^2}{2}\right)(4) = 45.0$ FT-Kips



Determine the area of reinforcement steel required to resist the moment in the short direction of the footing.

To determine the area of reinforcement steel required five quantities in addition to the moment value must be known:

From Table 21.2.2 $\phi = 0.90$, $f'_c = 3\ \text{Kips/in}^2$,

$f_y = 60\ \text{Kips/in}^2$, b = 69 inches and $\boldsymbol{d} = 8.25$ inches

the equation shown below will provide the solution for ρ,

where ρ represents the ratio of the $\frac{\text{area of steel}}{\text{area of concrete}}$

$R_u = \frac{M_{u-short}}{\emptyset\ b\ d^2} = \frac{27.3\ X\ 12{,}000}{0.9\ X\ 69\ X\ 8.25^2} = 77.5$ psi

$\rho = (\frac{0.85\ f'_c}{f_y})\ (1-\sqrt{1-\frac{2\ R_u}{0.85\ f'_c}})$

$\rho = (\frac{0.85\ X\ 3{,}000}{60{,}000})\ (1-\sqrt{1-\frac{2\ X\ 77.5}{0.85\ X\ 3{,}000}}) = 0.0013$

$A_s = \rho bd = 0.0013\ X\ 69\ X\ 8.25 = 0.74\ in^2$

The minimum allowable value for ρ is ρ_{min} which is equal to the greater of

(a) $\frac{3\sqrt{f'c}}{f_y} = 0.0027$ or **(b)** $\frac{200}{f_y} = 0.0033$

$A_{s,min} = \rho_{min}bd = 0.0033\ X\ 69\ X\ 8.25 = 1.88\ in^2$; $A_{s\ min}$ is greater than the calculated A_s, therefore, the required area of reinforcement steel for the footing in the short direction is 1.88 in^2.

Checking Temperature and Shrinkage reinforcement area for gross area of footing from ACI 318-14 Table 24.4.3.2 shown below results in

$A_{s,\ min} = 0.0018\ X\ 69\ X\ 12 = 1.49\ in^2$

Table 24.4.3.2 Minimum ratios of deformed shrinkage and reinforcement area to gross concrete area

Reinforcement Type	f_y, psi	Minimum reinforcement ratio	
Deformed bars	< 60,000	0.0020	
Deformed bars or welded wire reinforcement	$\geq$ 60,000	Greater of:	$\frac{0.0018\ X\ 60{,}000}{f_y}$
			0.0014

Use the A_s value of 1.88 in^2 of reinforcing steel. The area of a #4 rebar is 0.20 in^2, use 10 - #4@ 6" OC.

Determine the area of reinforcement steel required to resist the moment in the long direction of the footing, b = 48 inches.

$R_u = \frac{M_{u-long}}{\emptyset\ b\ d^2} = \frac{45.0\ X\ 12{,}000}{0.9\ X\ 48\ X\ 8.25^2}$ = 183.6 psi

$\rho = \left(\frac{0.85\ f'_c}{f_y}\right)\left(1-\sqrt{1-\frac{2\ R_u}{0.85\ f'_c}}\right) = \left(\frac{0.85\ X\ 3{,}000}{60{,}000}\right)\left(1-\sqrt{1-\frac{2\ X\ 183.6}{0.85\ X\ 3{,}000}}\right) = 0.0032$

$A_s = \rho bd$ = 0.0032 X 48 X 8.25 = 1.26 in^2

The minimum allowable value for ρ is ρ_{min} which is equal to the greater of

(a) $\frac{3\sqrt{f'c}}{f_y} = 0.0027$ or **(b)** $\frac{200}{f_y} = 0.0033$

$A_{s,min} = \rho_{min}bd$ = 0.0033 X 48 X 8.25 = 1.31 in^2

$A_{s,min}$ is greater than the calculated A_s, therefore, the required area of reinforcement steel for the footing in the long direction is 1.31 in^2.

Checking Temperature and Shrinkage reinforcement area for gross area of footing from ACI 318-14 Table 24.4.3.2 shown below results in

$A_{s,min}$ = 0.0018 X 48 X 12 =1.04 in^2

Use the $A_{s,min}$ value of 1.31 in^2 of reinforcing steel.

The area of a #4 rebar is 0.20 in^2, use 7 - #4's @ 6" OC.

Note: *Although it is not required by the code, some practitioners distribute half of the required bars in the middle third of the footing and distribute the remaining bars equally on both sides.*

Checking development length l_d for short direction steel, using ACI 318-14 Tables 25.4.2.2 and 25.4.2.4 shown above. Reinforcement development is calculated at the location of the maximum factored moment, which occurs at the column face. In the short direction, the bars have a distance of $\left(\frac{48'' - 9''}{2}\right) - 3$ in. =16.5 in. for development.

Using the formula from Table 25.4.2.2 under "No. 6 bars and smaller" and to the right of "Other cases"

$$l_d = \frac{3 f_y \Psi_t \Psi_e d_b}{50\lambda\sqrt{f'c}} = \frac{(3)(60{,}000)(1.0)(1.0)(0.5)}{50(1.0)\sqrt{3{,}000}} = 32.8 \text{ in.} > 16.5 \text{ in.}$$

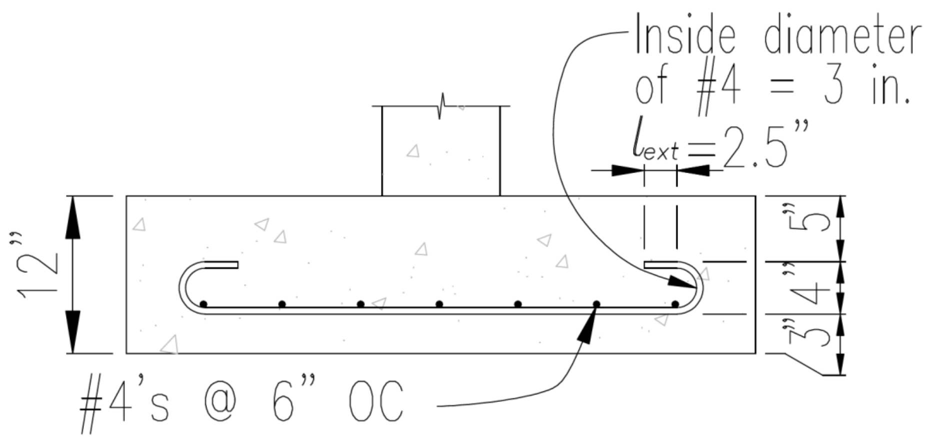
The development length is inadequate for #4's @ 6" OC and an alternate design needs to be considered. There are 3 alternatives or combinations thereof,

1.) Increase the width of the footing
2.) Use smaller rebar spaced more closely
3.) Use hooks to provide the required development length

The formula for standard hooks in tension per ACI 318-14 Section 25.4.3.1 is

$$l_{dh} = \frac{f_y \Psi_e \Psi_c \Psi_r d_b}{50\lambda\sqrt{f'c}} = \frac{(60{,}000)(1.0)(0.7)(1.0)(0.5)}{50(1.0)\sqrt{3{,}000}} = 7.7 \text{ in.} < 16.5 \text{ in.}, \textit{\textbf{OK}}$$

The development length available is 16.5 in. which is inadequate for a 90° hook as the l_{ext} length per Table 25.3.1 is $12d_b$ or 12 X 0.5 = 6 in. Therefore, a 180° hook is considered, the minimum inside diameter for a #4 rebar per the above Table 25.3.1is $6d_b$ or 6 x 0.5 = 3.0 in. and the required straight extension length is the greater of $4d_b$ or 2.5 in., $4d_b$ = 4 X 0.5 = 2.0 in. or 2.5 in. See figure below for hook details.



Checking development length l_d for long direction steel, using ACI 318-14 Tables 25.4.2.2 and 25.4.2.4 shown above. Reinforcement development is calculated at the location of the maximum factored moment, which occurs at the column face. In the long direction, the bars have a distance of $\left(\frac{69''-9''}{2}\right)$-3"=27 in. for development.

Using the formula from Table 25.4.2.2 under "No. 6 bars and smaller" and to the right of "Other cases"

$$l_d = \frac{3f_y\Psi_t\Psi_e d_b}{50\lambda\sqrt{f'c}} = \frac{(3)(60{,}000)(1.0)(1.0)(0.5)}{50(1.0)\sqrt{3{,}000}} = 32.9 \text{ in.} > 27 \text{ in.}$$

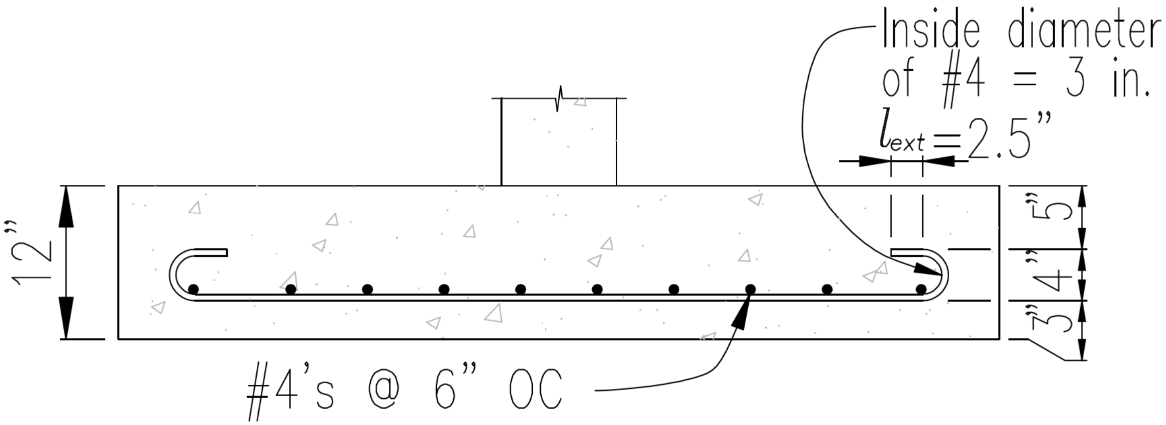
The development length is inadequate for #4's @ 6" OC and an alternate design needs to be considered. There are 3 alternatives or combinations thereof,

1.) Increase the width of the footing
2.) Use smaller rebar spaced more closely
3.) Use hooks to provide the required development length

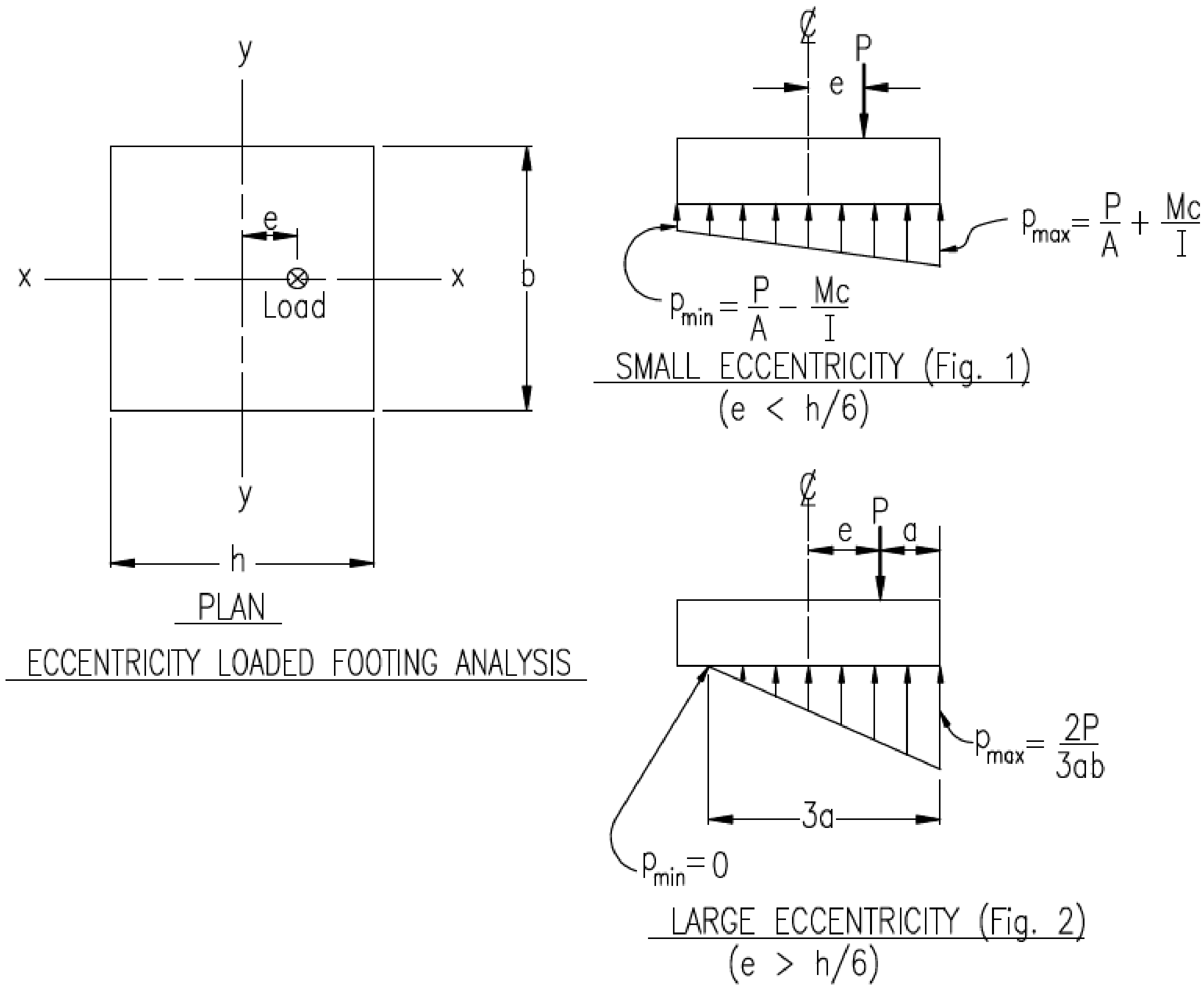
The formula for standard hooks in tension per ACI 318-14 Section 25.4.3.1 is

$$l_{dh} = \frac{f_y\Psi_e\Psi_c\Psi_r d_b}{50\lambda\sqrt{f'c}} = \frac{(60{,}000)(1.0)(0.7)(1.0)(0.5)}{50(1.0)\sqrt{3{,}000}} = 7.7 \text{ in.} < 27 \text{ in.}, \textbf{\textit{OK}}$$

The development length available is 27 in. which is inadequate for a 90° hook as the l_{ext} length per Table 25.3.1 is $12d_b$ or 12 X 0.5 = 6 in. Therefore, a 180° hook is considered, the minimum inside diameter for a #4 rebar per the above Table 25.3.1is $6d_b$ or 6 x 0.5 = 3.0 in. and the required straight extension length is the greater of $4d_b$ or 2.5 in., $4d_b$ = 4 X 0.5 = 2.0 in. See figure below for hook details.
Note: *The analysis of the transfer of column forces to the footing are not developed in this course.*


Inside diameter
of #4 = 3 in.
l_{ext} = 2.5"
12"
5"
4"
3"
#4's @ 6" OC

. Bearing Pressure under Eccentrically Loaded Footings



For an eccentrically loaded foundation where the eccentricity is small (i.e., $p_{min} > 0$) the soil pressure can be determined by superimposing the direct stress P/A due to the axial load and the bending stress Mc/I created by the moment.

Because tensile stresses cannot be transmitted between soil and concrete, superposition of stresses is valid only when the tensile bearing stresses do not exceed the direct compression stresses. For a rectangular footing, the maximum eccentricity for which superposition holds can be established from the limiting case of a triangular stress distribution on the base of a foundation. Although compression stresses develop over the entire base for this case, the stress is zero at the edge, where the tensile bearing and direct compression stresses are equal. Expressing the moment as Pe and setting $P_{min} = 0$ in the below equation yields:

$P_{min} = 0 = \frac{P}{A} - \frac{Mc}{I}$ or $\frac{P}{A} = \frac{Mc}{I} = \frac{Pe}{I}$; Solving for the eccentricity e gives: $e = \frac{I}{Ac}$

For a rectangular foundation of length h and width b, the above equation becomes $e = \frac{bh^3/12}{bh(\frac{h}{2})} = \frac{h}{6}$

If the eccentricity of the vertical load is large and the tensile bearing stresses exceed the direct stress, a triangular stress distribution will developed over a portion of the base. The maximum pressure associated with this distribution can be established by recognizing that the centroid of the soil pressure is located directly under the vertical component of the applied load. With the dimensions of the foundation established and with the eccentricity of the vertical load known, the distance between the resultant of the applied load P and the outside edge (denoted by "**a**", See Fig. 2) can be established. The length of base on which the triangular distribution of soil pressure acts is then **3a**. Equating the resultant of the soil pressure to the applied force gives

$\frac{Pmax}{2}3ab = P$; Solving for P_{max} , gives $P_{max} = \frac{2P}{3ab}$ where $a = \frac{h}{2} - e$

For a square foundation $e = \frac{M}{P}$ and $\frac{h}{2} = \frac{L}{2}$ and b = L where L = the side dimension of the square foundation and M = the total moment applied to the foundation

$$P_{max} = \frac{2P}{3((L/2)-(M/P)) \times L} = \frac{2P}{\frac{(3L^2 P) - (6LM)}{2P}} = \frac{4P^2}{((3L^2 P)-(6LM))}$$

Knowing the soil pressure distribution as shown in Figures 1 or 2 above, the design of the footing is similar to the previous examples.