A SunCam online continuing education course

Basics of Energy, Momentum, and Power for All Engineers

Part 1 – Basics of Energy for All Engineers

by

Bill Wells PE

Part 1 – Basics of Energy for All Engineers

A SunCam online continuing education course

Table of contents

1. Introduction

Energy is basic to all engineering disciplines. Part 1 of this course provides a broad overview of energy concepts and principles. As such, it will be very broad and not too deep. Many engineering disciplines overlap, and we should understand each other's terminology and basic concepts. Part 2 will continue by covering the related subjects of power and momentum. This course is intended for engineers, not physicists. Derivation of equations will only be used where useful.

2. Symbols used in this course

Quantity	Symbol
acceleration, angular	α (alpha)
acceleration, linear	a
distance	s
energy, work	E, W (Joule, ft-lb_f)
energy per unit volume	E_v
energy, kinetic	K
energy, potential	U
fluid density	γ (gamma)
force	F (lb_f, Newton)
gravitational acceleration (Earth)	g_e
mass	m (lb_m, kilogram)
moment of inertia	I
potential energy per unit volume	U_v
power	P
spring constant	k
torque	τ (tau)
velocity	v
velocity, angular	ω (omega)
Volume	V
Weight	W (lb_f)

3. Increasing Importance of energy in today's world

Worldwide sources of energy are changing. Renewable energy production is increasing. Vehicles are changing from internal combustion engines to electric motors. Energy from natural gas is being discouraged. We as engineers are dealing with increasing energy related issues and emerging technologies. We all need to stay proficient with the terms, units, and fundamentals of energy and power.

4. Move to Metric System

In this course, we will use the term **S I** to mean "International System of Units", (French for Système International d'Unités) and not the more familiar Metric System. Likewise, we will use the term **US** to mean the correct "United States Customary System" rather than Imperial System, English System, English Engineering, or other terms.

It is often said that the U.S. is one of the few countries globally which still uses the "Imperial" system of measurement. This is certainly not true, in the U.S. we are very comfortable using SI units in science, manufacturing, medicine, and much of engineering. Conversion is not mandatory in the U.S., and many industries choose not to convert for commercial reasons. US units remain common in many industries such as construction. The International Building Code uses US units almost exclusively. Engineers in North America will need to easily switch between both systems for many years to come.

In 1999, NASA lost a $125 million Mars orbiter satellite because of a mix-up using different measurement units. An investigation indicated that the failure resulted from a navigational error due to commands from Earth being sent in US units (in this case, pound-seconds) without being converted into the SI standard (newton-seconds). There likely have been many more errors caused by a "mix-up" of measurement systems.

When engineers switch back and forth between systems, we need to be knowledgeable about both. The most common source of confusion is regarding units of mass in the US system. In countries using exclusively SI units the basic unit of mass everywhere is the kilogram. Not so in US units, where we deal with lb_f (pounds force), lb_m (pounds mass), and slugs (lb_f/32.2 ft/s^2). Slugs will not be used in this course, and the relationship between calculations using SI units versus US units will be made clear. Content and example problems will use both systems: SI content and examples first, US units second as a comparison.

5. Energy units

Although both SI and US units will be used, there will be a distinction. The U S Customary System was adopted in 1832[(1)], and was based upon the British Imperial System. The British Gravitational System and English Engineering Units System have also been used, and are similar to the Imperial System [(2)]. All these systems of measurement (except SI) have one thing in common: they are *gravitational* systems based upon the standard gravity on Earth of 32.2 ft/s^2. Use of a gravitation-based system gets complicated when used in non-terrestrial applications. Therefore, in this course we will use US units only for Earthly examples.

5.1 Metric, or SI System

The unit for energy in the SI system is the joule. By definition, one joule (J) = one newton-meter (N-m). One joule equals the energy expended by a force of one newton (N) acting over a distance of one meter (m) in the direction of the force. The joule is named for James Prescott Joule (1818 - 1889), who studied the relation between mechanical and heat energy. As with every metric unit named for a person, its symbol uses an upper-case letter (J), but when written in full it is not capitalized.

5.2 U.S. Customary System, or US System

The unit for energy in the US Customary System is foot-pound (ft-lb_f). By definition it is a unit of energy that equals a force of one pound (lb_f) acting through a distance of one foot (ft).

Convenient conversions to know:

- One joule = 0.738 foot-pound (ft-lb_f) (on Earth)
- One foot-pound (ft-lb_f) = 1.36 joule (on Earth)
- One pound mass (lb_m) = one pound force (lb_f) (on Earth)
- One-pound mass (lb_m) = 0.454 kg
- One kg = 2.20 lb_m

6. Energy basics

Energy is defined as the capacity for doing work. In fact, the term *work* is often used in engineering as synonymous with *energy*, and in this course we will use both terms interchangeably. The first law of thermodynamics states that energy cannot be created or destroyed, but can be transferred. Much of this course will cover ways in which energy can be transferred from one form to another. Energy units are scalar, not vector.

$$\mathbf{W = F \cdot s} \qquad \text{(SI) (US)} \qquad \text{Equation 1}$$

Where:

- W is work, energy (ft-lb_f)
- **F** is force (lb_f or newtons)
- **s** is distance, (ft or meters)

Equation 1 describes work from linear movement, using either SI or US units.

Since force is not always aligned with movement, (Figure 1) we can rewrite equation 1 as follows:

$$\mathbf{W = Fs \cos \Theta} \qquad \text{(SI) (US)} \qquad \text{Equation 2}$$

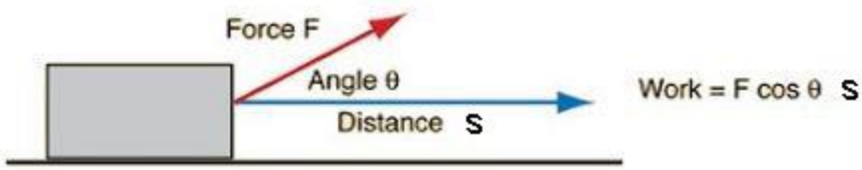

Figure 1 Basic illustration demonstrating concept of energy (work)

The following example demonstrates equation 2.

Example 1

Bob pulls a shipping box by a rope for a distance of 30 meters across the horizontal warehouse. He measures the force required to pull the box, and the scale reads 120 newtons. What work is done if the angle between the rope and the floor is 20 degrees?

Solution:

We use equation 2. Although Bob is pulling with a force of 120 newtons, only the horizontal component does useful work. Is energy conserved? Yes, converted to heat.

$$W = (F)(s)(\cos \Theta) = (120N)(30m)(0.94) = 3385\text{ J}$$

Re-work the example using US units:

$$F = 120n = (120N)\frac{lbf}{4.45N} = 27\ lb_f$$

$$s = (30m)\frac{3.28ft}{m} = 98.4\ ft$$

$$W = (27\ lb_f)\ (98.4\ ft)\ (0.94) = 2497\ ft\text{-}lb_f$$

$(1.36)\ (2497) = 3385\ \mathrm{J}$

If the force varies, we can use integration to find total work, as shown in equation 3. An example of this is a compression spring, which will follow later in this course.

$$\mathbf{W} = \int_{\mathbf{a}}^{\mathbf{b}} \mathbf{f}(\mathbf{x}) \cdot \mathbf{dx} \qquad \text{(SI) (US)} \qquad \text{Equation 3}$$

Work can also be performed from angular movement (rotation).

$$\mathbf{W} = \boldsymbol{\tau} \cdot \boldsymbol{\Theta} \qquad \text{(SI) (US)} \qquad \text{Equation 4}$$

Where:

- **τ** (tau) is torque (meter-newton or lb_f -ft)
- **Θ** (theta) is angular rotation, (radians)

Note:

Θ must be in radians. If you are working with angles measured in degrees, one radian = 57.3 degrees. (There are 2π radians in 360 degrees).

Example 2

Jose the millwright has been asked to remove a frozen hex nut on a boiler flange. He finds an M20 stainless steel nut has galled onto the stainless stud. He uses a ½ meter wrench and needs a continuous pull of 240 newtons to turn the nut. Assume the force acts at right angles to the wrench at a distance of 450 mm from the nut. If 6 full turns are required to remove the nut, how much work is done?

Solution:

We assume Jose pulls with a force of 240 N perpendicular to the handle.

$\tau\ = (240\text{ N})\,(0.45\text{ m}) = 108\text{ m-N}$

$\Theta = 6\text{ full turns} = (6)(2\boldsymbol{\pi}\text{ radians}) = 12\boldsymbol{\pi}\text{ radians}$

$W = (108\text{ m-N})\,(12\,\boldsymbol{\pi}) = 4071\text{ J}$

Using degrees, $\boldsymbol{\Theta} = (6)(360)/57.3 = 37.7\text{ radians}$

$W = (37.7)(108) = 4071\text{ J}$

7. Types of energy

The first law of thermodynamics is the *law of the conservation of energy*, which states that, although energy can change form, it can be neither be created nor destroyed. So, keep in mind that within a closed system, the various types of energy that are discussed below are conserved. A good example is that potential energy can be converted **to** kinetic energy. As engineers we know that in the real world there also will be friction involved.

7.1 Mechanical energy

The three primary types of mechanical energy are potential, kinetic, and elastic.

7.1.1 Mechanical potential energy

Potential Energy is energy stored as the result of its position. It is often gravitational potential energy; raising a mass vertically in a gravitational field from elevation h_1 to elevation h_2. The general expression for gravitational potential energy is equal to the work done against gravity to bring a mass to a given height. If the object is lifted straight up at constant speed, then the force required to lift it is equal to **mg.** Note that the symbol **g** is generic and does not necessarily refer to Earth's gravitational acceleration. When used for Earth, we use symbol $\mathbf{g_e}$ where the gravitational acceleration can be assumed to be constant at about 9.8 m/s^2 (SI) or 32.2 ft/s^2 (US).

$$U_p = m\,g\,h \qquad \text{(SI) (US)} \qquad \text{Equation 5}$$

Where:

- $\mathbf{U_p}$ is potential energy
- **m** is mass (kilograms or lb_m)
- **g** is acceleration due to gravity
- **h** is height the mass is raised from elevation h_1 to h_2 (meters or ft)

In US units, gravitational potential energy often is shown in a "shortcut", because the factor $\mathbf{mg_e}$ equals force, lb_f., which we relate to weight on Earth, W_e.

$$U_p = W_e\,h \qquad \text{(US)} \qquad \text{Equation 6}$$

Where:

- $\mathbf{W_e}$ is weight (on Earth, where acceleration due to gravity, 32.2 ft/s^2)
- **h** is the height **Δh**, the mass is raised from elevation h_1 to elevation h_2 (feet)

Note that W_e, weight on Earth, can only be used for terrestrial calculations where $\mathbf{g_e}$ = 32.2 ft/s^2.

.

Example 3

After setting up a base on Mars, NASA plans to put a satellite into orbit around the planet. They have asked you to estimate the energy required to raise the satellite from the surface of the planet to a stable orbit. Assume the orbit altitude height is 17,000 km, the satellite mass is 6,000 kg, and the gravity of Mars is 3.72 m/s^2.

Solution:

Use equation 5, and tell NASA that you are not a rocket scientist but can give them the minimum theoretical value.

g_m = 3.72 m/s^2

m = 6,000 kg

h = 17,000 km

$$U = \mathrm{mg_m h} = (6000\ \mathrm{kg})\left(3.72\frac{\mathrm{m}}{\mathrm{s}^2}\right)(17{,}000{,}000\ \mathrm{m}) = 3.79 \text{ x } 10^{11}\ \mathrm{J}$$

Example 4

Mary raises a bowling ball from the floor (h_1) up to a storage shelf (h_2). When she bought the ball, she was told it was a “14-pound ball”, but now she is in a locality where units are SI. The shelf is 1.5 meters above the floor. What is the bowling ball’s increase in potential energy?

Solution: use equation 5

This is a typical mixed-unit problem. Since we are dealing with mass (the bowling ball) it is usually best to convert everything to SI units.

Convert bowling ball mass (14 lb_m) to SI: m = (14 lb_m) (1 kg/2.20 lb_m) = 6.35 kg

h=1.5m

g_e=9.8 m/s^2

$$U_p = mgh = (6.35\ kg)\ (9.8\ m/s^2)\ (1.5\ m) = 93.4\ J$$

7.1.2 Mechanical kinetic energy

Kinetic energy is energy that an object has due to its motion, which may be linear or angular. If work is done on the object by applying a net force or torque, the object's velocity or RPM increases, thereby gaining energy. In all kinetic energy cases, energy is proportional to the square of velocity, both linear and angular.

7.1.2.1 Linear (translational) kinetic energy

$$K_L = \frac{1}{2}mv^2 \qquad \text{(SI)} \qquad \text{Equation 7}$$

Where:

- **K_L** is linear kinetic energy (J)
- **m** is mass (kg)
- **v** is velocity (m/s)

In US units, when using typical units of mass (lb_m), the equation for linear kinetic energy includes the factor **g_e** (32.2 ft/s^2).

$$K_L = \frac{mv^2}{2g_e} \qquad \text{(US)} \qquad \text{Equation 8}$$

Where:

- **K_L** is linear (translational) kinetic energy, ft-lb_f
- **m** is mass (lb_m)
- **v** is velocity (ft/s)
- **g_e** is 32.2 (ft/s^2)

We will calculate the following example problem in both SI and US, units to demonstrate equations 7 and 8.

Example 5

A 1500 kg car travels down the road at 100 km/h. How much kinetic energy does the car have?

Solution: use equation 7

v = 100 km/h = 27.8 m/s

m = 1500 kg

$$K_L = \frac{mv^2}{2} = \frac{(1500)(27.8)^2}{2} = 580{,}000 \text{ J}$$

Now we solve the same problem using US units.

Solution: use equation 8

v = 100 km/h = 91.1 ft/s

m = 1500 kg = 3307 lb_m

$$K_L = \frac{mv^2}{2g} = \frac{(3307)(91.1)^2}{(2)(32.2)} = 426{,}200 \text{ ft-lb}_f = (426{,}200)\left(\frac{1.36\text{J}}{\text{ft}-\text{lb}_f}\right) = 580{,}000 \text{ J}$$

7.1.2.2 Angular (rotational) kinetic energy

$$\mathbf{K_r = \frac{1}{2} I\omega^2} \qquad \text{(SI)} \qquad \text{Equation 9}$$

Where:

- $\mathbf{I}$ is angular moment of inertia, kg-m^2
- $\boldsymbol{\omega}$ is angular velocity in radians/s
- $\mathbf{K_r}$ is angular kinetic energy, joules

Example 6

A grindstone with angular moment of inertia 1600 kg-m^2 is rotating at 60 RPM. What is the grindstone's angular kinetic energy?

Solution: use equation 9.

Convert RPM to radians/s: one RPM = 0.105 radian per second

(60 RPM) (0.105) = 6. 3 radians/sec

Use equation 9

Moment of inertia, I = 1600 kg-m^2

Angular velocity, w = 6.3 radians/s

$$K_r = \frac{1}{2} I\omega^2 = (\frac{1}{2})\ (1600)\ (6.3)^2 = 31{,}750 \text{ J}$$

In US units, when using typical units of mass (lb$_m$), the equation for angular kinetic energy includes the factor $\mathbf{g_e}$ (32.2 ft/s^2).

$$\mathbf{K_r = \frac{I\omega^2}{2g_e}} \qquad \text{(US)} \qquad \text{Equation 10}$$

Where:

- $\mathbf{I}$ is angular moment of inertia, lb$_m$·ft^2
- $\boldsymbol{\omega}$ is angular velocity in radians/s
- $\mathbf{K_r}$ is angular kinetic energy, ft-lb$_f$

Example 7

Now we solve the grindstone example using US units.

Solution: use equation 10

Convert RPM to radians/s: one RPM = 0.105 radian per second

(60 RPM) (0.105) = 6. 3 radians/sec

Moment of inertia, I = 1600 kg-m^2 = 37,968 lb_m-s^2

$K_r = I\omega^2 / 2g_e$ = (37,968) (6.3)^2 / (2) (32.2) = 23,400 ft-lb_f

$K_r = \left(\frac{1.36\text{J}}{\text{ft}-\text{lb}_\text{f}}\right)$ (23,400) = 31,800 J

7.1.3 Putting it all together

Equations 5 (potential energy), 7 (translational kinetic energy), and 9 (angular kinetic energy) can be combined into one equation describing total mechanical energy:

$$\mathbf{mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2} \qquad \text{(SI)} \quad \text{Equation 11}$$

Example 8

A 20 kg, 0.5 m diameter, wooden disk is placed on a horizontal surface 12 m above the ground. The disk then rolls without slipping down a ramp inclined at an angle **ϴ** = 23°. What is its velocity v when it reaches the bottom of the ramp?

Solution: use equation 11

For this problem we disregard slope angle. To solve for velocity, the angle of incline is used only where friction is involved, such as a block sliding on a surface with friction. Since the disc is rolling without slipping; friction is negligible and the angle can be disregarded. The velocity at the base of the ramp will be the same regardless of the angle. The time required will change with slope angle, but not velocity.

As the disk rolls down the ramp, it gains both angular and translational kinetic energy. This energy is being transferred from the disk's potential energy **mgh**. Since the disk is not slipping, its angular velocity **ω** and its translational velocity **v** are both increasing as the disk rolls down the slope. These two velocities are not independent, and are related by the equation **v = r ω**, where **r** = radius of the disk and **ω** is angular velocity. This will simplify equation 11 and we can solve for **v**. We also need to know that for a disk or a cylinder, $\mathbf{I = \frac{1}{2} mr^2}$.

Substituting **v/r** for **ω**, and using $\mathbf{\frac{1}{2} mr^2}$ in place of **I,** we can write:

$$\mathrm{mgh} = \frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}\mathrm{I}\omega^2 = \frac{1}{2}\mathrm{mv}^2 + \left(\frac{1}{2}\mathrm{mr}^2\right)\left(\frac{1}{2}\left(\frac{v}{r}\right)^2\right)$$

$$\mathrm{mgh} = \frac{1}{2}\mathrm{mv}^2 + \left(\frac{1}{2}\mathrm{mr}^2\right)\left(\frac{1}{2}\left(\frac{v}{r}\right)^2\right) = \frac{1}{2}\mathrm{mv}^2 + \frac{1}{4}\mathrm{mv}^2 = \frac{3}{4}\mathrm{mv}^2$$

$$\cancel{\mathrm{m}}\mathrm{gh} = \frac{3}{4}\cancel{\mathrm{m}}\mathrm{v}^2 \qquad \mathrm{v}^2 = \frac{4}{3}\mathrm{gh} \qquad \mathrm{v} = \sqrt{\frac{4}{3}gh} \qquad \text{where: g= 9.8, h = 12}$$

v = 12.5 m/s

7.1.4 Mechanical elastic potential energy

Elastic potential energy is energy stored as a result of deformation of an elastic object, such as the stretching or compressing a spring. It is equal to the work done to stretch or compress the spring, which depends upon the spring constant **k** as well as the distance stretched. The force required to stretch or compress the spring is directly proportional to the amount of stretch**: F = kx**.

$$E_s = \frac{1}{2}\,kx^2 \qquad \text{(SI) (US)} \qquad \text{Equation 12}$$

Where:

- $\mathbf{E_s}$ is potential energy of spring

- **k** is the spring constant
- **x** is spring displacement (**Δs**)

Example 9

A machine pulls a spring with a spring constant k = 250 N/m, stretching it from its rest length of 0.10 m to 0.20 m. What is the elastic potential energy stored in the spring?

Solution: Use equation 12

$E_s = ½\ kx^2$

Displacement: $x_1 = 0.20$ m to $x_2 = 0.10$ m

x= 0.10 m

k = 250 N/m

$E_s = (1/2)\ (250\ N/m)\ (.10m)^2 = 1.25\ J$

7.2 Thermal energy

Thermal energy, also called heat energy, is a type of internal energy an object possesses owing to the kinetic energy of its constituent particles. In 1843 James Joule demonstrated the mechanical equivalent of heat in a series of experiments. In his most famous experiment, he made a device to determine the mechanical equivalent of heat. (See Figure 2). In this experiment, he compared the change in gravitational potential energy to the increase in temperature of water. This is an example of the law of conservation of energy, the first law of thermodynamics.

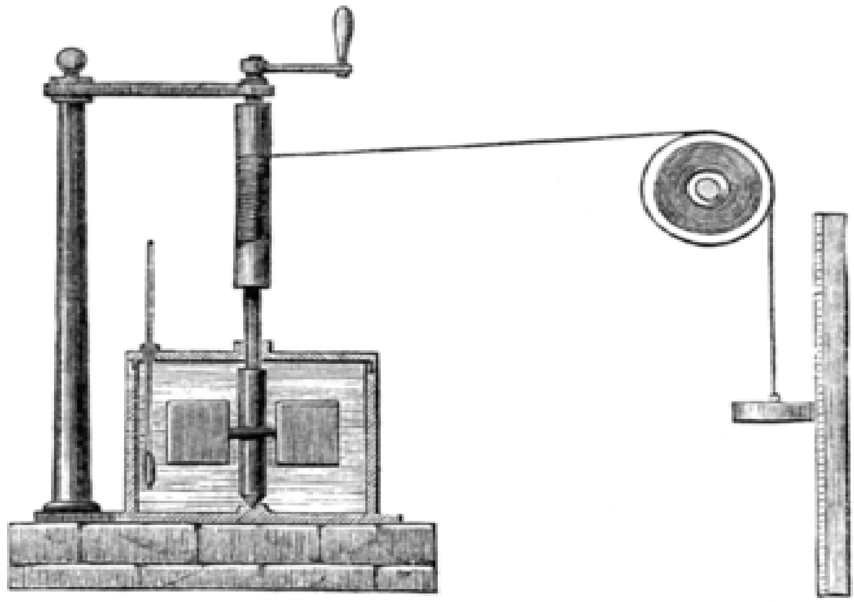

Figure 2 [3] The equivalence of heat and mechanical energy was by James Joule in 1843, who compared the change in potential energy to the increase in temperature of water.

Example 10

Let's say you want to re-create Joule's demonstration shown in Figure 2. The mass is 15 kg, drops 2 meters, and is quickly repeated 20 times. There is one liter of water in the insulated chamber. What is the theoretical temperature increase of the water?

Solution:

Potential energy released from falling mass = mgh = (15) (9.8) (2) (20) =5880 J. Water has a specific heat capacity of 4182 J/kg°C, meaning 4182 J will raise one litter (one kilogram) of water one-degree C. So, at 100% efficiency the water temperature would increase by 5880/4182 = 1.4 °C.

Note: For an interesting account of Joule's experiments, see https://royalsocietypublishing.org/doi/10.1098/rsta.2014.0348

Joule was able to obtain good accuracy with this particular experiment, an average value of 772.692 ft-lb$_f$/Btu, compared with the actual value of 778 ft-lb$_f$/Btu.

(Courtesy of The Royal Society Publishing) [(4)]

Convenient thermal energy conversions to know:

- 1 British Thermal Unit (Btu) = 1055 J
- 1 British Thermal Unit (Btu) = 252 calories
- 1 British Thermal Unit (Btu) = 0.000293 kWh

7.3 Radiation energy

Radiant energy is energy resulting from electromagnetic radiation, usually observed as it radiates from a source into the surrounding environment. Sometimes this energy is incorrectly referred to as "infra-red" radiation, but radiant energy is emitted from the entire electromagnetic spectrum, not just the infrared portion. The most obvious source of radiant energy is the sun. The amount of this energy which hits Earth has been measured and even though it varies is called the Solar Constant, with a value of 1368 watts/m^2. For practical purposes, the approximate value of solar radiation reaching Earth's surface is valued at a maximum of 1,000 watts/m^2. This reduced value is primarily due to atmospheric interference. These values are for radiation energy received from the Sun per second per m^2 area on a surface perpendicular to the Sun's rays. Stefan Boltzmann law states that the total amount of radiation energy emitted from the surface is directly proportional to the fourth power of its absolute temperature.

$$\mathbf{P = e\sigma A\, T^{4}} \qquad \text{(SI)} \qquad \text{Equation 13}$$

Where:

- **P** is Power radiated joules/meter^2 /s
- **σ** is Stefan-Boltzmann constant is 5.67x10^{-8} watts/meter^2
- **T** is temperature of hot item, degrees Kelvin
- **e** is emissivity
- **A** is area in m^2

Note that equation 13 is in terms of *power*, (joules/second), not *energy* (joules). To calculate total energy radiated, multiply by time in seconds.

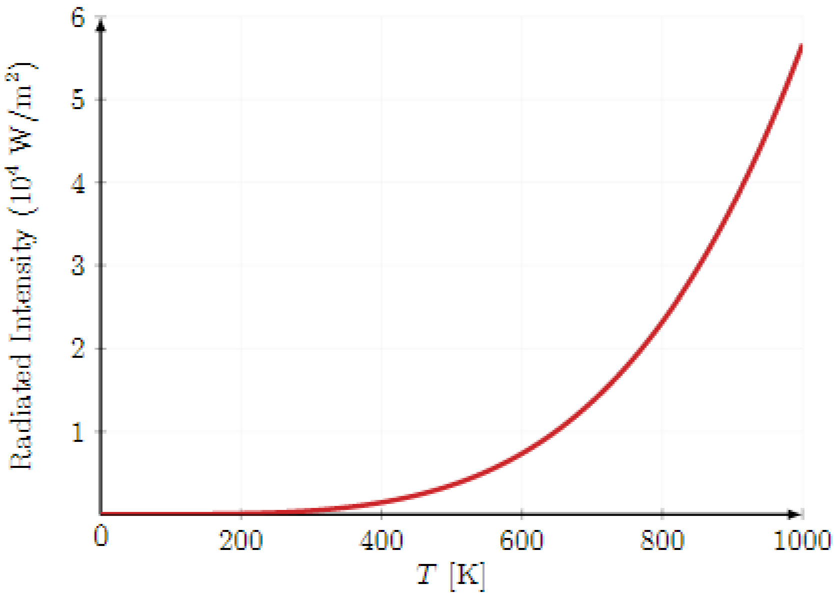

Figure 3 The radiation energy emitted per unit time per unit surface is proportional to the fourth power of its absolute temperature in degrees Kelvin

Example 11

A steel cube of emissivity e = 0.75 has a surface area of 2.5 m^2 and temperature 227ºC. Using the Stefan-Boltzmann law, calculate the value of net power emitted by the body.

Convert temperature in Celsius to Kelvin: T = 227 + 273 = 500° K

Area = 2.5 m^2

Emissivity (e) = 0.75

$P = e\sigma A (T^4)$

$P = (0.75)\ (5.67 \times 10^{-8})\ (2.5)\ (500)^4 = 2313\ J/m^2/s$
$(2313\ J/m^2/s)\ (2.5\ m^2) = 6645\ J/s = 6645$ Watts

7.4. Hydraulic energy

Fluid energy can be either hydraulic (liquid) or pneumatic (gasses). In this course we will consider only hydraulic energy, which can exist as either potential or kinetic energy.

7.4.1 Hydraulic potential energy

Potential Energy of a liquid

The potential energy of a liquid is the energy the liquid obtains as a result of being at some elevation. The "head" difference (**Δh**) of a liquid is what results in potential energy. Following is the equation for hydraulic energy *per unit volume*:

$$U_V = \rho g_e h \qquad \text{(SI)} \qquad \text{Equation 14}$$

Where:

- $\mathbf{U_v}$ is the potential energy of the liquid per unit volume (joules/m^3)
- **ρ** (rho) is the density of the liquid in kilograms per cubic meter (kg/m^3)
- $\mathbf{g_e}$ is the acceleration due to gravity on Earth (9.81 m/s^2)
- **h** is height difference **Δh** of the liquid between elevation h_1 to h_2 (m)

The relationship for the potential energy per unit volume of liquid is thus proportional to Δh and density ρ.

To obtain *total* potential energy we multiply by volume (V). Height, h, is to the center of mass of the liquid.

$$U_t = \rho g_e h V \qquad \text{(SI)} \qquad \text{Equation 15}$$

Where:

- $\mathbf{U_t}$ is total potential energy of a volume of liquid, joules
- $\boldsymbol{\rho}$ is density of the of the liquid, kg/m^3
- g_e is acceleration due to gravity on Earth (9.81 m/s^2)
- **h** is height difference Δh of the liquid between elevation h_1 to h_2 (m)
- **V** is volume of liquid, m^3

Note that equation 14 and 15 apply only to terrestrial applications, thus include g_e, acceleration of gravity on Earth. In the unlikely event that you are calculating potential energy of liquid on another planet, that planet's acceleration of gravity must be used.

Example 12

Calculate total potential energy of water in an elevated tank. Center of gravity of the water is 30 meters above ground level, and the tank holds 2,000,000 liters of water.

Use equation 15

Total potential energy = $\rho g_e hV$.

V = 2,000,000/1000 = 2000 m^3

ρ = 1000 kg/m^3

h = 30 m (Δh measured from ground to center of gravity of elevated water)

g_e is acceleration due to gravity on Earth (9.81 m/s^2)

$U_t = \rho ghV$ = (1000 kg/m^3) (9.8m/s^2) (30m) (2000 m^3) = 5.9 x 10^8 joules

In US units, total potential energy for a volume of liquid at an elevation **h** above datum is often given in term of total weight:

$$U_t = Wh \qquad \text{(US)} \qquad \text{Equation 16}$$

Where:

- $\mathbf{U_t}$ is total potential energy, ft-lb_f
- **W** is weight of the body of liquid, lb_f
- **h** is height (or change in height, **Δh**) of the liquid in feet

Example 13

We will re-calculate example 12 using US units

Use equation 16

Total potential energy = Wh

V = 2000 m^3 = 70,630 ft^3

W (weight) = (70,630 ft^3) (62.4 lbf/ft^3) = 4,407,000 lb_f

h = 98.4 ft,

U_t = (4,407,000lbf) (98.4) = 433,679,500 ft-lbf = 5.9 x 10^8 joules

7.4.2 Hydraulic kinetic energy

Energy is needed to accelerate a stationary body. Therefore, a moving mass of liquid possesses more energy than an identical, stationary mass. The energy difference is the kinetic energy of the fluid.

$$E_v = \frac{1}{2}\rho v^2 \qquad \text{(SI)} \qquad \text{Equation 17}$$

Where:

- **E_V** is kinetic energy per cubic meter of liquid, joules/m^3
- **ρ** is density of the liquid, kg/m^3
- **v** is velocity (or change in velocity, **Δv**), m/s.

7.4.3 Hydraulic pressure energy

Pressure energy is the energy stored in a liquid due to the force per unit area applied.

$$E_v = P \qquad \text{(SI)} \qquad \text{Equation 18}$$

Where:

- **E_v** is energy of a volume of liquid, joules/m^3
- **P** is pressure of the liquid, kg/m^2

Looking at equation 18, how can pressure be a unit of energy? Equation 18 is pressure in terms of newtons/m^2. To obtain *total* pressure energy, we must multiply by volume of liquid, so E_t = (kg/m^2) (m^3) = kg-m = joules.

7.4.4 Bernoulli equation

Bernoulli's equation combines the above three types of hydraulic energy: potential, kinetic, and pressure. It is a statement of the conservation of energy. The equation reflects the idea that energy is conserved along a streamline because the three terms of the equation can be thought of as representing the pressure energy, kinetic energy and potential energy of the fluid. Bernoulli's equation states that the overall sum of these energies doesn't change along a streamline – the energy of the fluid is just transferring between these different forms. If the elevation of the fluid increases, for example, the pressure or the velocity must decrease in proportion. This equation assumes no friction loss, which can be significant in piping systems.

Daniel Bernoulli presented his important theory in 1738.

$$\mathbf{P_1 + \frac{1}{2}\rho v_1^2 + \rho g_e h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g_e h_2} \qquad \text{(SI)} \qquad \text{Equation 19}$$

Where:

- **P** is pressure, newtons/m^2
- **ρ** (rho) is density of the liquid, kg/m^3
- **v** is liquid velocity, m/s
- **h** is elevation above a reference elevation, m
- $\mathbf{g_e}$ is acceleration due to gravity on Earth (9.81 m/s^2)

Example 14

Water is flowing in a fire hose with a velocity of 1.0 m/s and a pressure of 200,000 newtons/m^2 at the nozzle. Discharging from the nozzle the pressure decreases to atmospheric pressure (10,130 newtons/m^2), and there is no change in height. Use equation 19 to calculate the velocity of the water exiting the nozzle.

Solution: use equation 19

ρ =1000 kg/m^3

v_1 = 1.0 m/s

P_1 =200,000 newtons/m^2

P_2 = 10,1300 newtons/m^2

Since there is no elevation change, we can eliminate the potential energy term (ρgh).

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

200,000 + ½ (1000) (1.0)^2= 101,300 + ½ (1000) (v_2)^2

200,000 + 500 =101,300 + 500v_2^2

v = 14.1 m/s

8. The most famous energy equation

$$E = mc^2 \qquad \text{(SI)} \qquad \text{Equation 20}$$

Where:

- **E** is energy, joules
- **m** = mass, kilograms
- **c** = speed of light, 3×10^8 m/s

Example 15

The mass of a pencil is 12 grams. Calculate energy in joules if the pencil were totally converted to energy.

Solution:

m = 12 grams = 0.012 kg

$E = mc^2 \qquad E = (0.012)\,(3 \times 10^8)^2 \quad = 1.08 \times 10^{15}$ J

9. Industry standards, conversion factors, and online tools

> *"For those who want some proof that physicists are human, the proof is in the idiocy of all the different units which they use for measuring energy."*
> Richard P. Feynman

Physicists are not alone in this; engineers also have developed idiosyncratic units, so have many industries. Following are some examples.

Btu (British thermal unit)

The British thermal unit (Btu) is a traditional Imperial and US unit of energy. It is the amount of energy needed to cool or heat one pound of water by one-degree Fahrenheit. To raise one pound of water by one-degree Fahrenheit requires one Btu of heat, equivalent to 1055 joules.

Engineers accustomed to using the Btu find valuable to use this measurement of heat. It is very intuitive, easy to visualize. Its counterpart in SI is the calorie, which is defined as the amount of heat required to raise the temperature of one gram of water by one degree Celsius. Do not confuse the standard SI unit of calorie (cal) with kilocalorie (kcal or Calorie). The kilocalorie is an unofficial unit used mainly by the food industry.

Natural gas industry

Natural gas energy is sold either in units of energy or by volume. A common unit for selling by energy content is by therm. One therm is equal to about 1.05×10^8 joules. When selling natural gas by volume, units are cubic feet or cubic meter. In the US it is common to measure natural gas in units of 100 cubic feet, or ccf. We know, of course, that this unit, ccf, is not a unit of energy. So, the gas utility multiplies ccf usage by what is known as the Btu factor, which is Btu content per ccf. The primary constituent of natural gas is methane, which has a heat content of about 1,010 Btu/cubic foot. The gas utility measures the actual energy content of their supply and assigns a Btu factor. This converts your ccf energy usage into therms by adjusting it for actual heat content.

Food industry

We know that the food industry uses the unit Calorie to measure energy content of their products. However, in the food industry Calorie (with a capital C) is actually a kilocalorie (1000 calories).

The "small" calorie is defined as the amount of energy needed to increase the temperature of 1 gram of water by 1 °C. The “large” Calorie is therefore the amount of energy needed to increase the temperature of 1 kilogram of water by 1 °C.

Refrigeration industry.

In the U.S., one ton on of refrigeration is a unit of cooling capacity, or heat removal capacity. One ton of refrigeration is the heat required to convert 1 ton (US short ton of

2,000 lb. or 910 kg) of water into ice at the same temperature. So, it is the latent heat of ice (heat required to convert water into ice at the same temp.)

The British thermal unit (Btu) is commonly used in the power, steam generation, heating and air-conditioning industries. Although it is still used 'unofficially' in English-speaking countries (such as USA, Canada, UK), its use has declined or has been replaced in other parts of the world. The capacity of small cooling systems and heat pumps is often given in Btus – meaning the number of Btus per hour, i.e., Btu/h.

A few useful conversion factors for measurement of energy.

Unit	Symbol	Equivalent in joules	Other units
Kilowatt hour	kWh	3,600,000	3413 Btu
Calorie	cal	4.186	0.004 Btu
Kilocalorie (food Calorie)	kcal	4186	1000 cal
British Thermal unit	Btu	1055	252 cal
Horsepower-hour	HP-hour	2,684,520	0.75 kWh
Therm		1.05×10^8	100,000 Btu
Ton of refrigeration	Ton	3.08×10^8	288,000 Btu
Watt-hour	Wh	3600	3.41 Btu

Online conversion between units.

There is no longer any need for extensive conversion tables to manually convert energy (or any other) units. There are now many websites available to do this, often more reliable than crunching the numbers yourself. Which of the many sites is best for engineering use? The author's favorite is simply named "Convert", available at joshmadison.com or search online for Convert for Windows. (5)

Online calculators

There are also many websites available to perform our calculations. Most are aimed at students. Many are simply not suited for engineering use. The author uses Omni Calculator (6), which features many engineering and physics calculators.

10. Trends in production and consumption of energy

All charts courtesy of U.S. Energy Information Administration (7)

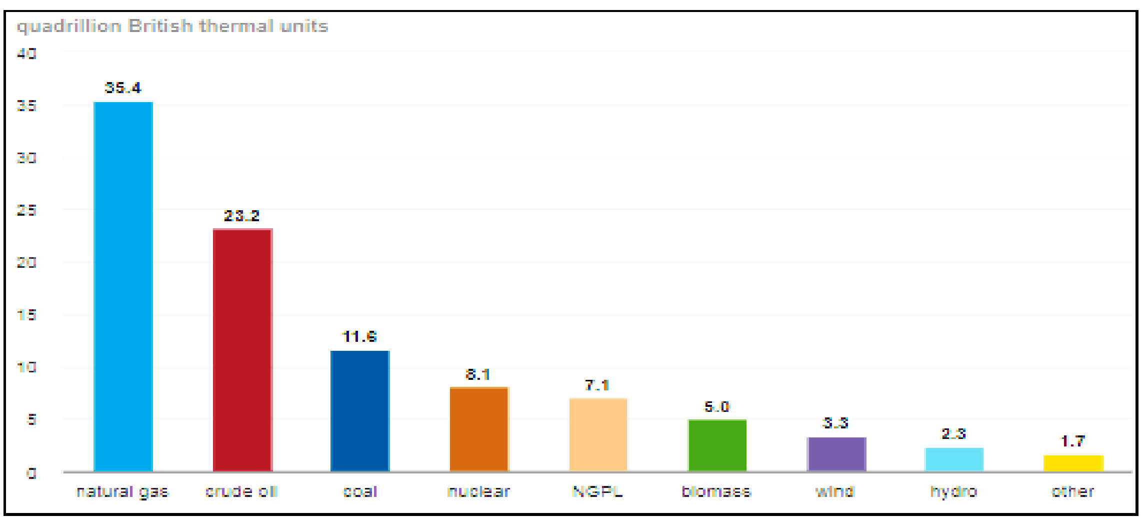

Figure 4 U.S. primary energy production by major source, 2021

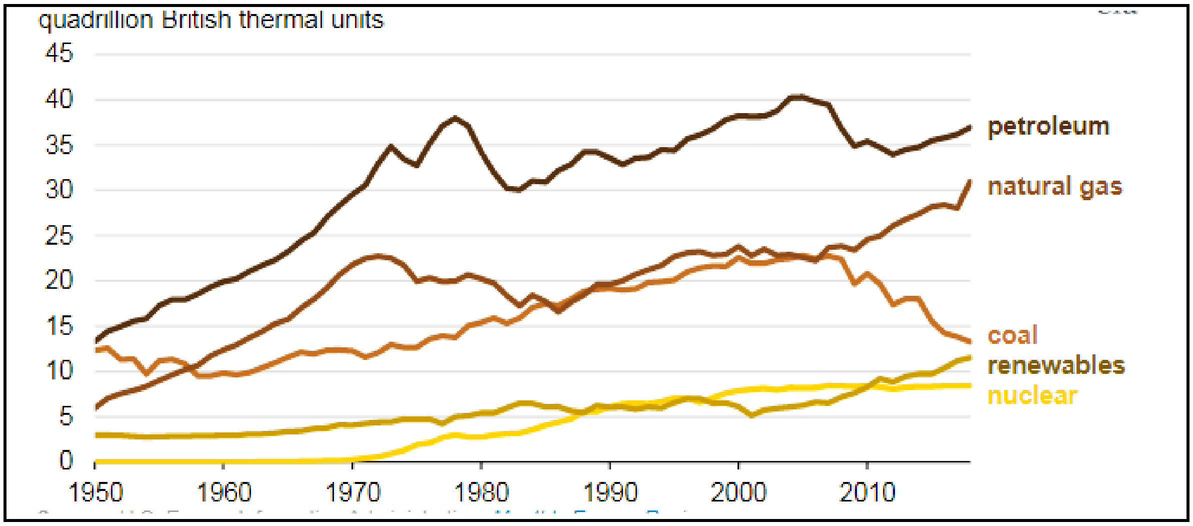

Figure 5 U.S. total energy consumption (1950 - 2018)

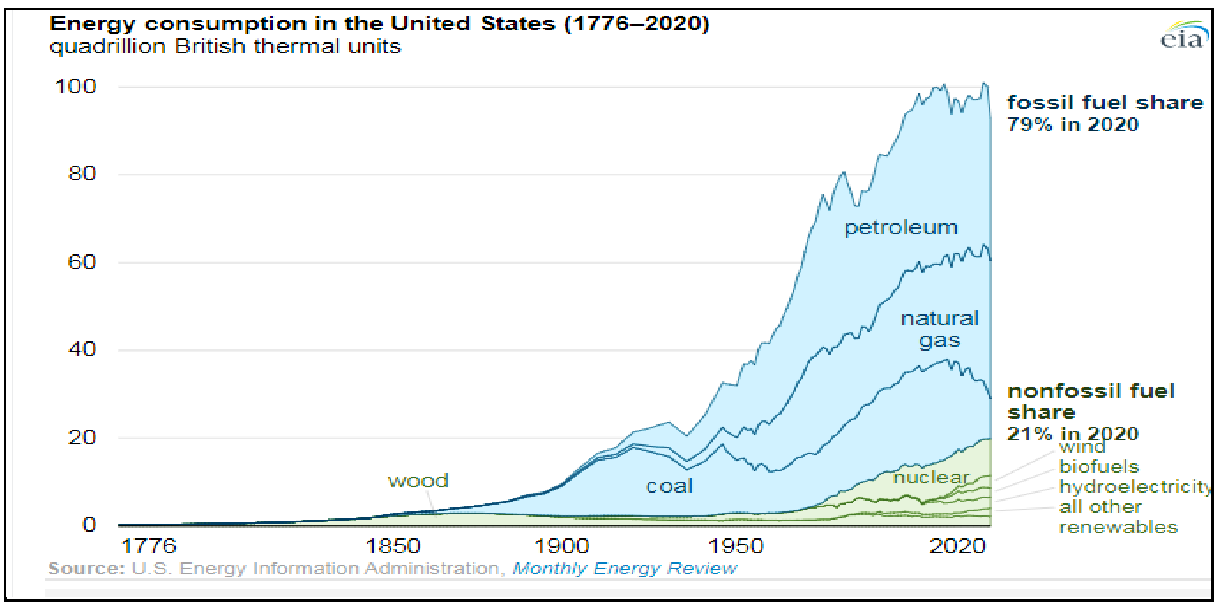

Figure 6 Energy consumption in the U.S. (1776-2020)

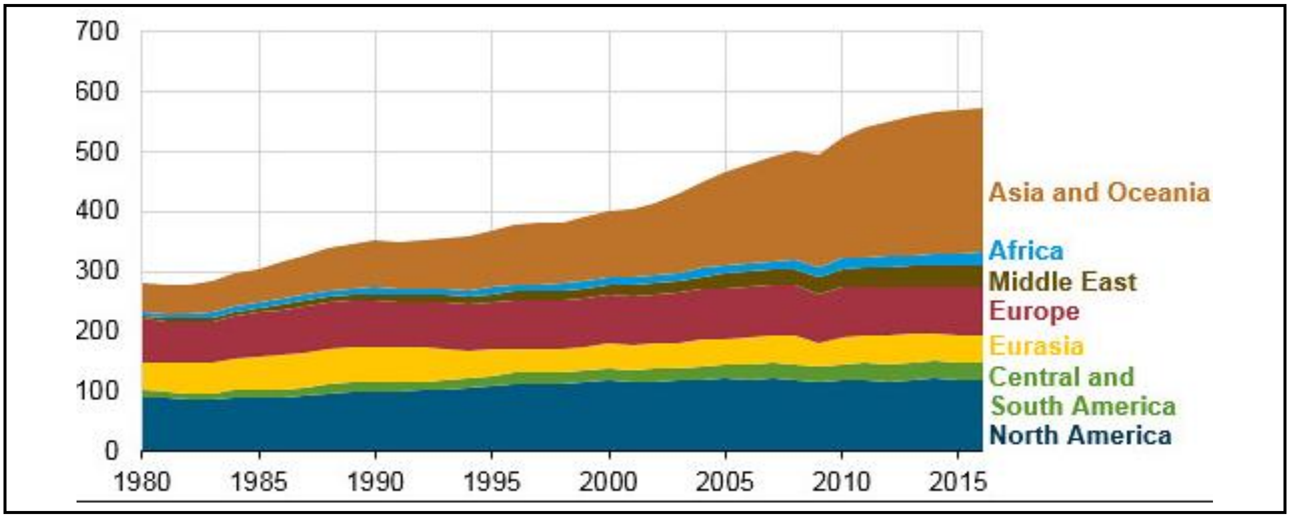

Figure 7 Energy use by world region (1980 - 2016)

11. Conclusion

It has been difficult to cover the vast subject of energy in a relatively brief course. I hope it has summarized many of the key concepts and achieved its objective of providing a conceptual understanding of energy as it applies to engineering. My engineering education sixty years ago was taught entirely using the US System of Units. I tried to write this course in a way that explained the subject in terms of both SI units and US units. I sincerely hope this course has been helpful. Feel free to contact me with questions or corrections. My contact information can be found on my author's page.

Bill Wells PE

Olympia WA

12. References used

1. Barbrow, L.E. and Judson, L. V. (1976) *Weights and Measures of the United States*. National Bureau of Standards Special Publication 447. p. 5–6
2. Comings, E. W. (1940). "English Engineering Units and Their Dimensions". Ind. Eng. Chem. 32
3. Wikimedia Commons [Online], Available
4. Royal Society, UK
5. “Convert is free to use and distribute”.
6. https://www.omnicalculator.com
7. https://www.eia.gov/

