A SunCam online continuing education course

Basics of Energy, Momentum, and Power for All Engineers

Part 2 – Basics of Mechanical Power and Momentum

by

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1. Introduction

The concept of power is basic to all engineering disciplines. Part 2 of this course provides a broad overview of the concepts and principles of generating and using mechanical power. Also included is a review of linear and angular momentum. For those who studied Part 1, *Basics of Energy*, you will see a few important concepts repeated. These fundamental concepts are required to provide a coherent introduction to the study of power.

This course is for engineers. Derivation of equations will be used only where useful.

2. Symbols used in this course

Quantity	Symbol
acceleration, angular (rotational)	α (alpha)
acceleration, linear	a
distance	s (meter, m; foot, ft)
energy, work	E, W (joule, ft-lb_f)
energy, kinetic	K (joule, ft-lb_f)
fluid density	γ (gamma)
force	F (lb_f, newton)
gravitational acceleration (Earth)	g_e
mass	lb_m, kilogram
moment of inertia	I ($lb_f \cdot ft \cdot s^2$, $kg \cdot m^2$)
power	P (watts, ft-lb_f/s)
time	t, s, (second)
torque	τ (tau)
velocity, linear	v
velocity, angular	ω (omega)
linear momentum	p
angular momentum	L

3. Increasing Importance of Power and Energy

Worldwide sources of energy are changing. Renewable energy production is increasing. Vehicles are changing from internal combustion engines to electric motors. We as engineers are dealing with increasing power related issues and emerging technologies. We need to stay proficient with the terms, units, and fundamentals of energy and power.

4. Move to Metric System

In this course, we will use the abbreviation SI to mean “International System of Units”, (French for Système International d'Unités) and not the more familiar name “Metric System”. Likewise, we will use the abbreviation US to mean the correct “United States Customary System” rather than the commonly used name “Imperial System”. (1)

It is often said that the U.S. is one of the few countries globally which still uses the “Imperial” system of measurement. This is certainly not true, in the U.S. we are very comfortable using SI units in science, manufacturing, medicine, and much of engineering. Conversion is not mandatory in the U.S., and many industries choose not to convert for commercial reasons. US units remain common in many industries such as construction. The International Building Code uses US units almost exclusively. Engineers in North America will need to easily switch between both systems for many years to come.

In 1999, NASA lost a $125 million Mars orbiter satellite because of a mix-up using different measurement units. An investigation indicated that the failure resulted from a navigational error due to commands from Earth being sent in US units (in this case, pound-seconds) without being converted into the SI standard (newton-seconds). There likely have been many more errors caused by a “mix-up” of measurement systems.

When engineers switch back and forth between systems, we need to be knowledgeable about both. The most common source of confusion is regarding units of mass (lb_m) and force (lb_f) in the US system. In countries using exclusively SI units the basic unit of mass is the kilogram. In the US system, we deal with lb_f (pound force), lb_m (pound mass), and slugs (lb_f/32.2 ft/s^2). Slugs will not be used in this course, and the relationship between calculations using SI units versus US units will be made clear. Content and example problems will use both systems.

5. Units for Energy and Power

Both SI and US units will be used in this course. The US Conventional System was adopted in 1832[(1)], and was based upon the British Imperial System. The British Gravitational System and English Engineering Units System are similar to the Imperial System [(3)]. All these systems of measurement (including the current US but not the SI system) have one thing in common: they are *gravitational* systems based upon the standard gravity on Earth of 32.2 ft/s^2. Use of a gravitation-based system gets complicated when used in non-terrestrial applications. Therefore, in this course we will use US units only for Earthly examples.

5.1 Metric, or SI System

The unit for energy in the SI system is the joule. By definition, one joule (J) = one newton-meter (N-m). One joule equals the energy expended by a force of one newton (N) acting over a distance of one meter (m) in the direction of the force. The joule is named for James Prescott Joule (1818 – 1889).

The basic unit for power in the SI system is the watt. Power is defined as work done per unit time; P = $\frac{\boldsymbol{energy}}{\boldsymbol{time}} = \frac{\boldsymbol{joules}}{\boldsymbol{seconds}}$ = watts. One kilowatt (kW) = $1000\,\frac{\boldsymbol{joules}}{\boldsymbol{second}} =$ 1000 W. The watt is named for James Watt (1736 – 1819).

As with every metric unit named for a person, the symbols for energy (J) and power (W) use upper-case letters but when written in full are not capitalized.

5.2 U.S. Customary System

The unit for energy in the US System is foot-pound (ft-lb_f). It is a unit of energy that equals a force of one pound (lb_f) acting through a distance of one foot (ft). [(2)] The term *work* is often used in engineering as synonymous with *energy,* and in this course, we will use the terms *work* and *energy* interchangeably. They are exactly the same.

The unit for power in the U.S. Customary System is $\frac{ft-lbf}{s}$. A more common unit of power is the familiar unit of mechanical horsepower, or simply horsepower (HP). One horsepower is defined as the energy required to raise 550 pounds one foot in one second, so one HP = $\frac{550\ ft-lbf}{s}$. And yes, the unit of horsepower is indeed based upon gravitational acceleration on Earth of 32.2 ft/s^2.

Convenient energy and power conversions to know:

- One joule = 0.738 foot-pound energy (ft-lb_f) (on Earth)
- One foot-pound energy (ft-lb_f) = 1.36 joule (on Earth)
- One pound mass (lb_m) = 0.4536 kg (by definition, same everywhere)
- One kg = 2.2046 pound mass (lb_m) (by definition, same everywhere)
- One horsepower (HP) = 746 W = 0.746 kW
- One kW = 1.34 HP

In the above list, why the qualifier "on Earth"? This is because in the US system, the unit for pound-force, lb_f, is based upon the acceleration of gravity on Earth of 32.2 ft/s^2.

6. Power Basics

Power is defined as the rate at which work is produced or expended; it is the $work/time$ ratio. Power units are scalar, not vector.

$$\mathbf{P_{avg} = W/\Delta t} \quad = \qquad \text{(SI) (US)} \qquad\qquad \text{Equation 1}$$

Where:

- **P** is power, (watts or ft-lbf/s)
- **W** is total work, energy, (ft-lb_f or joules)
- **Δt** is time in seconds

If a constant force F is applied throughout a distance s, the work $\mathrm{W} = \mathrm{F} \cdot s$. Since F=ma, Equation 1 can be rewritten as: $P = \frac{\boldsymbol{mas}}{\boldsymbol{\Delta t}}$

Therefore, Equation 1 can be re-written:

$$P = \frac{W}{\Delta t} = \frac{d}{\Delta t}(F \cdot s) = F \cdot \frac{\Delta s}{\Delta t} = \mathrm{F} \cdot v$$

So now we have another useful Equation for power:

$$\mathbf{P} = \mathbf{F} \cdot \mathbf{v} \qquad \text{(SI) (US)} \qquad \text{Equation 2}$$

Where:

- **P** is power, (watts or ft-lb_f/s)
- **F** is force (N or lb_f)
- **v** is velocity (m/s or ft/s)
- **m** is mass, (kg or lb_m)
- **s** is distance (m or ft)

Since force is not always aligned with movement, (see Figure 1 below) we can rewrite Equation 2 as follows:

$$\mathbf{P} = \mathbf{Fv}\cos\Theta \qquad \text{(SI) (US)} \qquad \text{Equation 3}$$

Where **Θ** is the angle between F and v, and F is a constant force.

Note that Equation 1 represents *average* power due to the total work W expended over time duration Δt. That means the rate of work can vary over time duration **Δt**; in that case, work could be done by a variable force. Equations 2 and 3 represent instataneous power. The difference between average and instantaneous power is important. For example, where a drive system is being designed for an application with instances of high power demand. In that case, it is important to evaluate $\boldsymbol{P} = \boldsymbol{F} \cdot \boldsymbol{v}$ for the situations where $\boldsymbol{F}$ is maximum. If $\boldsymbol{F}$ is constant, Equations 1, 2, and 3 yield the same result.

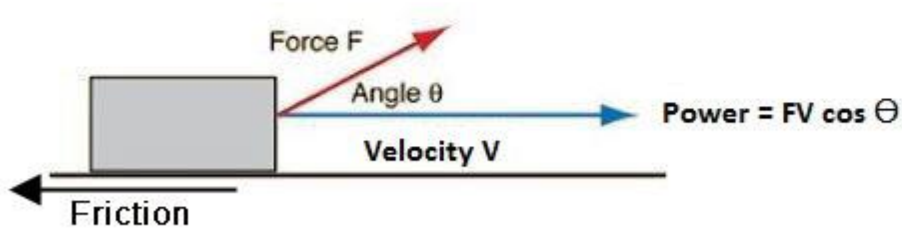

Figure 1. Basic illustration demonstrating concept of power

The first law of thermodynamics is the law of the conservation of energy, which states that, although energy can change form, it can be neither be created nor destroyed.

7. Types of power

In this course we consider mechanical and hydraulic power, which is often defined as a type of mechanical power

7.1 Mechanical power

Mechanical energy is the energy a body has due to its movement or position. Mechanical power is the rate that this energy is produced or used.

7.1.1 Mechanical power: linear motion, constant velocity

In this case, a force is moving an object in a straight line at constant velocity. This is called rectilinear motion. The body is not accelerating. The following examples illustrate linear motion power.

Example 1

What is the power output if a constant force of 12N is applied on a shipping box to slide it 5 meters horizontally in 3 seconds? The force F is applied at an angle of 25°.

Solution:

Using Equation 3;

velocity = 5/3 = 1.67 m/s

$P=FV \cos \Theta$

F = 12N

$\cos 25° = 0.91$

$P=(12)(1.67)(\cos 25°) = (12)(1.67)(0.91) = 18.2\ W$

Note that the box moves at constant velocity and does not accelerate. The resisting frictional force (Figure 1) is equal to (cos 25°) (12N) = 10.9N. Energy is conserved; the work done dragging the box across the floor is converted into heat energy.

Power can be calculated from work due to gravitational force. In other words, potential energy created or expended over duration **Δ**t. Energy is required to raise an object from elevation h_1 to elevation h_2. Note that here the elevation change is linear. This change in potential energy is expressed as **E=mgh**. The power required to perform this work is variable depending on the time duration ΔT of the work.

$$\mathbf{P = \frac{mg\Delta h}{\Delta t}}$$ (US) (SI) Equation 4

Where:

- **P** is power, (watts or ft-lb_f/s)
- **m** is mass, (kg or lb_m)
- **g** is gravity, (9.8 m/s^2 or 32.2 $ft/s^{2)}$
- **Δh** is elevation change (meters or ft)
- **Δt** is time in seconds

Example 2

What is the power required for a passenger elevator if it has a total mass of 3000 kg and is required to move upward at a constant velocity of 2.5 m/s?

Solution: First use Equation 4, then check results by using Equation 2.

In this example we are given neither Δt nor Δh, but we can quickly deduce both from the constant velocity; v = 2.5 m/s: Δt = 2.5 m, Δt = 1 sec.

Using Equation 4: **P =** $\mathbf{\frac{mg\Delta h}{\Delta t}}$ = $\mathbf{\frac{(3000)\ (9.8)\ (2.5)}{1}}$ = 73,500 joule/second = 73.5 kW

Now use Equation 2**:** $\mathbf{P = F \cdot v}$ = (3000) (9.8) (2.5) = 73.5 kW

Note that this calculation does not include the power to accelerate the elevator, only the theoretical power required to keep it moving up at 2.5 m/s. Power due to acceleration is covered in section 7.1.2.

Example 3

The force required to power a boat is proportional to its constant velocity. If a velocity of 5 km/hr requires 8.0 kW, what power is required at 15 km/hr?

$P_1 = F_1 \times 5 = 8.0$ kW, $F_1 = 8/5 = 8000/5 = 1600$ N

Since F is proportional to v, $F_2 = (15/5)\ (1600) = 4800$ N

$P_2 = F_2\ V_2 = (4.8)\ (15) = 72$ kW

Example 4

This problem involves an inclined flat belt conveyor. The conveyor belt slides upon a flat surface. In conveyor design, power calculations are usually done using belt pull and belt speed.

A 10 ft long inclined slider belt conveyor is required to convey boxes weighing 50 lbs. each at a speed of 50 ft/min. The conveyor will handle no more than 5 closely spaced boxes at a time. The change of elevation is upwards 3 ft. Belt weight is 3 lb_f/ft. Assume coefficient of friction is 0.5. What theoretical power is required?

Solution:

First calculate frictional force (it is a slider conveyor).

Conveyor is 10 ft long and rises 3 ft. Therefore, angle of incline of conveyor

= arc sin (3/10) = 17.5°.

Total load on conveyor belt = (5) (50) + (3) (10) = 280 lbs.

Force of load normal to conveyor surface = (280) (cos 17.5) = (280) (0.95) = 266 lb_f

Friction force = (0.5) (266) = 133 lb_f.

Belt velocity = 50 ft/m = 0.83 ft/s

Force (pull) to raise boxes = (sin 17.46) (250 lb_f) = (0.30) (250) = 75 lb_f.

Total belt pull = 133 + 75 = 208 lb_f

Total power = Fv = (208 lb_f) (0.83 ft/s) = 172.6 ft-lb_f/s = 0.31 HP

7.1.2 Mechanical Power: linear acceleration (rectilinear motion)

In this case, a force is accelerating an object in a straight line. We know that acceleration is defined as the rate of change in velocity with respect to time (**Δv/Δt**). We also know that velocity is a vector quantity that has both magnitude and direction. As a result, any change in velocity, meaning a change in either its magnitude or direction, is considered acceleration. We will not address acceleration due to changes in direction. Power is defined as the rate of work done by an object within a specific time interval. The most intuitive way to calculate power due to acceleration is to determine change in kinetic energy between time 1 and time 2, and then divide this change in energy by time, Δt.

As a review of linear kinetic energy, following is the equation:

$$\mathbf{K_L = \frac{1}{2}(mv^2)} \qquad \text{(SI)} \qquad \text{Equation 5}$$

Where:

- **K_L** is linear (translational) kinetic energy (J)
- **m** is mass (kg)
- **v** is velocity (m/s)

In US units, when using typical units of mass (lb_m), the equation for linear kinetic energy includes the factor g_e (32.2 ft/s^2).

$$K_L = \frac{1}{2ge}(mv^2) \qquad \text{(US)} \qquad \text{Equation 6}$$

Where:

- **K_L** is linear (translational) kinetic energy, (ft-lb_f)
- **m** is mass (lb_m)
- **v** is velocity (ft/s)
- **g_e** is 32.2 ft/s^2

So, to calculate linear energy for a given time duration (from time 1 to time 2), we calculate the difference in kinetic energy and divide by Δt:

$$P = \frac{1}{2}(mv_2^2 - mv_1^2) / \Delta t \qquad \text{(SI)} \qquad \text{Equation 7}$$

$$P = \frac{1}{2ge}(mv_2^2 - mv_1^2) / \Delta t \qquad \text{(US)} \qquad \text{Equation 8}$$

Where:

- **P** is power due to acceleration, (watts or ft-lb_f/s)
- **m** is mass (kg or lb_m)
- **v_1** is velocity (m/s or ft/s) at time 1

- **v_2** is velocity (m/s or ft/s) at time 2
- **K_1** is kinetic energy at time 1
- **K_2** is kinetic energy at time 2
- **Δt** is time elapsed between time 1 and time 2.

Example 5

A car of mass 1500 kg accelerates from 50 to 100 km/h in 6 seconds. What power is required to accelerate the car from 50 to 100 km/h?

Using Equation 7, we first calculate kinetic energy at 50 km/h and 100 km/h:

50 km/h = 13.9 m/s; $K_1 = ½ (1500) (13.9)^2 = 145$ kW

100 km/h = 27.8 m/s; $K_2 = ½ (1500) (27.8)^2 = 580$ kW

P = (580 – 145) / 6 = 72.5 kW

Note that example 5 assumes mass m remains constant. In this case change in mass is negligible. This is not always the case, such as in rocket powered craft.

Example 6

Now we re-work example 5 using US units. Use Equation 8:

$g_e = 32.2$ ft/s^2; $1/2g_e = 1/64.4$

Convert mass units: 1500 kg = 3307 lb_m

50 km/h = 45.6 ft/s; $K_1 = (1/64.4) (3307) (45.6)^2 = 106{,}900$ ft-lb_f

100 km/h = 91.2 ft/s; $K_2 = (1/64.4) (3307) (91.2)^2 = 428{,}000$ ft-lb_f

P = (428,000 – 106,900) / 6 = 53,600 ft-lb_f/s = 72.5 kW

Since power is defined as $\frac{W}{\Delta t}$ we can evaluate linear acceleration power in a second way. We know that acceleration is defined as the rate of change in velocity with respect to time.

$$\mathbf{P = \frac{W}{\Delta t} = (\ Fd)/\Delta t = Fv = ma\Delta v} \quad \text{(SI) (US)} \qquad \text{Equation 9}$$

Where:

- **P** is power due to acceleration, (watts, ft-lb_f/s)
- **m** is mass (kg or lb_m)
- **Δv** is change in velocity (m/s or ft/s)
- **a** is linear accelerating, (m/s^2 or ft/s^2)

Example 7

What power is needed to keep a 12.0 kg object moving at a constant acceleration of 8.0 m/s^2 for 14 s?

Use Equation 9: $P = \frac{W}{\Delta t} \ = \frac{Fd}{t} = \mathrm{Fv} = ma\Delta v$

$\Delta v = (a)(\Delta t)$ = (8 m/s^2) (14s) = 112 m/s

P = (12) (8) (112) = 10752 W = 10.75 kW

7.2 Mechanical power: angular (rotational) motion

Power can also be produced or expended from angular (rotational) motion such as torque on a shaft twisting a propeller.

As we saw back in Section 6, power is defined as the rate at which work is done with respect to time; **P = W/Δt** (Equation 1). Work can also be performed from angular motion (rotation), and is calculated using the following equation:

$$\mathbf{W = \tau \cdot \Theta} \qquad \text{(SI) (US)} \qquad \text{Equation 10}$$

Where:

- $\boldsymbol{\tau}$ (tau) is torque (newton-meter or lb_f -ft)
- $\boldsymbol{\Theta}$ (theta) is angular rotation, (radians)

  Note:

  $\boldsymbol{\Theta}$ must be in radians. If you are working with angles measured in degrees, one radian = 57.3 degrees. (There are 2π radians in 360 degrees).

Since we can calculate work produced by angular motion (Equation 10), we can also calculate angular power, the rate at which angular work is done with respect to time.

7.2.1 Mechanical power: angular, constant velocity

In this case, a torque $\boldsymbol{\tau}$ is turning an object, a disk or shaft for example, at constant angular velocity $\boldsymbol{\omega}$**.** The object is not accelerating. Angular power is defined as the rate at which angular energy is being transferred.

Angular power can be expressed as $P = \frac{\tau \cdot \Theta}{t}$; analogous to linear power, $P = \frac{f \cdot s}{t}$

$$\mathbf{P = \frac{\tau \cdot \Theta}{t}} \qquad \text{(SI) (US)} \qquad \text{Equation 11}$$

Where:

- **P** is power, energy per second (watts, ft-lb_f/s)

- **t** is time in seconds
- **Θ** (theta) is angular rotation, (radians)
- **τ** (tau) is torque (N-m or lb_f -ft)

We can also compare the angular power Equation 11 to the similar linear power equation. From Equation 2: $P = F \cdot v$

The angular power equivalent is:

$$\mathbf{P} = \boldsymbol{\tau} \cdot \boldsymbol{\omega} \qquad \text{(SI) (US)} \qquad \text{Equation 12}$$

Where:

- **P** is power, (watts or ft-lb_f/s)
- **τ** is torque (N-m or lb_f-ft)
- **ω** is angular velocity in radians/s

Note:

Angular rotation **Θ** (theta) and angular velocity **ω** (omega) must be in radians. If you are working with angles measured in degrees, one radian = 57.3 degrees. (There are 2π radians in 360 degrees).

In industrial applications, rotational speed is rarely expressed in radians/second, we usually RPM (revolutions per second).

A very useful conversion to know is the following:

Radians/second = (RPM) (0.105)

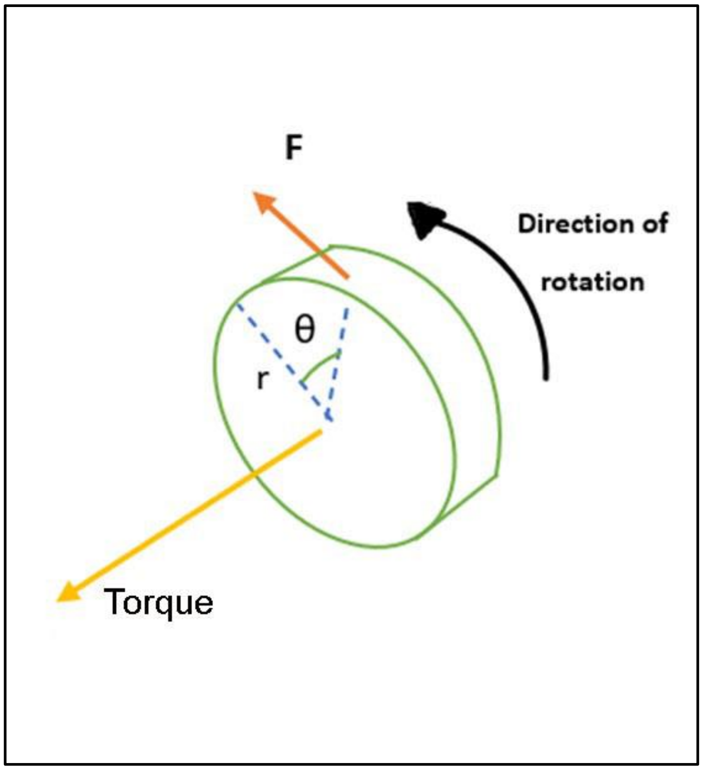

Figure 2. Visual example of torque

Example 8

A turbine is being used to drive an electric generator. A torque of 5000 N-m is required to drive the generator, which is rotating at 285 RPM. Determine the power needed to drive the generator.

Convert RPM to rad/s: (285) (0.105) = 30 rad/s

Use Equation 12, $P = \tau \cdot \omega$

P = (5000 N-m) (30 rad/s) = 15 x 10^4 joules/sec = 15 x 10^4 watts =150 kW

Example 9

An electric motor runs at 3600 rpm with a measured power consumption of 2000 W. What is the torque created by the motor?

Use Equation 12, $P = \tau \cdot \omega$

Radians/sec = (3600) (0.105) = 378 rad/s

τ = 2000/378 = 5.3 N-m

Example 10

When drilling a deep well, a motor is connected to a gear box which drives a large drill bit. The gear box reduces the drive speed to 120 RPM. The torque at that speed is measured to be 2000 N-m. What would be the required power?

Use Equation 12, $P = \tau \cdot \omega$

120 RPM = (120) (0.105) = 12.6 radians/sec

P = (2000 N-m) (12.6) = 25,200 W = 25.2 kW

7.2.2 Mechanical Power: angular (rotational) acceleration

As discussed in section 7.1.2, Linear Kinetic Energy $= \frac{1}{2}\mathrm{mv}^2$. The equation for angular kinetic energy is very similar.

Angular Kinetic Energy:

$$\mathbf{K_R = \frac{1}{2} I\omega^2} \qquad \text{(SI)} \qquad \text{Equation 13}$$

Where:

- **K_R** is angular (rotation) kinetic energy (J)
- **I** is moment of inertia (kg-m^2)
- **ω** (omega) is angular velocity (radians/s)

An intuitive way to calculate power due to angular acceleration is to calculate change in angular kinetic energy between time 1 and time 2.

So, to calculate average power of angular acceleration we evaluate: $(K_2 - K_1)/t$:

$$P = \frac{1}{2}\left(I\omega_2^2 - I\omega_1^2\right)/t \qquad \text{(SI)} \qquad \text{Equation 14}$$

Where:

- **t** is time in seconds
- I is moment of inertia (kg-m^2)
- **ω** (omega) is angular velocity (radians/s)
- **P** is angular power (watts)

Example 11

A blower with moment of inertia I = 250 kg-m^2 is accelerated by a motor from 0 to 1200 RPM in 12 seconds. What power is required?

We could use Equation 14, but since initial RPM = zero, might as well use Equation 13 to get kinetic energy at 1200 RPM, then divide by time duration t.

Radians/sec = 1200 RPM x 0.105 = 126 radians/sec

P= ½ ((250) (126)^2)/12 = 165,000 W = 165 kW

8. Horsepower and Power - Torque Curves

8.1 Horsepower

The term "horsepower" originated in the late 18th century. James Watt coined the term after he invented and built greatly improved versions of the first commercially available steam engine. He wanted a simple way to explain the power of his improved steam engine in a way that his potential customers could easily understand. They had previously been reliant on horses to do most of the mechanical work that his engines would replace. While selling his engines he was often asked a question. How many horses could one engine replace? While working with coal miners he set up an experiment. He would have horses walk around a wheel connected to a pulley system that would lift a load of coal. Through his calculations he determined that a horse could on average lift around 33,000 ft- lbf/minute. Therefore, he determined that 33,000 ft-lb_f/minute, or 550 ft- lb_f/second, would equal 1 horsepower.

8.2 The Power, Torque, RPM connection

We know from Equation 12 that power = (torque) (angular velocity). We are also aware that most engineers do not use radians/second when dealing with shaft speed, instead we use RPM. Therefore, an often-used equation relates power, torque, and RPM:

$$\mathbf{HP} = \frac{(\boldsymbol{\tau})\,(\mathbf{RPM})}{\mathbf{5252}} \qquad \text{(US)} \qquad \text{Equation 15}$$

Where torque $\boldsymbol{\tau}$ is measured in lb_f-ft.

We can do the same in SI units:

$$\mathbf{kW} = \frac{(\boldsymbol{\tau})\,(\mathbf{RPM})}{\mathbf{9550}} \qquad \text{(SI)} \qquad \text{Equation 16}$$

Where torque $\boldsymbol{\tau}$ is measured in N-m.

8.3 Power-Torque Curves

The best way to visualize the relationship between torque and power is to study a power/torque curve. As we see in Equations 15 and 16, power is directly related to torque and shaft speed (RPM). The two equations are generic, and apply power being produced (by electric motor, internal combustion engine, hydroelectric turbine), and power being consumed (by boat propeller shaft or vehicle driveshaft). In many cases shaft speed of an engine will be a variable, vehicle engines are an obvious example. Usually, engine manufacturers publish power and torque characteristics (curves) at full load. Torque and power curves indicate the maximum torque and power distribution through the whole range of engine speed, not just at one optimum RPM.

What is a power-torque curve?

A torque curve plots the torque of an engine or electric motor against the speed, usually RPM, that the engine is running. The power curve values are calculated, based upon torque and engine speed. These power-torque curves are generally available for all commercially manufactured electric motors and internal combustion engines.

A power/torque curve can help determine the application of an engine to specific requirements. Going only by the engine supplier's "power rating" can be very misleading. The manufacture's advertised power rating is likely to be at an unrealistically high RPM (see Figure 3 below). This engine will produce 110 kW but only at 5500 RPM. Also, you may prefer to select a drive motor based upon torque rating at a specific RPM.

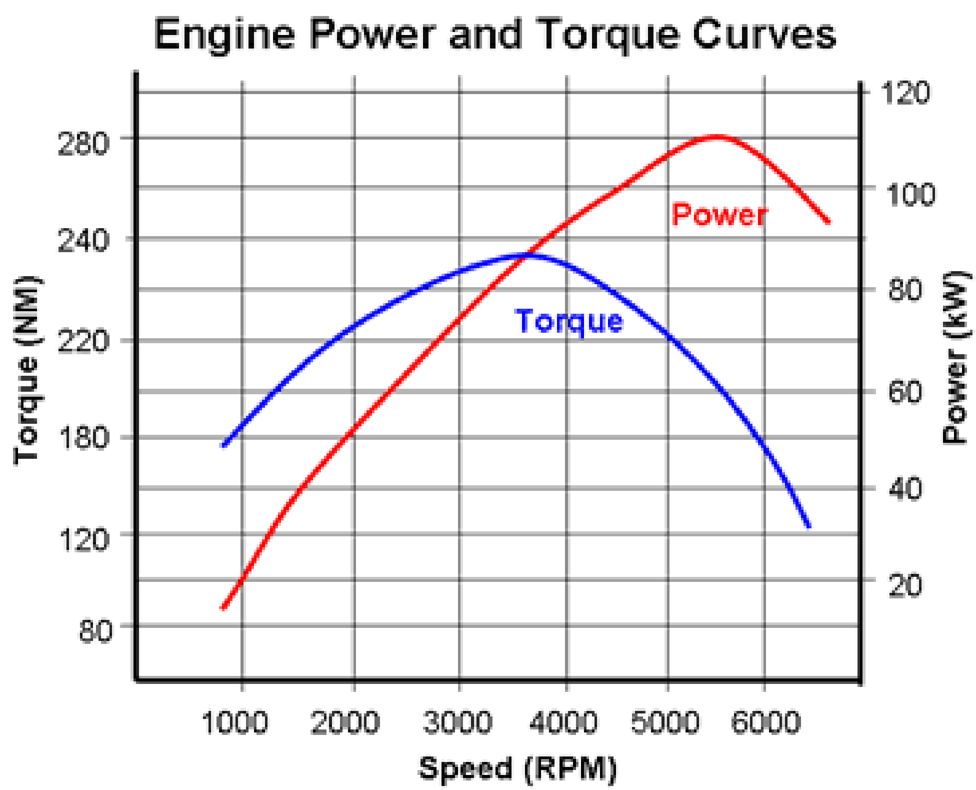

Figure 3. Typical Internal combustion gasoline engine power/torque curve, SI units [(4)]

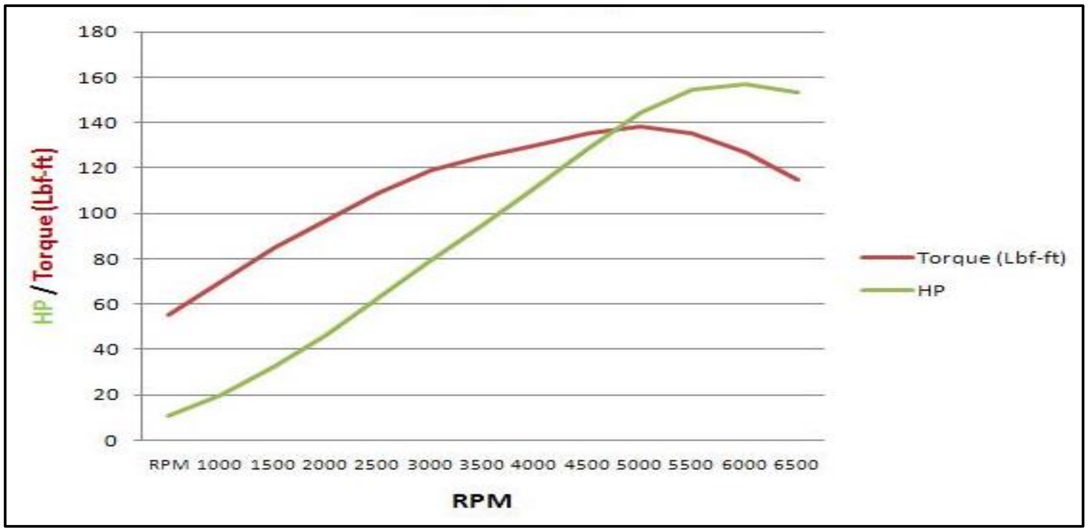

Figure 4. Typical Internal combustion gasoline engine power/torque curve, US units [(5)]

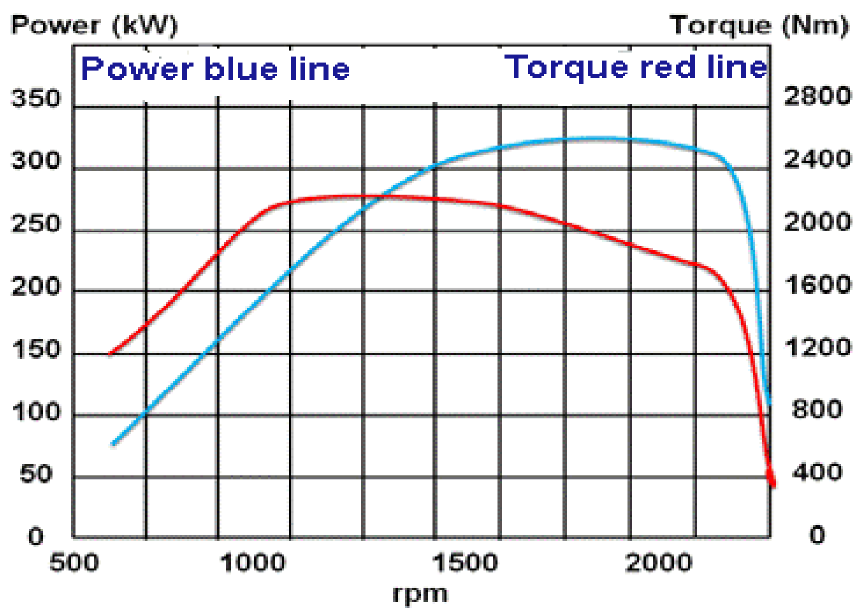

Figure 5. Typical curve for internal combustion Diesel engine, SI units, kW

Torque - power - RPM characteristics for electrical motors vary greatly depending upon type. Figure 6 is a generic curve depicting typical motors used in electric vehicles. Many of these motors use sophisticated electronic controls to achieve desired performance. Note the difference in low RPM torque between electric vehicle motors (Figure 6) versus internal combustion engines (Figure 3).

Torque-power characteristics for the many different types of electric motors will not be discussed in this basic course.

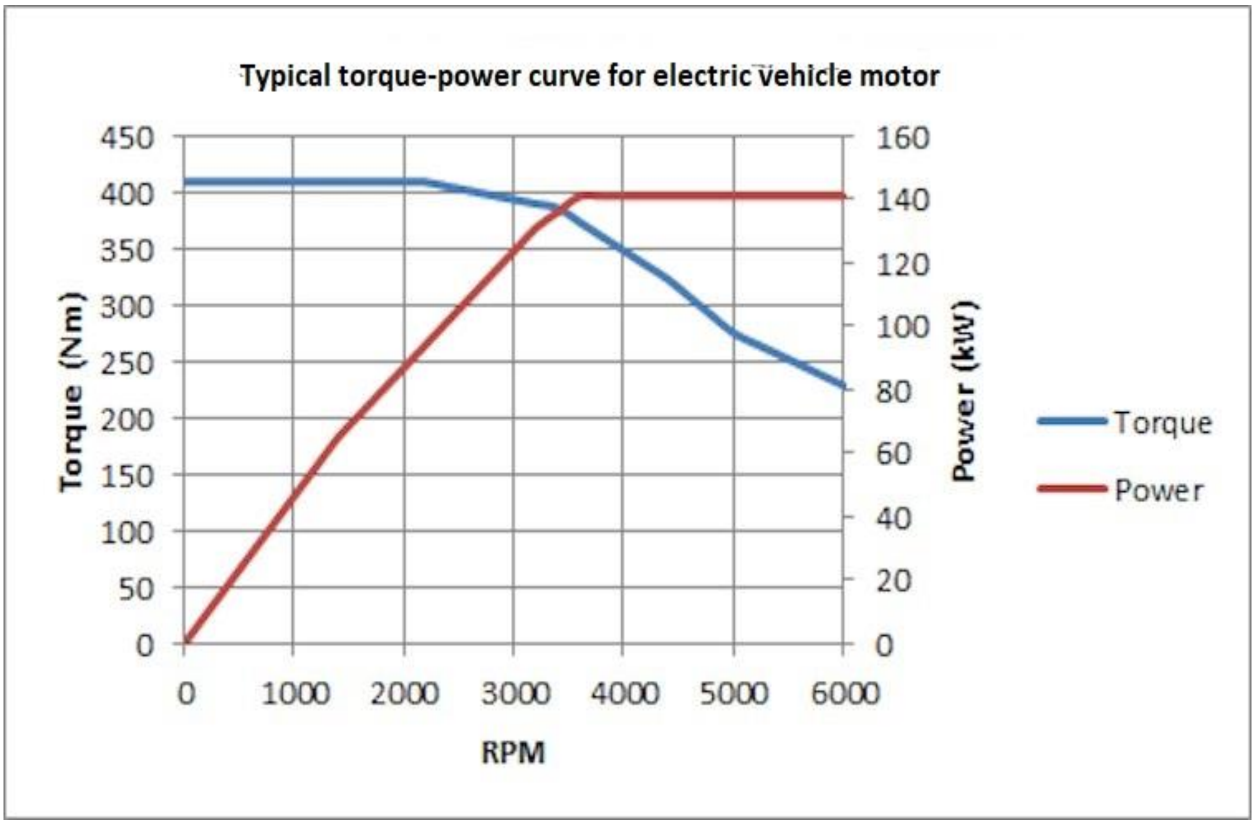

Figure 6. Typical power/torque curve for electric motor-powered automobile [(4)]

Power output efficiency of a gasoline engine versus Diesel engine

Most internal combustion engines are inefficient at turning fuel into usable energy. The energy efficiency of gasoline engines averages around 30 percent. In an internal combustion engine, efficiency is determined in part by the compression ratio. The compression ratio of a gasoline engine is considerably lower than that of a Diesel engine. The combustion ratio of a typical gasoline engine is between 8:1 and 12:1. Diesel engines have a much higher compression ratio, between14:1 and 25:1. Diesel engines typically have a higher energy efficiency, around 40 percent and higher. The higher fuel efficiency of a Diesel engine is mainly due to its higher compression ratio. In a Diesel engine, the fuel is ignited by the high temperature achieved in the combustion chamber from compression, not from a spark plug

9. Hydraulic Power

Also referred to as "turbomachines," this section will be limited to pumps and turbines. A simple distinction is that turbines extract useful energy from fluids, pumps add useful energy to fluids.

9.1 Pump Power

The hydraulic power required for a pump has long been calculated by the following familiar equation in US units and HP. For most hydraulic equipment, efficiency is a major factor and especially for pumps. Pump efficiency can be in a wide range, from above 80% down to 50% and lower.

$$\mathbf{P_{HP} = \frac{Q\gamma h}{550e}} \qquad \text{(US)} \qquad \text{Equation 17}$$

Where:

- **P_{HP}** is hydraulic power (in horsepower)
- **Q** is CFS (cubic feet/sec)
- **γ** is weight of fluid (lb_f/ft^3) (water is 62.4 lb_f/ft^3)
- **h** is total head of fluid (ft)
- **e** is efficiency (decimal)

A similar equation in SI units:

$$\mathbf{P_{kW}} = \frac{\mathbf{Q\,\rho\,g\,h}}{\mathbf{(1000)e}} \qquad \text{(SI)} \qquad \text{Equation 18}$$

Where:

- **P_{kW}** is hydraulic power (in kW)
- **Q** is flow (m^3/sec)
- **ρ** (rho) is density of fluid (kg/m^3) (water is 1000 kg/m^3)
- **g** is acceleration of gravity (g_e = 9.81 m/s^2)
- **h** is total differential head (m)
- **e** is efficiency of pump (decimal)

Example 12

At a desalinization plant, you are asked to calculate the power required for a submerged centrifugal pump that will be used to transfer brine from a holding pond to an elevated process tank. You are given these requirements:

Brine density = 1230 kg/m^3

Flow rate 0.025 m^3/sec

Elevation difference from pond level to top of tank 45 m

Using Equation 18, calculate P_{kW} = (0.025) (1230) (9.8) (45) /1000 e

P_{kW} = 13.6/e

You consult with the supplier and select a suitable pump, which has a hydraulic efficiency of 82% in the range of operating flow conditions. (see section 9.1.1 below for selecting pump power requirements from pump curves).

Calculated P_{kW} = 13.6/.82 = 16.6 kW

Likely a 20-kW motor will be selected by the electrical engineer.

9.1.1 Centrifugal Pump Curves and Power Requirement

Centrifugal pumps are the most common type of pump. Centrifugal pump curves are unique for hydraulic equipment documents in that the essential component of efficiency (e) is usually supplied by equipment supplier. In the typical pump curve shown below (*Figure 7*), both efficiency and power required are provided for a range of operating conditions. Note that power to drive the pump under various conditions is provided as a guide to selecting an appropriate drive motor.

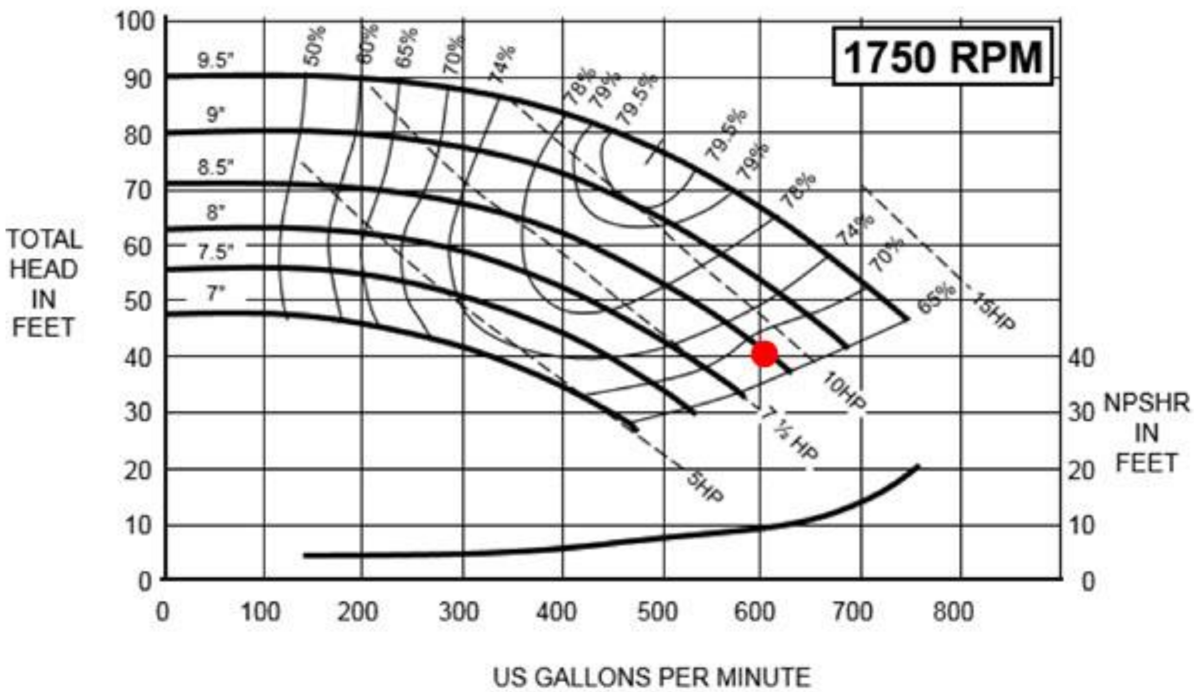

Figure 7. Typical centrifugal pump curve

Example 13

Compare power calculated by Equation 17 with the power requirement provided by the pump curve in Figure 7. Here are the operating conditions for the pump (also shown as the red dot on the pump curve).

Flow rate: 600 US GPM

Total head on pump: 40 feet (this is pressure at pump discharge)

Liquid pumped: water

Q = 600 GPM = 1.34 ft^3/sec

$\gamma = 62.4\ lb_f/ft^3$

e = .68 (from pump curve)

h = 40 ft

HP = $\frac{(1.34)(62.4)(40)}{(550)(.68)}$ = 8.9 HP

With a calculated 8.9 HP demand, you would select a 10 HP motor. Same as shown on the pump curve.

9.2 Power from water turbines

There are two main types of hydropower turbines: reaction and impulse.

9.2.1 Reaction turbine

A reaction turbine generates power from the combined forces of pressure energy and kinetic energy of moving water. A rotor is placed directly in the water stream, allowing water to flow over the blades rather than striking each individually. Reaction turbines are generally used for sites with moderate head and higher flows, and are the most common type currently used for power generation. Francis turbines, a type of reaction turbine, are used at Hoover and Grand Coulee dam, these turbines have a hydraulic efficiency of about 95%. Reaction turbines give greater efficiency at high flow rate and moderate head.

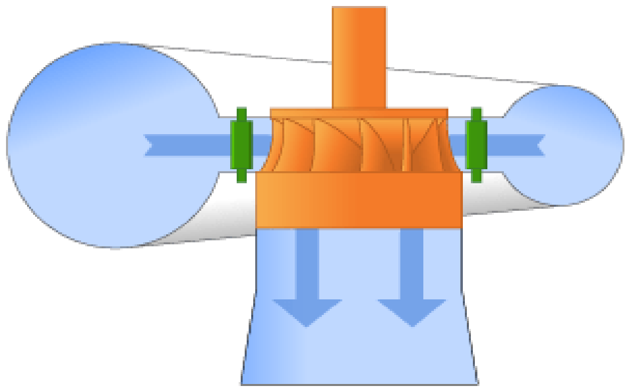

Figure 8. Reaction turbine (6)

9.2.2 Impulse turbine

An impulse turbine generally uses the velocity of the water to move the runner and discharges at atmospheric pressure. A water stream hits each bucket on the runner. With no suction on the down side of the turbine, the water flows out the bottom of the turbine housing after hitting the rotor. An impulse turbine is generally suitable for high-head, low-flow applications.

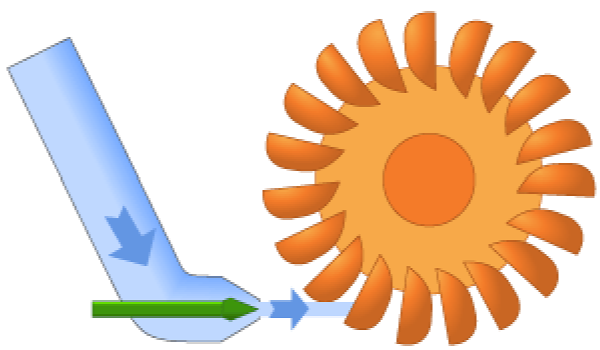

Figure 9. Impulse turbine (6)

The Pelton turbine is an example of an impulse turbine. A Pelton turbine has one or more free jets discharging water into an aerated space and impinging on the buckets of a rotor. Pelton turbines are generally used for very high heads and lower flows.

The ideal rotor speed of a Pelton turbine will cause all the kinetic energy in the jet to be transferred to the wheel. In this case the final jet velocity must be zero. The optimal rotor speed is $V_i/2$, or half the initial jet velocity.

Power produced by hydraulic turbines is determined by net pressure (head) at entrance to turbine, and the volumetric flow rate. The density of the liquid is a factor also, but in this section, we assume water of constant density. If the two equations below look familiar, they are. Equations for power produced by turbines are similar to equations for power required by pumps.

$$\mathbf{P_{kW} = \frac{Q\,\rho g h\,e}{1000}} \qquad \text{(SI)} \qquad \text{Equation 19}$$

Where:

- **Q** is flow (m³/sec)
- **ρ** (rho) is density of fluid (kg/m³) (use 1000 kg/m³ for water)
- **g** is acceleration of gravity (g_e = 9.81 m/s²)
- **h** is total differential head (m)
- **e** is efficiency of pump (decimal)

And in US units, hydraulic power from turbines is:

$$\mathbf{P_{HP} = \frac{Q\gamma h e}{550}} \qquad \text{(US)} \qquad \text{Equation 20}$$

Where:

- Q is flow (cubic feet/second, CFS)
- γ is weight of fluid (lb_f/ft^3) (for water, 62.4 lb_f/ft^3)
- h is head of fluid (ft)
- e is efficiency (as decimal)

Example 14

A Francis turbine has a hydraulic efficiency 0f 83%. The turbine develops 500 kW power under a net head of 30 meters, what is the flow rate through the turbine?

Use Equation 19 and rearrange to find Q

Q = (500) (1000) / (1000) (9.8) (30) (.83) = 2.05 m^3/s

10. A few words on electrical power and energy

A thorough discussion of electrical power is outside the scope of this course. Complete continuing education courses on the subject are available. You can consider this section as optional; there will be no test questions from Section 10.

Electrical units are based on the SI system. The definitions of potential (volt), electric current (ampere), electrical resistance (ohm) were defined in terms of SI units when the electrical industry started. So, there are no US nor Imperial electrical units! Electrical power, with units of watts is the same as described in section 5.1. Power is defined as work done per unit time; P = $\frac{energy}{time} = \frac{joules}{seconds}$ = watts. In terms of electrical units,

Pw = VI = $\frac{joules}{\cancel{coulomb}} \times \frac{\cancel{coulomb}}{seconds}$ = volts x amperes = watts.

$$P_w = VI \qquad \text{Equation 21}$$

Where:

- P_w is power, watts
- **V** is voltage, volts
- **I** is amperage, amps

Electrical Parameter	Measuring Unit	Symbol	Description
Voltage	Volt	V	Unit of Electrical Potential **V = I × R**
Current	Ampere	I or i	Unit of Electrical Current **I = V ÷ R**
Resistance	Ohm	R or Ω	Unit of DC Resistance **R = V ÷ I**
Power	Watts	W	Unit of Power **P = V × I** or **I^2 × R**

Table 1. A few electrical terms used with power calculations

11. Momentum

Momentum is not energy. However, it is important to know how kinetic energy and momentum are similar and how they differ. The difference is primarily because of the presence of friction: kinetic energy is always conserved, but some is conserved in ways such as heat energy, which is not always easy to quantify, for example in collisions. Momentum is always conserved and is not concerned with friction or heat.
Momentum is a measurement of mass in motion: how much mass is in motion. The standard unit for momentum is kg-m/s. Momentum is a vector quantity. Doubling either the mass or velocity of an object will simply double the momentum.

For clarity, this section will use only SI units.

Differences between kinetic energy and momentum

Kinetic energy	Momentum
Is a scalar quantity	Is a vector quantity
Is always positive (>0)	Can be either positive or negative (>0 or <0)
Depends quadratically on velocity	Depends linearly on velocity
Depends on even powers of velocity (in special relativity)	Depends on odd powers of velocity (in special relativity)
Is conserved only in special cases	Is always conserved

Table 2. Important differences between kinetic energy and momentum

11.1 Law of conservation of momentum

The total momentum of a closed system is constant. This law is valuable for solving many kinetics problems. The general equation is:

$$\mathbf{m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2} \qquad \text{(SI)} \qquad \text{Equation 22}$$

Where:

- $\mathbf{m_1}$ and $\mathbf{m_2}$ are mass of objects 1 and 2, kg
- $\mathbf{u_1}$ and $\mathbf{u_2}$ are initial velocities of objects 1 and 2, m/s
- $\mathbf{v_1}$ and $\mathbf{v_2}$ are final velocities of objects 1 and 2, m/s

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Example 15

Find the velocity v_1 of a bullet of mass 5 grams which is fired from a pistol of mass 1.5 kg. The recoil velocity of the pistol is 1.5 m/s

Solution:
Mass of bullet, m_1 = 5 gram = 0.005 kg
Mass of pistol, m_2 = 1.5 kg
Recoil velocity of pistol, v_2 = 1.5 m/s

Using Equation 22 (law of conservation of momentum)
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
Initial velocity of the bullet, $u_1 = 0$
Initial recoil velocity of pistol, $u_2 = 0$

(0.005 kg) (0) + (1.5 kg) (0) = (0.005 kg) (v_1) + (1.5 kg) (1.5 m/s)
0 = (0.005 kg) (v_1) +(2.25 kg-m/s)
v_1 = -450 m/s

The above example illustrates a welcome simplification to Equation 22 in cases when initial velocities of both objects are zero:

$$\mathbf{m_1v_1 = -m_2v_2} \qquad \text{(SI)} \qquad \text{Equation 23}$$

11.2 Linear momentum

Linear momentum is the product of the mass and velocity of an object. It is a vector quantity, possessing magnitude and direction. If m is an object's mass and v is its velocity, then the object's momentum p is:

$$\mathbf{p = m \cdot v} \qquad \text{(SI)} \qquad \text{Equation 24}$$

Where:

- **p** is linear momentum (kg-m/s)
- **m** is mass (kg)
- **v** is velocity (meters/s)

The relationship between kinetic energy and momentum is given by the equation $K=p^2/2m$, where K is kinetic energy, p is momentum and m is mass. This relationship comes directly from the definitions of momentum ($p=mv$) and kinetic energy ($K=½mv^2$).

Kinetic energy is a scalar quantity, since it involves the square of the velocity. Momentum is clearly a vector since it involves the velocity vector.

If the force varies over time, the change in momentum is $\Delta p = \int_{t1}^{t2} F(t)dt$

We can calculate change of moment as follows:

$$\mathbf{\Delta p = F\,\Delta t} \quad \text{(SI)} \qquad \text{Equation 25}$$

Where:

- **Δp** is the change in linear momentum,
- **F** is the force applied on the body (average or consistent force)
- **Δt** is the time interval for which force acts on the object.

The concept of momentum can be demonstrated by the following example problem:

Example 16

The recoil of a cannon is a classic problem in momentum conservation. Consider a wheeled, 500 kg cannon firing a 2 kg cannonball horizontally from a ship. The ball leaves the cannon traveling at 200 m/s. At what speed does the cannon recoil as a result?

Solution:

Here we use “simplified” law of conservation of momentum, Equation 23:

The cannon and ball are both initially stationary (nothing is moving). Momentum is always conserved, so we know that $p_c + p_b = 0$ (momentum of cannon + momentum of cannonball = 0). In this case, the cannon is horizontal, so we can use scalar quantity for both cannon and ball.

$$(m_c)(v_c) + (m_b)\,(v_b) = 0$$

$$\frac{m_b v_b}{m_c} = -v_c$$

$$(2)\frac{200}{500} = -0.8\ m/s$$

A good way to understand the use of both momentum and kinetic energy is the ballistic pendulum problem, described in the example below:

Example 17

A ballistic pendulum is a device used to measure the speed of a bullet. A bullet of mass m is fired at a block of wood of mass M hanging from a string. The bullet embeds itself into the block, and causes the combined block plus bullet system to swing up a height h. What is v_o, the speed of the bullet before it hits the block?

Note that a completely inelastic collision takes place, resulting in a large loss of kinetic energy, converted largely to heat. The initial kinetic energy is larger than the final gravitational potential energy. So, we must include linear momentum in solving the problem.

Start with the swing of the pendulum just after the collision until it reaches its maximum height. Conservation of mechanical energy applies here, so:

So, the speed of the system immediately after the collision is: $v_f = \sqrt{(2gh)}$

Apply conservation of momentum to determine the relationship between v_f and the bullet's speed before the collision. The wood block is stationary before the collision.

Total momentum before the collision = momentum after the collision $mv_o = (m+M)\ v_f$

Therefore $v_o = \frac{(M+m)(\sqrt{2gh})}{m}$

Take the case where mass of bullet = 50 g, mass of block = 3.0 kg, h =1.50m, g= 9.8.

$\sqrt{(2gh)} = \sqrt{((2)\ (9.8)\ (1.5))} = 5.4$, mass of bullet = 0.05 kg

$v_o = (3.05)\ (5.4)/0.05 = 329.4$ m/s

11.3 Angular (rotation) momentum

Angular momentum is a vector quantity that represents the product of a body's rotational inertia and rotational velocity (in radians/sec) about a particular axis. This equation is an analog to the definition of linear momentum as p = mv. Units for linear momentum are kg-m/s while units for angular momentum are kg - m^2/s.

$$\mathbf{L} = \mathbf{I\omega} \qquad \text{(SI)} \qquad \text{Equation 26}$$

Where:

- **L** is angular momentum (kg-m^2/s)
- **I** is moment of inertia (kg-m^2)
- **ω** is velocity (radians/s)

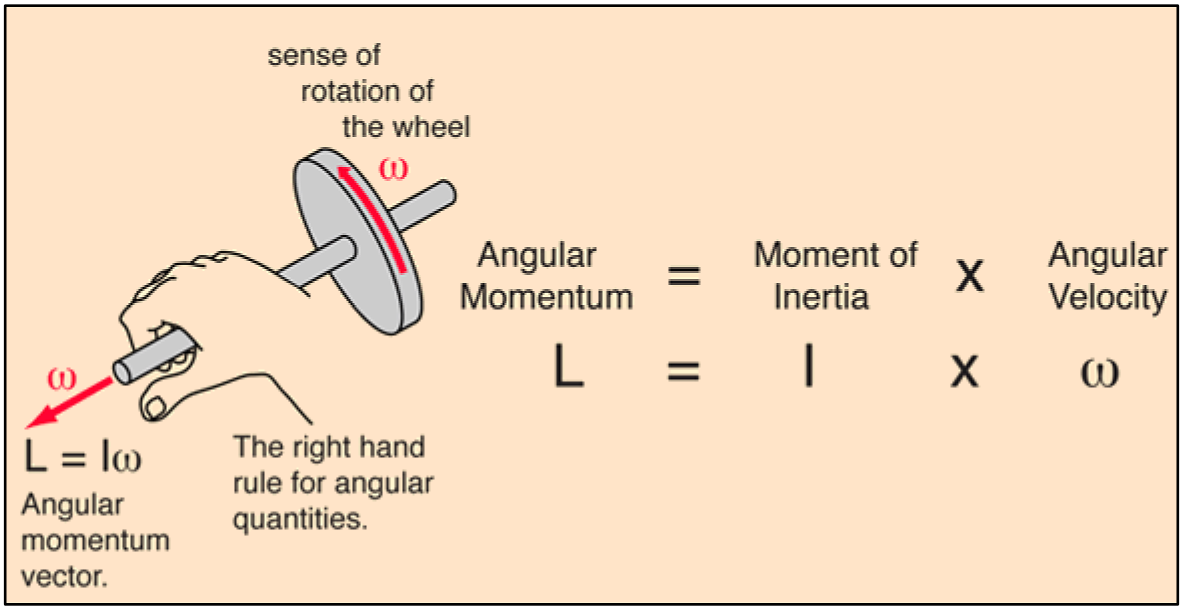

Figure 10. Illustration of angular momentum [7]

Example 18

A satellite is rotating once per minute. It has a moment of inertia of 10,000 kg-m^2. Julia, an astronaut, extends the satellite's solar panels, increasing its moment of inertia to 30,000kg-m2. How quickly is the satellite now rotating?

The equation for angular momentum is ***L=Iω*** (Equation 26)

Remember that one rotation per minute = 1 RPM = 0.105 radian/sec

Where:

- L is angular momentum
- The initial period of rotation of the satellite is 1 minute
- Initial angular speed of satellite = 1.0 RPM = 0.105 radian/s

Solve for initial angular momentum:

L_i= (10,000 kg-m^2) (0.105 radian/s) =1050 kg-m^2/s

After Julia extends the solar panels, angular momentum is the same (conservation of momentum), but the moment of inertia is now 30,000 kg-m^2.

1050 = (30,000 kg-m^2) (ω)

ω = 0.035 rad/s

Plugging this back into the equation, we get:

ω=2π/t

0.035=2π/t

Therefore, t =180 s.

The satellite now rotates once every 3 minutes.

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12. Industry standards, conversion factors, and online tools

"For those who want some proof that physicists are human, the proof is in the idiocy of all the different units which they use for measuring energy." Richard P. Feynman

Physicists are not alone in this; engineers also have developed idiosyncratic units, so have many industries. Following are some examples.

12.1 Btu (British thermal unit) and Btu per hour.

The British thermal unit (Btu) is a traditional Imperial and US unit of energy. It is the amount of energy needed to cool or heat one pound of water by one-degree Fahrenheit. To raise one pound of water by one-degree Fahrenheit requires one Btu of heat, equivalent to 1055 joules. The unit of power is Btu *per hour* (Btu/h).

12.2 Refrigeration industry.

In the U.S., one ton of refrigeration is a unit of cooling, or heat removal capacity. or heat removal capacity. One ton of refrigeration is the heat required to convert 1 ton (US short ton of 2,000 lb. or 910 kg) of water into ice at the same temperature, in a 24-hour period. So, it is the latent heat of ice (heat required to convert water into ice at the same temp.) equates to 12,000 Btu/hr.

A few useful conversion factors for measurement of energy and power.

Unit	Symbol	Equivalent in joules	Other units
Kilowatt hour	kWh	3,600,000	3413 Btu
Calorie	cal	4.186	0.004 Btu
Kilocalorie (food Calorie)	kcal	4186	1000 cal
British Thermal unit	Btu	1055	252 cal
Horsepower-hour	HP-hour	2,684,520	0.75 kWh
Therm		1.05×10^8	100,000 Btu
Ton of refrigeration	Ton	3.08×10^8	288,000 Btu/hr.
Watt-hour	Wh	3600	3.41 Btu

12.3 Online conversion of units

There are many websites available to do unit conversions. Which of the many sites is best for engineering use? The author's favorite is simply named "*Convert*," available at joshmadison.com or search online for Convert for Windows. (8)

12.4 Online calculators

There are also many websites available to perform our calculations. Most are aimed at students. Many are simply not suited for engineering use. The author uses *Omni Calculator* (9), which features many engineering and physics calculators.

13. Trends in production and consumption of energy

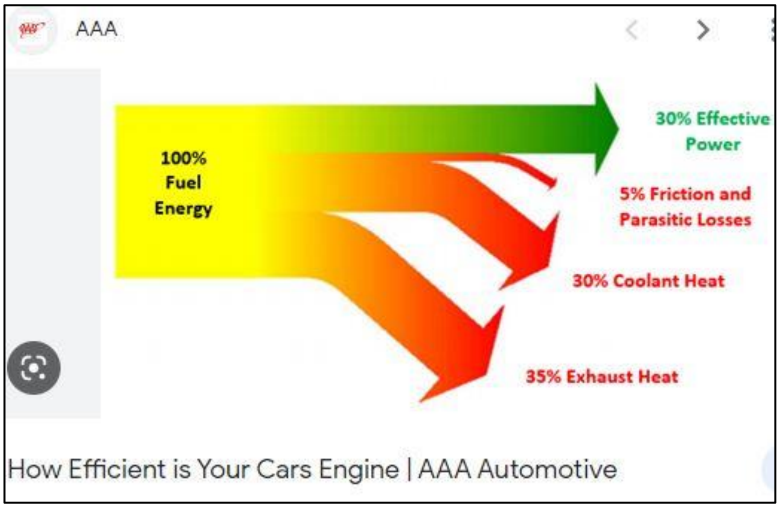

Figure 11. Internal combustion gasoline engine efficiency (10)

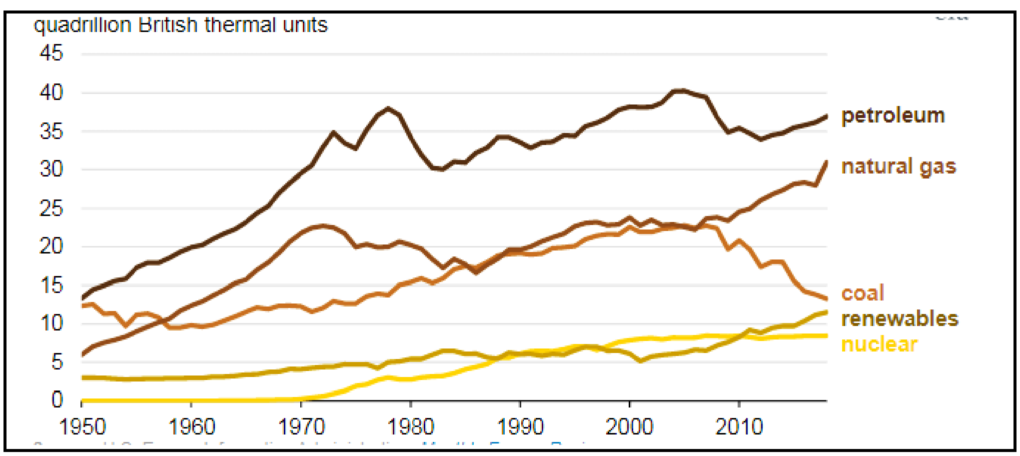

Figure 12. U.S. total energy consumption (1950 – 2018) (11)

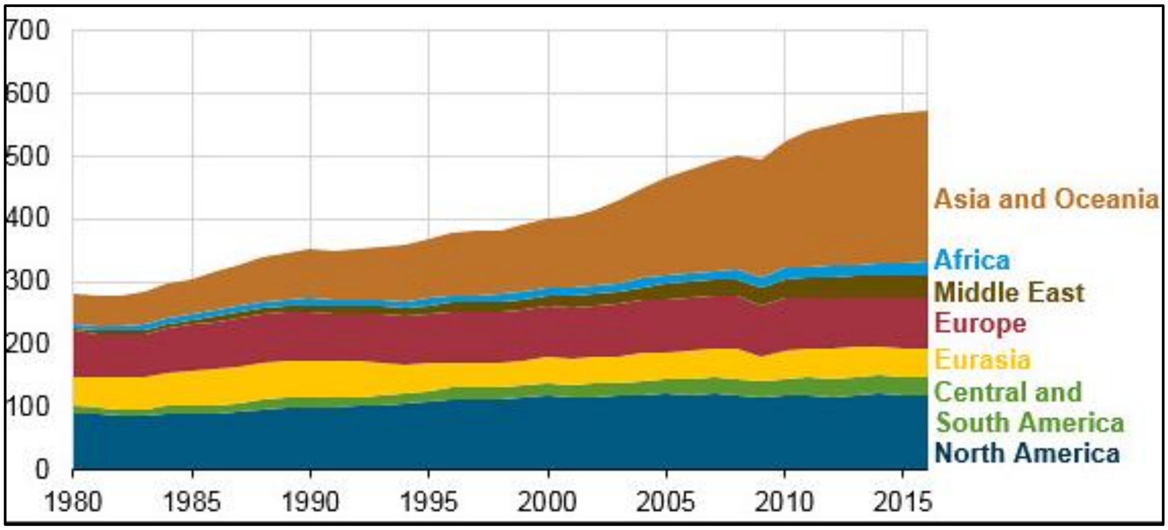

Figure 13. Energy use by world region (1980 - 2016) [11]

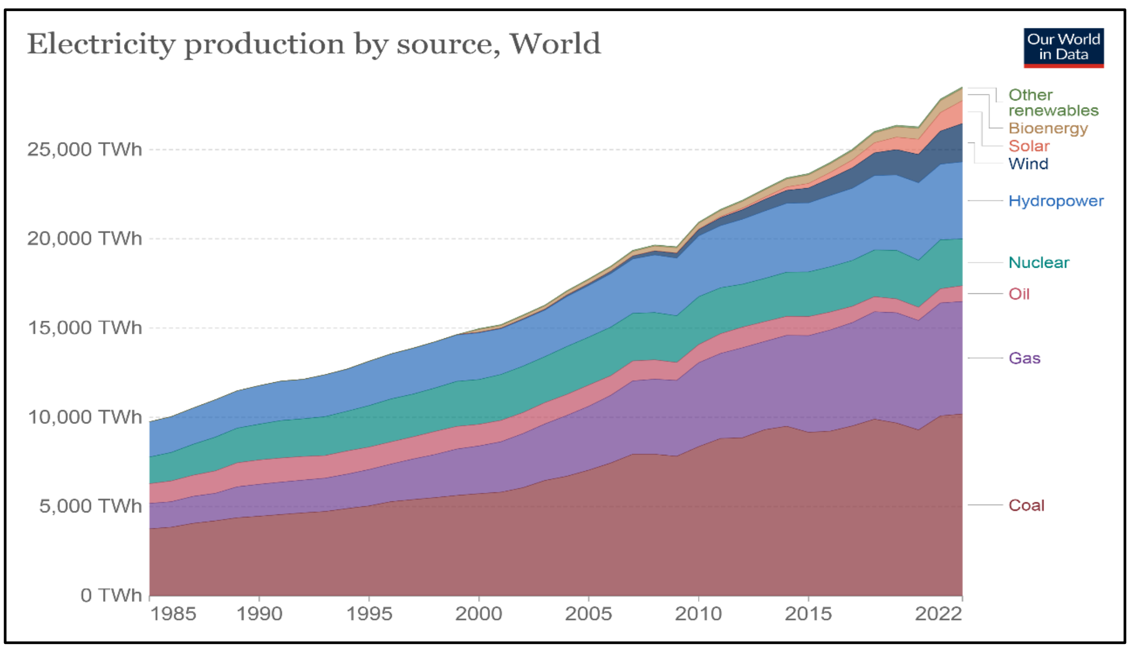

Figure 14. Electrial energy by souce [12]

14. Conclusion

It has been difficult to cover the vast subject of mechanical power and momentum in a relatively brief course. I hope it has summarized many of the key concepts and achieved its objective of providing a conceptual understanding of power and momentum as they apply to engineering. My engineering education sixty years ago was taught entirely using the US System of Units. I tried to write this course in a way that explained the subject in terms of both SI units and US units. I sincerely hope this course has been helpful. Feel free to contact me with questions or corrections. My contact information can be found on my author's page.

Bill Wells PE
Olympia WA

15. References Used

1. This terminology is used consistently in Mechanical Engineering Review Manual
2. Barbrow, L.E. and Judson, L. V. (1976) *Weights and Measures of the United States*. National Bureau of Standards Special Publication 447. p. 5–6
3. Comings, E. W. (1940). "English Engineering Units and Their Dimensions
4. TheConversation.com
5. MechGuru.com
6. U.S. Department of energy
7. HyperPhysics, Georgia State University
8. Convert
9. Omni Calculator
10. AAA Automotive
11. United States E.I.A.
12. BP statistics

