Electrical Power

Part I: Generation
Part II: Distribution Systems
Part III: Transformers
Part IV: Transmission Lines
Part V: The National Electrical Safety Code (NESC)

Theory / Rating / Capacity / Configuration / Testing / Markings / Types of Transformers

by

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Nomenclature[1]

A	ABCD parameter	-
a	phase	-
A	area	m^2
B	ABCD parameter	-
B	magnetic flux density	T
B	magnetic flux density	T
B	susceptance	S, Ω^{-1}, or mho
b	phase	-
c	speed of light	m/s
C	capacitance	F
C	ABCD parameter	-
c	phase	-
D	ABCD parameter	-
D	distance	m
E	electric field strength	V/m
E	energy	J
E	voltage (generated)	V
f	frequency	Hz, s^{-1}, cycles/s
f_{droop}	frequency droop	Hz/kW
G	conductance	S, Ω^{-1}, or mho
GMD	geometric mean distance	m
GMR	geometric mean radius	m
h	specific enthalpy	kJ/kg
I	effective or DC current	A
I	rms phasor current	A
K	correction factor	-
K	skin effect ratio	-
l	length	m
L	inductance	H
m	mass	kg

[1] Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related, and may be found when reviewing the references listed for further information. Further, all the nomenclature, symbols, or subscripts will be found in all "Parts" of this complete course. For guidance on nomenclature, symbols, and electrical graphics: IEEE 280-2021. IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering. New York: IEEE; and IEEE 315-1975. Graphic Symbols for Electrical and Electronics Diagrams. New York: IEEE, approved 1975, reaffirmed 1993.

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M	mutual inductance	H
n	Steinmetz exponent	-
N	number of turns	-
n_s	synchronous speed	r/min or min^{-1}
p	pressure	Pa
P	number of poles	-
P	power	W
pf	power factor	-
pu	per unit	-
Q	heat	J
r	radius	m
R	resistance	Ω
s	specific entropy	$\text{kJ/kg} \cdot \text{K}$
S	apparent power	kVA
SWR	standing wave ratio	-
T	temperature	ºC or K
v	wind velocity	km/hr
V	effective or DC voltage	V
v	velocity (speed)	m/s
V	rms phasor voltage	V
V_{droop}	voltage droop	V/kVAR
VR	voltage regulation	-
W	work	kJ
X	reactance	Ω
x	variable	-
Y	admittance	S, Ω^{-1}, or mho
y	admittance per unit length	S/m, $1/\Omega \cdot \text{m}$ Ω^{-1}, or mho/m [℧ / m]
Z	impedance	Ω
z	impedance per unit length	Ω/m
Z_0	characteristic impedance	Ω

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Symbols

α	turns ratio	-
α	attenuation constant	Np/m
α	thermal coefficient of resistance	1/°C
β	phase constant	rad/m
γ	propagation constant	rad/m
Γ	reflection coefficient	-
δ	skin depth	m
Δ	change, final minus initial	-
ε	permittivity	F/m
ε_0	free-space permittivity	8.854×10^{-12} F/m
ε_r	relative permittivity	-
η	efficiency	-
θ	phase angle	rad
κ	coupling coefficient	-
μ	permeability	H/m
μ_o	free-space permeability	1.2566×10^{-6} H/m
μ_r	relative permeability	-
ξ	ratio of radii	-
ρ	resistivity	$\Omega\cdot$m
σ	conductivity	S/m
ω	armature angular speed	rad/s

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Subscripts

ϕ	phase
0	zero sequence
0	characteristic
0	free space (vacuum)
0,o	initial (zero value)
1	positive sequence
1	primary
2	negative sequence
2	secondary
ab	a to b
AC	alternating current
avg	average
bc	b to c
c	controls or critical
c	core
C	capacitor
ca	c to a
Cu	copper
d	direct
DC	direct current
e	eddy current
e	equivalent
eff	effective
ext	external
f	final / frequency
fl	full load
g	generator
h	hysteresis
int	internal
l	line
l	line
l	per unit length

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L	inductor
ll	line-to-line
m	motor
m	maximum
m	mutual
max	maximum
n	neutral
nl	no load
O	origin
oc	open circuit
p	phase
p	primary
ps	primary to secondary
pu	per unit
q	quadrature
R	receiving end
R	resistance
s	synchronous
s	secondary
S	sending end
sc	short circuit
sys	system
t	terminal
w	wave

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INTRODUCTION

Although this is a five part course, each individual part is meant to be stand-alone should one be interested in that topic. The overall purpose of the course is to provide an overview of electric power from generation, through the various distribution systems, including the vital transformer links that change the voltage from the high voltage required for minimum losses during transmission to medium- and low-voltage for the end-users. Additionally, the transmission lines connecting the system are covered. And, finally, the rule from the National Electric Safety Code® (NESC®) that govern it all completes the overview.

Part I, Generation, the more common type of plants producing the power. The basics of alternating current and direct current generators is explained include the principles of parallel operation. Finally, energy management and power quality are covered.

Part II, Distribution Systems, covers the classification of such systems, how the common neutral is utilized, overhead and underground distribution, along with fault analysis methods.

Part III, Transformers, informs on power transformers, their ratings, voltage regulation, testing methods and parameters used to analyze both transformers and transmission lines.

Part IV, Transmission Lines, discusses the electrical parameters of such line: resistance, inductance, and capacitance. Important effects such as the skin effect and reflection are explained. This part completes with an explanation of models for each type of transmission line: short, medium, and long.

Part V, The National Electrical Safety Code, covers organization of the code and some of the multitude of requirements for the transmission of electrical power.

The information is primarily from the author's books, Refs. [A] and [B] with the NESC information from the Handbook covering the code, Ref. [C]. The coverage of the NESC does not include end-users buildings—this is covered by the NEC, Ref. [D]. Information useful in many aspects of electric engineering may be found in [E] and [F] as well as the appendices. Reference [G] has detailed descriptions of analysis techniques. Reference [H] provides detailed engineering review for Parts I through IV of this course. Reference [I] provides indepth engineering regarding transformers applicable to this particular course, Part III.

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THEORY

A *transformer* is an electrical device consisting of two or more multiturn coils of wire placed in close proximity in order to link the magnetic field of one to the other. This linkage allows the transfer of electrical energy from one or more alternating circuits to one or more other alternating circuits through the process of magnetic induction. The transformer can be used to raise or lower the value of a capacitor, inductor, or resistor. It enables the efficient transmission of electrical energy at high voltage over great distances, and then is used to lower the voltage to safe values for industrial, commercial, and household use. An exact transformer model with the core parameters referred to the primary side is illustrated in Fig.1.

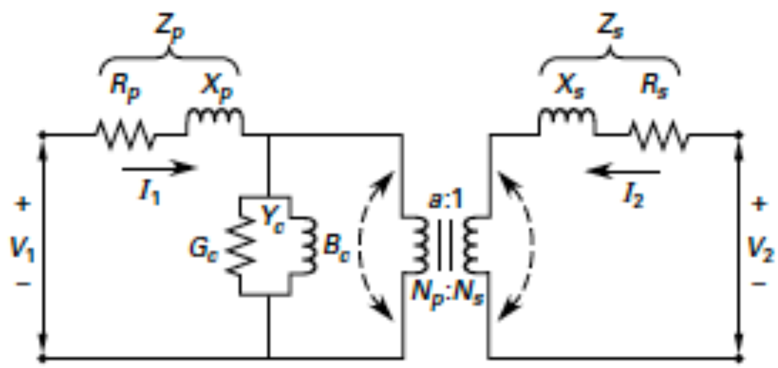

Figure 1: Transformer Model

The parameters are defined as follows.

• R_p is the *primary winding resistance*.

• X_p is the *primary winding reactance*. This is also called the *primary winding leakage reactance*. The *primary winding leakage inductance*, L_p, is sometimes used in the model as well. All the terms represent the inductance not mutually coupled with the secondary winding.

• Z_p is the *primary impedance*, that is, $R_p + jX_p$.

• G_c is the *core conductance*. The reciprocal is sometimes used here and is referred to as the *core resistance*, R_c.

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• B_c is the *core susceptance*. The reciprocal is sometimes used here and is referred to as the *core reactance*, X_c. The reciprocal is also called the *core magnetizing reactance*, X_m. The *core inductance*, L_c, is sometimes used in the model as well.

• Y_c is the *core admittance*, that is, $G_c + jB_c$. The reciprocal is the *core impedance*, Z_c, that is, $R_c + jX_c$.

• The *turns ratio*, N_p/N_s, is given the symbol a, primarily for ease of use in equations. The relationships based on the turns ratio given as, $a = N_p/N_s = V_p/V_s = I_s/I_p = \sqrt{Z_p / Z_s}$, are valid only for ideal transformers, though they are an excellent approximation for large transformers. The relationships are valid for the real transformer in Fig. 1, but only if the electrical parameters are referenced to the ideal portion of the real model, shown by the dashed lines.[2]

• X_s is the *secondary winding reactance*. This is also called the *secondary winding leakage reactance*. The *secondary winding leakage inductance*, L_s, is sometimes used in the model as well. All the terms represent the inductance not mutually coupled with the primary winding.

• R_s is the *secondary winding resistance*.

• Z_s is the secondary impedance, that is, $R_s + jX_s$.

TRANSFORMER RATING

Transformers are rated according to the winding voltages, frequency, and kVA rating. Continuous operation at rated values is possible without excessive heat build-up or malfunction of the transformer. The apparent power (measured in kVA) rather than the real power (measured in kW) is used to rate the transformer because the total heating effect depends on the square of the actual current flowing, including that moving in the system between reactive components.

The two main power losses in transformers are *core losses* (*iron losses*) and *copper losses* (*winding losses*). Core losses, represented in Fig. 1 by G_c, consist of *eddy current losses* and *hysteresis*

[2] The ideal transformer model is not used for power transformers because the efficiency, regulation, and power factor must be determined. In this course, the subscripts *p* and *s* used for ideal transformers are not used on the current and voltage in the real transformer. They are reserved for electrical parameters of the ideal transformer, that is, the parameters referring to the area of the dashed lines in Fig. 1.

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losses. Eddy current losses, P_e, are the result of microscopic circulating currents in the iron caused by the magnetic flux passing through. Hysteresis losses, P_h, are caused by the cyclic changes in the magnetic state of the iron. Both these losses are constant and independent of transformer load. That is, *they do not vary from no-load to full-load conditions*, but they do depend on the mass of iron, m, of the transformer core. Equation 1 and Eq. 2 can be used to calculate the losses. The maximum flux density, B_{max}, is calculated from Faraday's law. The exponent n is called the *Steinmetz exponent* and varies from 1.5 to 2.5 with a common value of 1.6. The coupling coefficient, k, depends on the transformer design.

Equation 1: Eddy Current Loss

$$P_e = \kappa_e B_{\max}^2 f^2 m$$

Equation 2: Hysteresis Loss

$$P_h = \kappa_h B_{\max}^n fm$$

The core losses, ignoring the small line loss caused by R_p, may be approximated from the transformer model in Fig. 1 using the following formula.

Equation 3: Core Loss

$$P_c = \frac{V_1^2}{R_c}$$

The term R_c is the reciprocal of the core conductance, G_c. Copper losses, P_{Cu}, are caused by the total wire resistance and can be calculated from the following.

Equation 4: Copper Loss

$$P_{Cu} = I^2 R = I_1^2 R_p + I_2^2 R_s$$

The *transformer efficiency* is the ratio of the output power to the input power and is at a maximum when the copper losses equal the core losses; that is, when the variable losses equal the constant losses. The *all-day efficiency* is the ratio of energy delivered by a transformer in a 24-hour period to the energy input during the same period.

Equation 5: Transformer Efficiency

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{in}} - \Sigma P_{\text{losses}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{out}} + P_c + P_{\text{Cu}}}$$

Example 1

Determine the hysteresis power loss at 60 Hz for a 500 kVA-rated 200 kg iron core transformer with a coupling coefficient of 4×10^{-4} and a Steinmetz exponent of 1.6 in a 1.4 T peak magnetic field.

Solution

The hysteresis loss is determined from Eq. 2. All the terms are given; but could be looked up in specification sheets or drawings.

$$\begin{aligned} P_h &= \kappa_h B_{\max}^n fm \\ &= \left(4 \times 10^{-4}\right)\left(1.4\ \text{T}\right)^{1.6}\left(60\ \text{Hz}\right)\left(200\ \text{kg}\right) \\ &= 8.22\ \text{W} \end{aligned}$$

See App. A to aid in unit analysis.

Example 2

The transformer in Ex. 1 uses a primary voltage of 345 kV. If the core resistance is 200 MΩ, estimate the total core power loss.

Solution

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The total core losses can be estimated using Eq. 3.

$$P_c = \frac{V_1^2}{R_c}$$

$$= \frac{\left(345\times10^3\right)^2}{200\times10^6\ \Omega}$$

$$= 595\ \text{W}$$

TRANSFORMER CAPACITY

The capacity of a given transformer is determined by its ability to handle the maximum current without changing the properties of the conducting material, which is normally copper, or overheating and breaking down the insulation. In addition to these concerns, each transformer has losses that reduce the available power.

The capacity is given in terms of the apparent power, *S*, in kVA. The total capacity of a three-phase transformer is √3 times larger than the capacity of a single-phase transformer at the same voltage and current.[3]

VOLTAGE REGULATION

Voltage regulation is the change in the output voltage from the no-load to the full-load condition.

Equation 6: Voltage Regulation

$$\text{VR} = \frac{V_{nl} - V_{fl}}{V_{fl}}$$

The voltage regulation can be expressed in terms of rated quantities. At no load, reflecting the primary voltage to the secondary side gives V_p/a. The rated conditions of the secondary represent full load. The voltage regulation is then

[3] Any three-phase circuit power equation includes the factor √3 , which is approximately 1.73.

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Equation 7: Voltage Regulation

$$\mathrm{VR} = \frac{\frac{V_p}{a} - V_{s,\text{rated}}}{V_{s,\text{rated}}}$$

The primary voltage, coming from the large generation source, can be considered constant. That is, the primary side is considered an infinite bus. This assumption is embedded in any regulation equation based on rated values and also in per-unit calculations of the regulation. (See Ref. [J] for an explanation of the per-unit system). With output voltage considered to have a 1.0 pu value, the difference between the primary voltage and the output voltage of the transformer secondary in per-unit values is the regulation. That is, to supply the load current and secondary voltage at rated values, the primary voltage must be greater than the secondary. The excess is termed the regulation and is found by using Eq. 7 with per-unit values instead of rated values.

Example 3

A single-phase transformer rated for 50 kVA operates at a rated secondary voltage of 440 V and 0.8 pf lagging. The primary winding impedance is

$$Z_p = R_p + jX_p = 0.014 + j0.026 \text{ pu}$$

What is the voltage regulation in percent?

Solution

To determine the regulation, the primary voltage is required. Because per-unit impedance is given, the per-unit method will be used. The output voltage, (i.e., the secondary voltage) is 1.0 pu at rated conditions. The output current is 1.0 pu as well. Because the power factor is 0.8 lagging, the current is 1.0 pu$\angle -36.9°$ per unit. The impedance in polar form is

$$Z_p = 0.014 + j0.026 \text{ pu} = 0.030 \text{ pu} \angle 61.7^\circ$$

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Using the model given in Fig. 1 and noting that the per-unit current in the secondary must also flow in the primary, the phase voltage at the primary of the transformer (the voltage at the dashed lines) is

$$\begin{aligned} V_{\text{primary phase}} &= I_{\text{pu}} Z_P \\ &= \left(1.0 \text{ pu} \angle -36.9^\circ\right)\left(0.030 \text{ pu} \angle 61.7^\circ\right) \\ &= 0.030 \text{ pu} \angle 24.8^\circ \end{aligned}$$

Reflecting this voltage to the secondary side does not require the turns ratio since per-unit values are in use.[4] Given that the secondary voltage is 1.0 pu, at an angle of 0° since it is the reference, the primary voltage is

$$\begin{aligned} V_{\text{primary reflected}} &= V_{\text{primary phase,pu}} + V_{s,\text{pu}} \\ &= 0.030 \text{ pu} \angle 24.8^\circ + 1.0 \text{ pu} \angle 0^\circ \\ &= 1.027 \text{ pu} \angle 0.7^\circ \end{aligned}$$

The voltage regulation is not concerned with the resulting angle. The regulation is given by

$$\begin{aligned} \text{VR} &= V_{\text{primary reflected,pu}} - V_{\text{rated,pu}} \\ &= 1.027 \text{ pu} - 1.0 \text{ pu} \\ &= 0.027 \text{ pu or } 2.7\% \end{aligned}$$

THREE-PHASE TRANSFORMER CONFIGURATIONS

Three-phase transformers can be connected as either delta or wye. When voltage or current ratios are given for three-phase transformers, they are *assumed to specify the line conditions*, regardless of the connection. If only the turns ratio, *a*, is given, the line quantities must be calculated. For delta connections, the line and phase voltage and current are given by Eq. 8 and Eq. 9, respectively.

[4] The advantage of the per-unit system is that by choosing base primary and secondary voltages related by the turns ratio, which occurs in this example by letting the secondary voltage be the base and relating the primary voltage to it, the transformer is no longer required in the electrical model.

Equation 8: Delta Line & Phase Voltage

$$V_l = V_\phi \quad \text{[delta]}$$

Equation 9: Delta Line & Phase Current

$$I_l = \sqrt{3}\, I_\phi \quad \text{[delta]}$$

For wye connections, the line and phase voltage and current are given by Eq. 10 and Eq. 11, respectively.

Equation 10: Wye Line & Phase Voltage

$$V_l = \sqrt{3}\, V_\phi \quad \text{[wye]}$$

Equation 11: Wye Line & Phase Current

$$I_l = I_\phi \quad \text{[wye]}$$

Because either the voltage or current contains a factor of √3, the power for either connection type is given by the following.

Equation 12: Transformer Power

$$P = \sqrt{3}\, I_l V_l \,(\text{pf}) = \sqrt{3}\, I_\phi V_\phi \,(\text{pf})$$

Figure 2 shows how the connections would be displayed in an electrical schematic. The line-phase relationships are illustrated in Fig. 3 for various connection types.[5]

[5] The turns ratio, *a*, is sometimes given as the ratio of the secondary to the primary turns. If this is the case, switch the *a* from the numerator to the denominator (or from the denominator to the numerator, depending on where *a* originally appears) to obtain the correct relationship.

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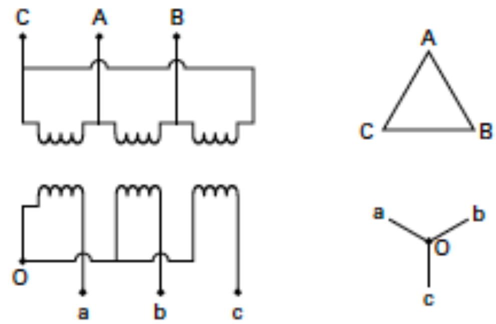

Figure 2: Standard Transformer Schematic

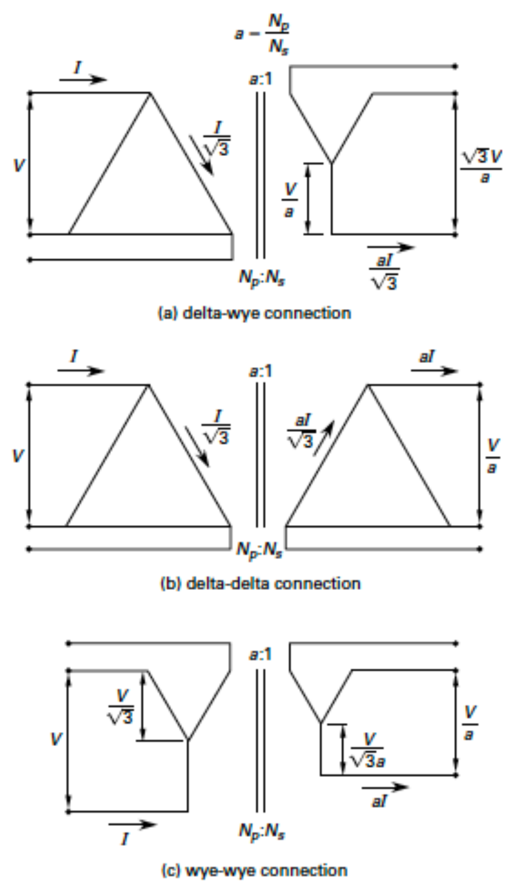

Figure 3: Transformer Connections

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Wye connections, with a common neutral, offer economic and operating advantages. Delta connections allow third-harmonic voltages common to all transformers to circulate within the delta so that line electrical parameters are unaffected. For this reason, transformers normally have at least one delta-connected winding. Additionally, delta connections can suffer a failure of a single phase and still provide 57.7% of their rated load.[6]

TRANSFORMER TESTING

Transformer testing is done to determine the parameters for a real transformer, a model of which is shown in Fig. 1. Based on this model, testing determines the rating, efficiency, and equivalent circuit values. Two standardized tests are performed: open circuit and short circuit. Open-circuit and short-circuit conditions are established on the low-voltage side of the transformer, which can be either the primary or the secondary side.[7]

The primary voltage and current, the secondary voltage and current, and the power are measured in each test. From these values, the transformer's properties are derived. The open-circuit test determines the core parameters and the turns ratio. The short-circuit test determines the winding impedances and verifies the turns ratio. The admittance parameter, Y, accounts for the power loss in the core. The susceptance parameter, B, accounts for energy storage in the core. The resistance parameter, R, accounts for power loss in the windings. The reactance parameter, X, accounts for the leakage (self-inductance) of the primary and secondary windings.

OPEN-CIRCUIT TEST

The open-circuit test determines the core parameters and the turns ratio. An open-circuit test is performed by opening the secondary terminals of the transformer. Actually, a voltmeter measures the secondary voltage, V_{2oc}, but the meter has such high resistance it can be considered an open circuit. The rated voltage, V_{1oc}, is applied to the primary terminals. (The rated voltage is used because the core losses are dependent on the magnetic flux density, B_{max}.) The input current, I_{1oc}, is measured as well as the input power, P_{oc}. Because the current flow I_{1oc} is small, the voltage drop across Z_p is negligible. The open-circuit power, P_{oc}, represents the core power loss. The input open-circuit current, I_{1oc}, is the exciting current that maintains the flux in the core. The open-circuit model is shown in Fig. 4.

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[6] When only two phases of a delta connection are purposefully used, it is called an *open delta* or *vee transformer*.
[7] Because of this, the terms "high-voltage" (HV) and "low-voltage" (LV) are sometimes used instead of the terms "primary" and "secondary." The terms "high-tension" and "low-tension," synonymous with high- and low-voltage, respectively, are also sometimes used.

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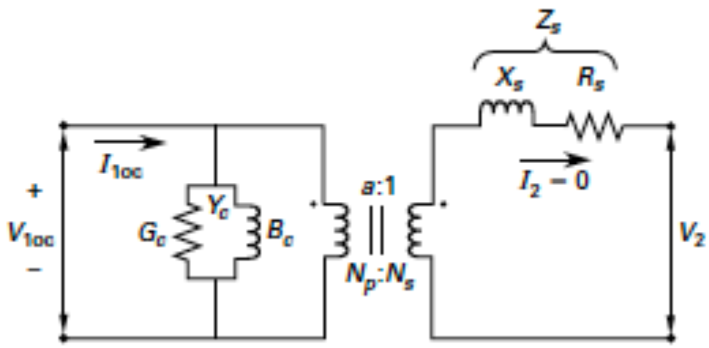

Figure 4: Transformer Open-Circuit Test Model

The admittance is

Equation 13: Open-Circuit Admittance

$$Y_c = G_c + jB_c = \frac{I_{1oc}}{V_{1oc}}$$

The conductance is

Equation 14: Open-Circuit Conductance

$$G_c = \frac{P_{oc}}{V_{1oc}^2}$$

The susceptance is

Equation 15: Open-Circuit Susceptance

$$B_c = \frac{1}{X_c} = \frac{-1}{\omega L_c} = -\sqrt{Y_c^2 - G_c^2} = \frac{-\sqrt{I_{1oc}^2 V_{1oc}^2 - P_{oc}^2}}{V_{1oc}^2}$$

The susceptance is negative ($-1/\omega L$) for a lagging condition.

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The turns ratio is[8]

Equation 16: Open-Circuit Turns Ratio

$$a_{ps} = \frac{V_{1\text{oc}}}{V_{2\text{oc}}}$$

The power is

Equation 17: Open-Circuit Power

$$P_{\text{oc}} = V_{1\text{oc}}^2 G_c$$

The reactive power is

Equation 18: Open-Circuit Reactive Power

$$Q_{\text{oc}} = V_{1\text{oc}}^2 B_c$$

The apparent power is

Equation 19: Open-Circuit Apparent Power

$$S_{\text{oc}} = V_{1\text{oc}}^2 Y_{\text{oc}} = V_{1\text{oc}} I_{1\text{oc}} = \sqrt{P_{\text{oc}}^2 + Q_{\text{oc}}^2}$$

Example 4

An open-circuit test is conducted on a 120 V transformer winding. The results of this test follow.

$V_{1\text{oc}}$ = 120 V
$V_{2\text{oc}}$ = 240 V
$I_{1\text{oc}}$ = 0.25 A
P_{oc} = 20 W

[8] The subscript "ps" is added to clarify the turns ratio as being from the primary to the secondary, as is standard for this course and the author's texts. The turns ratio is sometimes used to indicate the secondary to primary ratio. Care should be taken to determine the definition used in a particular situation.

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What are the transformer equivalent circuit element parameters given by this data?

Solution

The admittance is found from Eq. 13.

$$Y_c = G_c + jB_c = \frac{I_{1\text{oc}}}{V_{1\text{oc}}} = \frac{0.25 \text{ A}}{120 \text{ V}}$$
$$= 2.08 \times 10^{-3} \text{ S}$$

The core conductance is found from Eq. 14.

$$G_c = \frac{P_{\text{oc}}}{V_{1\text{oc}}^2} = \frac{20 \text{ W}}{(120 \text{ V})^2}$$
$$= 1.39 \times 10^{-3} \text{ S}$$

The core susceptance is found from Eq. 15.

$$B_c = -\sqrt{Y_c^2 - G_c^2}$$
$$= -\sqrt{(2.08 \times 10^{-3} \text{ S})^2 - (1.39 \times 10^{-3} \text{ S})^2}$$
$$= -1.55 \times 10^{-3} \text{ S}$$

The turns ratio is found from Eq. 16.

$$a_{ps} = \frac{V_{1\text{oc}}}{V_{2\text{oc}}} = \frac{120 \text{ V}}{240 \text{ V}}$$
$$= 0.5$$

Note: Appendix A may be helpful in verifying units.

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SHORT-CIRCUIT TEST

The short-circuit test determines the winding impedances and verifies the turns ratio. A short-circuit test is performed by shorting the secondary terminals of the transformer. Actually, an ammeter measures the secondary current, I_{2sc}, but the meter has such low resistance that it can be ignored in the analysis of the circuit. A voltage, V_{sc}, is applied to the primary such that the rated current, that is, the volt-amp rating divided by the voltage rating, flows. The voltage will be low because the secondary winding has minimal impedance. The voltage is normally less than 5% of the rated value. Because of this minimal secondary impedance, the core admittance, Y_c, is considered shorted. The effective circuit then consists of the primary impedance in series with the reflected secondary impedance. The input current, I_{1sc}, and output current, I_{2sc}, are measured as well as the input power, P_{sc}. The short-circuit power, P_{sc}, represents the copper loss or I^2R losses in the windings. The short-circuit test model is shown in Fig. 5.

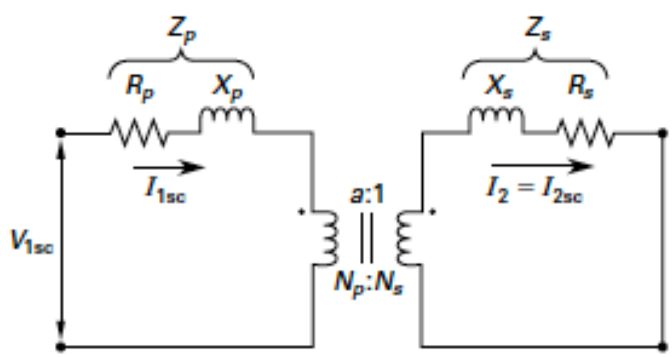

Figure 5: Transformer Short-Circuit Test Model

The total impedance, Z, is

Equation 20: Short-Circuit Total Impedance

$$Z = \frac{V_{1sc}}{I_{1sc}} = R_p + jX_p + a_{ps}^2\left(R_s + jX_s\right)$$

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The total resistance, *R*, is

Equation 21: Short-Circuit Total Resistance

$$R = \frac{P_{\text{sc}}}{I_{1\text{sc}}^2} = R_p + a_{\text{ps}}^2 R_s$$

To maximize efficiency, transformers are normally designed with $R_p = a_{\text{ps}}^2 R_s$. Therefore,

Equation 22: Max Efficiency Resistance

$$\begin{aligned} R_p &= a_{\text{ps}}^2 R_s \\ &= \frac{P_{\text{sc}}}{2I_{1\text{sc}}^2} \end{aligned}$$

The total reactance, *X*, is

Equation 23: Short-Circuit Total Reactance

$$\begin{aligned} X &= X_p + a_{\text{ps}}^2 X_s \\ &= \frac{\sqrt{I_{\text{sc}}^2 V_{\text{sc}}^2 - P_{\text{sc}}^2}}{I_{1\text{sc}}^2} \end{aligned}$$

To maximize efficiency, transformers are normally designed with $X_p = a_{\text{ps}}^2 X_s$. Therefore,

Equation 24: Max Efficiency Total Reactance

$$\begin{aligned} X_p &= a_{\text{ps}}^2 X_s \\ &= \frac{Q_{\text{sc}}}{2I_{1\text{sc}}^2} \end{aligned}$$

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The turns ration is

Equation 25: Short-Circuit Turns Ratio

$$a_{\text{ps}} = \frac{I_{2\text{sc}}}{I_{1\text{sc}}}$$

The power is

Equation 26: Short-Circuit Power

$$P_{\text{sc}} = I_{1\text{sc}}^2 R = I_{1\text{sc}}^2 \left(R_p + a_{\text{ps}}^2 R_s \right)$$

The reactive power is

Equation 27: Short-Circuit Reactive Power

$$Q_{\text{sc}} = I_{1\text{sc}}^2 X = I_{1\text{sc}}^2 \left(X_p + a_{\text{ps}}^2 X_s \right)$$

The apparent power is

Equation 28: Short-Circuit Apparent Power

$$S_{\text{sc}} = I_{1\text{sc}}^2 Z_{\text{sc}} = V_{1\text{sc}} I_{1\text{sc}} = \sqrt{P_{\text{sc}}^2 + Q_{\text{sc}}^2}$$

Example 5

A short-circuit test is conducted on a transformer rated at 15 kVA and 1300 primary volts.

What is the value of the input current, $I_{1\text{sc}}$?

Solution

Short-circuit tests are conducted at the rated current, that is, $I_{1\text{sc}} = I_{\text{rated}}$. The rated current is found from the following.

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$$I_{\text{rated}} = \frac{S_{\text{rated}}}{V_{\text{rated}}} = \frac{15\times10^3 \text{ VA}}{1300 \text{ V}} = 11.5 \text{ A}$$

ABCD PARAMETERS

ABCD parameters, also known as *transfer* or *chain parameters*, are analytic tools for transmission and distribution problem-solving. To use them on transformers, the secondary impedance is reflected to the primary as shown in Fig. 6. The result is a two-port network. The ABCD parameters for any two-port network are

Equation 29: Parameters A and B

$$V_{\text{in}} = \text{A}V_{\text{out}} - \text{B}I_{\text{out}}$$

Equation 30: Parameters C and D

$$I_{\text{in}} = \text{C}\text{V}_{\text{out}} - \text{D}I_{\text{out}}$$

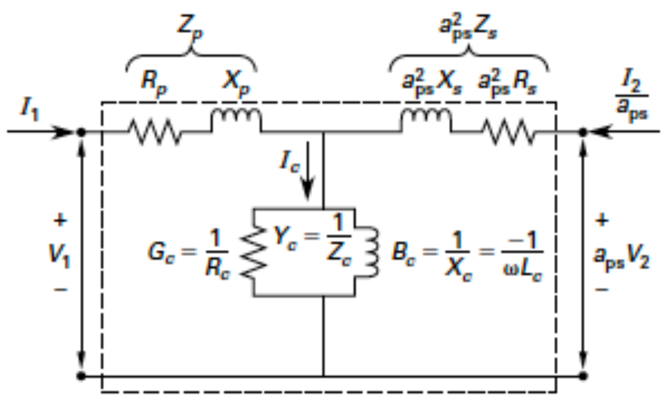

Figure 6: Transformer Two-Port Network

Circuit analysis of the transformer yields Eq. 31 through Eq. 35, which are used to determine the ABCD parameters.[9]

---

[9] Care should be used when dealing with the susceptance in Eq. 35. Susceptance is negative for an inductive element.

Equation 31

$$V_1 = a_{\text{ps}} V_2 (1 + Z_p Y_c) - \left(\frac{I_2}{a_{\text{ps}}} \right) \left(Z_p + a_{\text{ps}}^2 Z_s \left(1 + Z_p Y_c \right) \right)$$

Equation 32

$$I_1 = a_{\text{ps}} V_2 Y_c - \left(\frac{I_2}{a_{\text{ps}}} \right) \left(1 + a_{\text{ps}}^2 Z_s Y_c \right)$$

Equation 33

$$Z_p = R_p + jX_p$$

Equation 34

$$Z_s = R_s + jX_s$$

Equation 35

$$Y_c = G_c + jB_c$$

To *maximize efficiency*, transformers are normally designed with Z_p equal to $a_{\text{ps}}^2 Z_s$. In this case, the ABCD parameters are given by Eq. 36 through Eq. 39.

Equation 36: Max Efficiency Parameter A

$$\text{A} = 1 + Z_p Y_c$$

Equation 37: Max Efficiency Parameter B

$$\text{B} = Z_p \left(2 + Z_p Y_c \right)$$

Both the susceptance, B_c, and the associated reactance, X_c, carry any negative sign internally.

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Equation 38: Max Efficiency Parameter C

$$C = Y_c$$

Equation 39: Max Efficiency Parameter D

$$D = A$$

ABCD parameters can be used for two-port networks that are chained together, such as power distribution systems and cascaded amplifiers. For two such cascaded networks, the ABCD parameters are given by Eq. 40 through Eq. 43.

Equation 40: Cascaded Networks Parameter A

$$A = A_1A_2 + B_1C_2$$

Equation 41: Cascaded Networks Parameter B

$$B = A_1B_2 + B_1D_2$$

Equation 42: Cascaded Networks Parameter C

$$C = A_2C_1 + C_2D_1$$

Equation 43: Cascaded Networks Parameter D

$$D = B_2C_1 + D_1D_2$$

The ABCD parameters for common two-port network types are given in Fig. 7.

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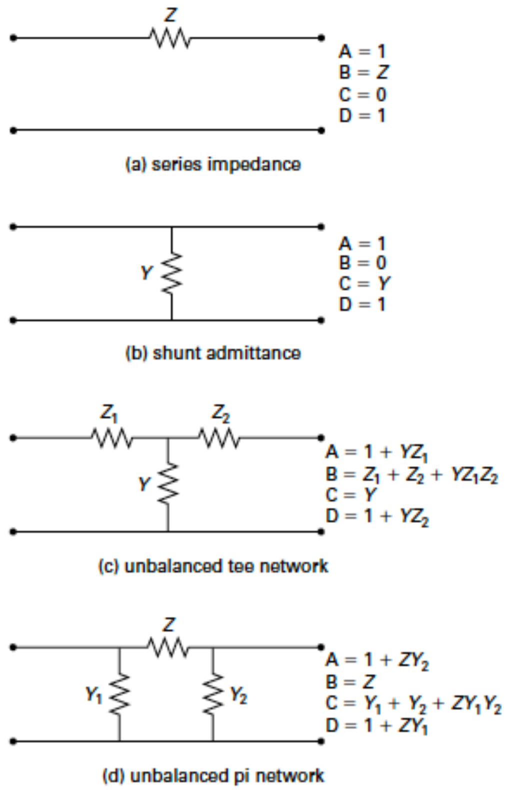

Figure 7: Two-Port Network ABCD Parameters

TERMINAL MARKING AND POLARITY

The term *connections* as used in regard to transformers refers to the terminal markings and connections of power transformers (as opposed to instrument transformers), distribution transformers, and regulating transformers.[10] A *power transformer* is any transformer that transforms energy between a generator and the distribution primary circuits. *Distribution transformers* transform energy from the primary distribution circuit during transmission to the secondary distribution circuit or to a consumer's service circuit.[11] *Regulating transformers* control the voltage, phase angle, or both to an output circuit and compensate for fluctuations of input voltage or load. *Instrument transformers* isolate and transform voltage or current levels, and are

[10] For more information on markings and connections for transformers, see IEEE Standard C57.12.70, *IEEE Standard for Standard Terminal Markings and Connections for Distribution and Power Transformers*. A related international standard is *Power Transformers—Part 1: General* (IEC 60076-1).
[11] Distribution transformers are generally rated for power in the range of 5 kVA to 500 kVA.

designed to reproduce the voltage or current of the primary circuit in the secondary circuit, with the phase relationship and waveform effectively unchanged.

The terminals on the high-voltage side of a transformer are marked HV or H. The terminals on the low-voltage side are marked LV or X. If additional windings exist, they are labeled Y and Z in order of decreasing voltage. If the voltages are the same, then the winding with the highest apparent power (i.e., kVA) rating is marked X, the next Y, and the lowest Z. If the voltages and apparent power ratings are identical between the windings, the designations are arbitrary. In that case, multiple terminals are marked H_1 and H_2, with the corresponding opposite sides marked X_1 and X_2.

The neutral of a wye connection is labeled H_0 or X_0, depending on the location of the neutral (that is, depending on whether the neutral is on the high-voltage side, H, or the low-voltage side, X).

If a two-terminal transformer winding is grounded, the side ground always has the subscript 2, as in H_2 or X_2.

Transformers generally have subtractive polarity, which is indicated by placing the H_1 and X_1 terminals directly across from each other. Subtractive polarity of a transformer (which is physically indicated on a transformer) is equivalent to the negative mutual inductance of a transformer (which is based on theory). Knowing the polarity and the mutual inductance sign ensures that Kirchhoff's voltage law (KVL) analysis of primary and secondary transformer circuits is performed correctly.11 Viewed from the high-voltage side, the H_1 terminal is always located to the right of the transformer case, as Fig. 8 illustrates.[12]

When a transformer has multiple taps (that is, multiple windings whose terminals are external to the transformer), the full winding extends from the lowest to the highest number (for example, from X_1 to X_5), and the intermediate numbers indicate the taps.

[12] The ground is optional and is used as a reference for the voltage.

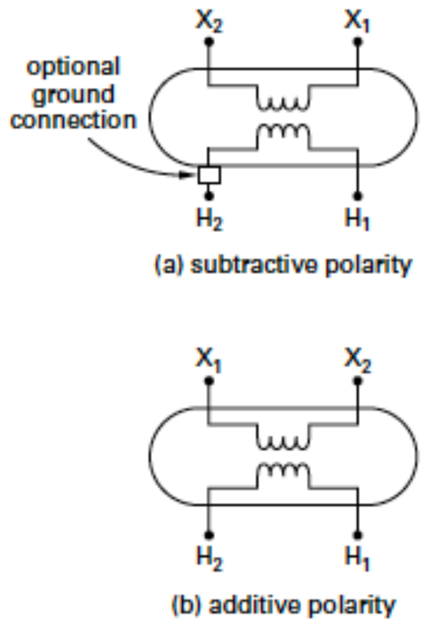

Figure 8: Transformer Polarity Types

TRANSFORMER PARALLEL OPERATION

To expand capacity, transformers may be connected for parallel operation if the following conditions between the two transformers are met: (1) similarly marked terminals must be connected; (2) ground connections must be compatible; and (3) the following values for the transformers must match.

- turns ratios
- primary voltages
- secondary voltages
- resistance and reactance (that is, impedance values)

Properly connected single-phase transformers that are operating in parallel are shown in Fig. 9. Three single-phase transformers connected as a three-phase transformer are shown in Fig. 10, with the primary side shown at the top of the figure and the secondary at the bottom, as is common practice. If combinations of additive and subtractive polarity are used, the delta and wye connections shown remain the same. That is, the same H and X terminals are connected to one another; only the physical locations of the X_1 and X_2 terminals change.

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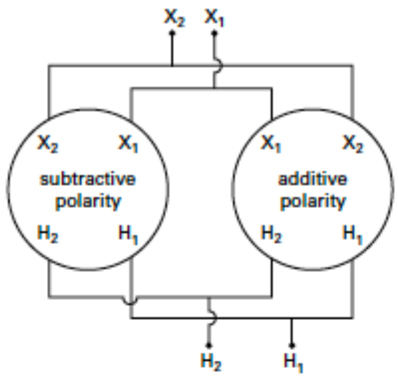
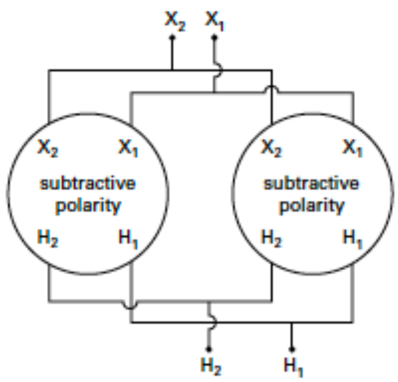
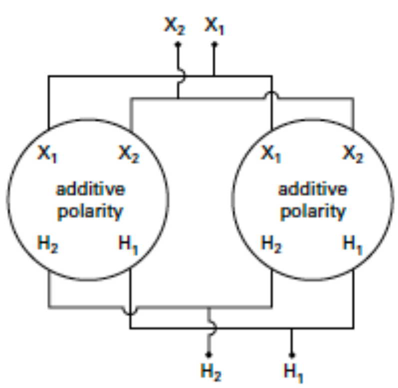


(a) subtractive-additive polarity



(b) subtractive-subtractive polarity



(c) additive-additive polarity

Figure 9: Parallel Transformer Connections[13]

[13] If one ever has to do this, a good review is in order and a second set of qualified eyes would also be useful.

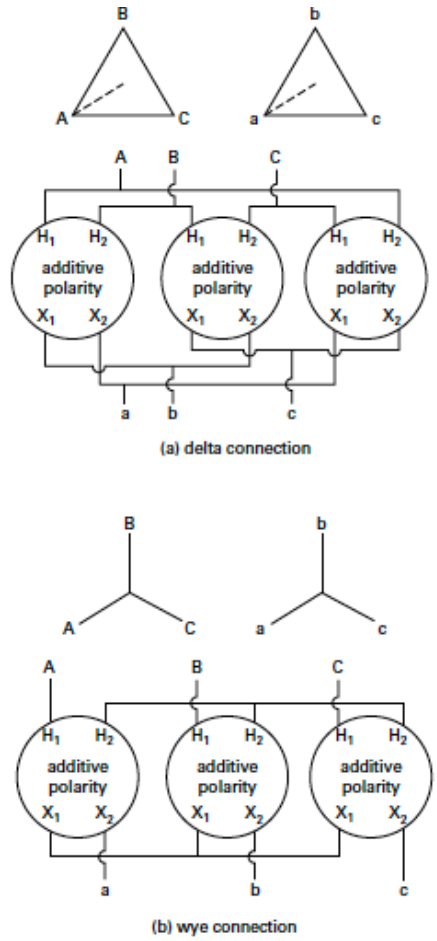


Figure 10: Three-Phase Transformer Connections

PHASE MARKINGS

Markings on transformers indicate the phase sequence and angular displacement of the transformer. For example, if the high voltage markings are H_1, H_2, and H_3, this indicates that the phase sequence is from winding 1, to winding 2, to winding 3. Such a sequence is known as an *a-b-c rotation*. The secondary windings will be marked X_1, X_2, and X_3, indicating the corresponding phases for an a-b-c rotation.

The angular displacement is the angle between the high-voltage sides and their corresponding low-voltage sides. It is measured clockwise starting from the line connecting H_1 to the neutral, to the

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line connecting X*1* to the neutral. The neutral of a delta connection is in the center of the triangle formed by the three windings. The neutral of a wye connection is at the center of the wye, where all connections meet. In a delta-delta or wye-wye connection, the angular displacement is 0°, as shown in Fig. 11. In Fig. 11(a), the dashed lines indicate the line connecting H_1 to the neutral on the high-voltage side, and connecting X_1 to the neutral on the low voltage side. Since the dashed lines are parallel to each other and in the same direction, the angular displacement between them is 0°. In Fig. 11(b), no dashed lines are used because one winding connection is from H_1 to the neutral, and the other is from X_1 to the neutral. The displacement between these two lines is also 0°. When a transformer is changed from delta to wye or from wye to delta, a 30° displacement is expected, as shown in Fig. 12. For delta-wye and wye-delta connections, the displacement between the high-voltage side and the low-voltage side is 30°.

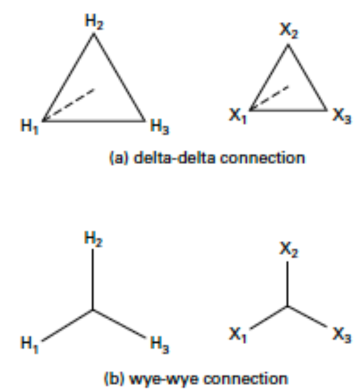

Figure 11: Angular Displacement of 0°

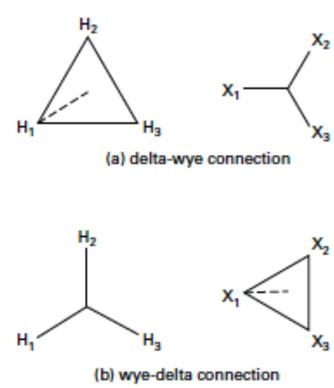

Figure 12: Angular Displacement of 30°

Transformer principles and the analytic methods in this course are applicable to all types of transformers. The transformer model, which is based on the transformer type and the analysis method used, changes based on the transformer rating and the relative magnitudes of the electrical parameters.

Power transformers are large transformers that transfer energy between a generator and primary distribution circuits. In such large transformers, the total leakage reactance, X, is five times the value of the total resistance, R. As a result, a power transformer can only be accurately represented in a diagram by a reactance as shown in Fig. 13(a).

Distribution transformers are transformers that transfer electrical energy from a primary distribution circuit to a secondary distribution circuit. Because the transformer primaries are connected to a large, essentially infinite bus, the voltage and power factors can be considered constant. The load on an individual distribution transformer does not change the primary voltage. Consequently, the distribution transformer can only be represented in a diagram by the series portion of the equivalent impedance (that is, by the total leakage reactance and the total winding resistance, as shown in Fig. 13(b)).

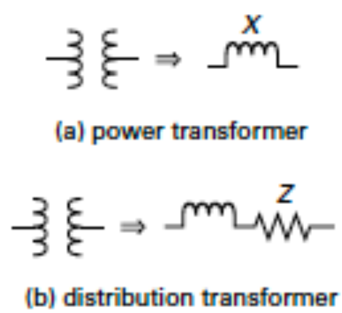

Figure 13: Special Transformer Models

Furnace transformers supply electric furnaces (e.g., welding machines) of the induction, resistance, open-arc, or submerged-arc types. The secondary voltage is in the range of a few hundred volts, with currents as high as 100,000 A.

Grounding transformers act as a neutral point for grounding purposes.

Instrument transformers adjust current and voltage to levels that metering devices utilize while isolating the metering circuit from the system being measured.

Typical per-unit values for transformer parameters are given in Table 1.

Table 1: Typical Transformer Values*

parameter (see Fig. 27.6)	typical per-unit values	
	3–250 kVA	1–100 MVA
R_p or R_s	0.009–0.005	0.005–0.002
X_p or X_s	0.008–0.025	0.030–0.060
R_c	20–50	100–500
X_c	20–30	30–50
I_c	0.05–0.03	0.03–0.02

*The values for transformers rated between 250 kVA and 1 MVA vary, and their value ranges overlap with the values for transformers rated between 3-250 kVA and 1-100 MVA.

BUCK / BOOST / AUTO TRANSFORMERS

An *autotransformer* has only one winding that is common to both the primary and secondary circuits associated with that winding. Current flowing in the output comes directly from the primary winding. (The primary voltage generates a current through the output load.) A portion comes from the inductive process, generating additional voltage and current through the output load. (Primary windings generate a voltage in the secondary windings.)

An autotransformer is also called a *buck* or *boost transformer*, depending on the connection.[14] Connections to the primary and secondary windings are made on one winding either to *boost* the input, which raises the output, or to *buck* the input, which lowers the output. Fig. 14(a) shows an example of a three-phase autotransformer configured to boost the input, and Fig. 14(b) shows one configured to buck the input. In the autotransformer in Fig. 27.14, the single winding has two endpoints and one or more terminals at intermediate points to provide the desired output. Such a transformer can also be automated to move from terminal to terminal in a given time frame in order to start large loads with a minimum starting current.

[14] The term autotransformer describes a unit, such as a large motor starter, that is designed to minimize starting transients by using a lower voltage at start-up and then automatically raising the voltage to the rated value. Buck or boost transformers often have taps that must be manually moved and are set once at the desired value for a given installation.

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The actual wiring configuration of boost and buck transformers is shown in Fig. 15. In terms of apparent power, the capacity of an autotransformer compared to a two-winding transformer is

Equation 44: Autotransformer vs. Two-Winding Transformer

$$S_{\text{at}} = S_{\text{tw}}\left(1+\frac{N_s}{N_P}\right) = S_{\text{tw}}\left(1+\frac{N_2}{N_1}\right)$$

N_2/N_1 is always greater than one, so the capacity of an autotransformer is always greater than that of a similar two-winding transformer. Generally, the boost (the increase in output voltage over the input voltage) and the buck (the decrease in output voltage under the input voltage) are limited to about 20% of the input.

An autotransformer is generally smaller, lighter, and less expensive than an equivalent traditional transformer with two separate windings, because less material is required for its fabrication.[15] One disadvantage of the autotransformer is the lack of electrical isolation between the primary and secondary windings.[16] In addition, if the insulation fails (i.e., if a short occurs), the full output voltage may be applied to the load. Finally, should there be a break in the winding (i.e., an opening), the transformer is then an inductor in series with the load, which under a light load may also result in the full output voltage being applied to the load.

As shown in Fig. 27.14, autotransformers can be used to regulate distribution system voltages to account for losses, or to change voltage from a standard distribution value (208 V or 240 V) to one required by an end user (240 V or 120 V).

[15] This is true for turns ratios up to about 3:1, after which a standard two-winding transformer is just as economical.
[16] Grounds on the secondary will be seen as a fault by the primary rather than being isolated as in a two-winding transformer. Also, any disturbances on the primary will be felt directly on the secondary loads.

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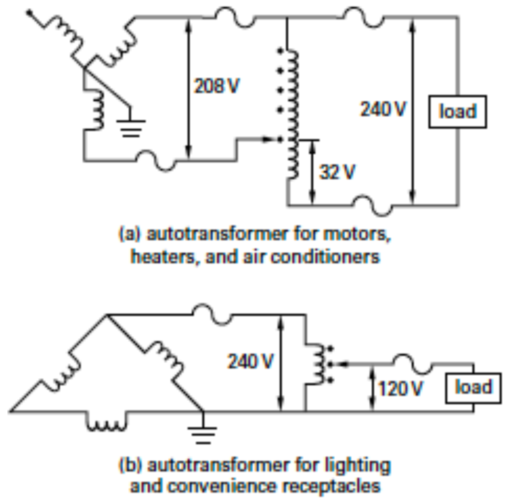


Figure 14: Autotransformers

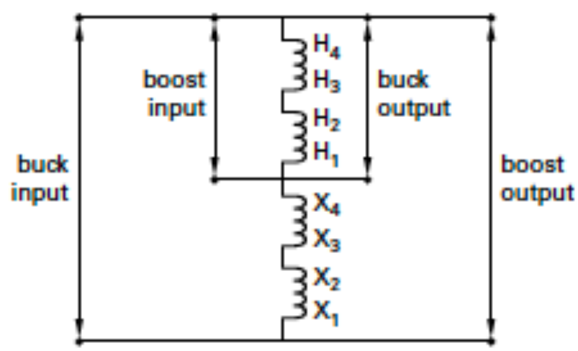
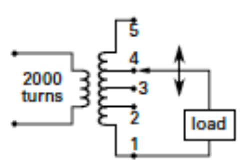


Figure 15: Autotransformer Connections

Example 6

An autotransformer with five evenly spaced taps is used to limit the surge of current to a load, as shown.



The primary voltage is 220 V. Find the voltage at tap 2 during the startup process.

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Solution

Z_1 is the impedance from the reference point (the ground point) to tap 1. Z_2 is the impedance from tap 1 to tap 2, and so on. Using the concept of a voltage divider, the voltage relationship at tap 2 is as follows.

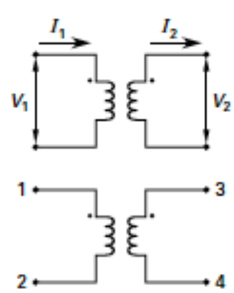
$$V_{\text{load}} = V_1 \left(\frac{Z_1 + Z_2}{Z_1 + Z_2 + Z_3 + Z_4 + Z_5} \right)$$

The taps are evenly space, so $Z_1 = Z_2 = Z_3 = Z_4 = Z_5$. Thus, the voltage relationship can be simplified to the following and solved.

$$\begin{aligned} V_{\text{load}} &= V_1 \left(\frac{2Z}{5Z} \right) \\ &= (220 \text{ V}) \left(\frac{2}{5} \right) \\ &= 88 \text{ V} \end{aligned}$$

Example 7

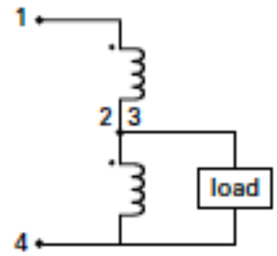
A two-winding transformer is shown. The terminals will be connected to create an autotransformer.



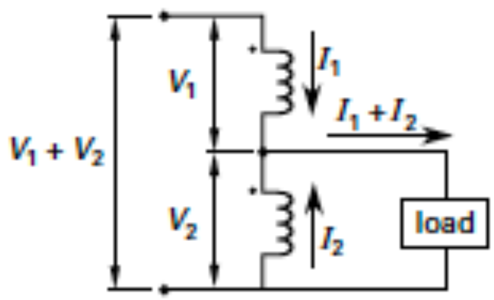
In terms of apparent power, what is the capacity of the autotransformer compared to the original two-winding transformer?

Solution

Connect the two-winding transformer to create an autotransformer.



To determine the capacity, label the voltage and current values as shown.



The capacity of the autotransformer is the following.

$$S_{at} = (V_1 + V_2)I_1 = (I_1 + I_2)V_2$$

Use the turns ratio to obtain the voltage for V_2 in terms of V_1.

$$\begin{aligned} S_{at} &= (V_1 + V_2)I_1 = I_1V_1 + I_1V_2 \\ &= I_1V_1 + I_1\left(V_1\frac{N_2}{N_1}\right) \\ &= I_1V_1\left(1 + \frac{N_2}{N_1}\right) \end{aligned}$$

The capacity of the original two-winding transformer is as follows.

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$$S_{\text{tw}} = I_1 V_1 = I_2 V_2$$

Substitute the primary side two-winding capacity into the autotransformer capacity calculated earlier.

$$S_{\text{at}} = I_1 V_1 \left(1 + \frac{N_2}{N_1}\right) = S_{\text{tw}} \left(1 + \frac{N_2}{N_1}\right)$$

For a step-up voltage transformer, the term N_2/N_1 is always greater than 1, so the capacity of the autotransformer is greater than that of the two-winding transformer from which it was formed.

OPEN-DELTA TRANSFORMERS

An open-delta transformer is identical to a normal delta transformer except that one of the transformers is not installed. The concept is shown in Fig. 16(a) for both the primary and the secondary windings. There is 0° displacement between the primary and secondary voltages for a delta-delta connection. The open-delta connection is used in transmission lines[17] and is typically achieved using two single-phase transformers connected as shown in Fig.,16(b). The same connections are shown in Fig. 16(c) using standard connection markings. The equivalent schematic is shown in Fig. 16(d) and the phasor diagram in Fig. 16(e). Even though the connection is open-delta, the primary side is sourced from all three phases. On the secondary side, the voltages are 120° apart, but the full line currents flow through the phases (which does not occur in a normal three-phase connection) and are out of phase with the voltages (by how much depends on the power factor of the load). This results in a reduced capacity of 57.7% for the open-delta transformer, compared to the 66.7% (two-thirds of the original value) that would be expected if one bank of a three-phase transformer bank were removed.[18] The two remaining phases must then carry the current of the missing phase.

[17] Transmission line transformers are known generically as *pots*, *kettles*, or *cans*.
[18] This is also referred to as the 86.6% limit for the capacity of the open-delta transformer. The value of 86.6% is referenced to the full three-phase transformer capacity. The 86.6% limit is determined by the maximum current through a single phase and is independent of the power factor of the load. 86% (referenced to three transformers) of 66.7% (referenced to two transformers) is equal to the stated 57.7% open open-delta capacity. This limit is sometimes called the *utility factor*.

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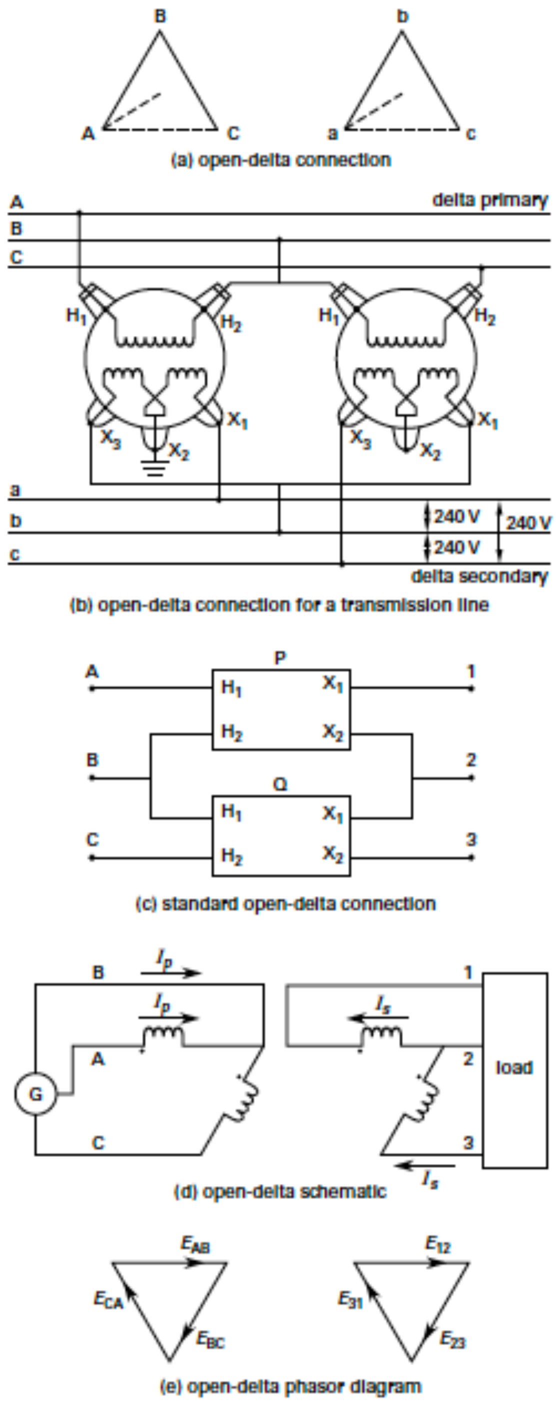

Figure 16: Open-Delta Transformer

Example 8

Two single-phase transformers, each rated for 200 kVA and 7200 V/600 V, are connected in an open-delta configuration.

What is the maximum capacity of the resulting installed transformer, expressed as a percentage of the installed transformer rating?

Solution

Each transformer is rated for 200 kVA, so the installed transformer rating is

$$S_{\text{installed}} = (2)(200\ \text{kVA}) = 400\ \text{kVA}$$

However, this is not the maximum capacity. The maximum capacity is limited by the maximum current through a given transformer. The transformer carries the current of the missing phase. The current capacity of each secondary transformer is thus

$$\begin{aligned} S &= IV \\ I &= \frac{S}{V} = \left(\frac{200\ \text{kVA}}{600\ \text{V}}\right)\left(1000\ \frac{\text{VA}}{\text{kVA}}\right) \\ &= 333.33\ \text{A} \end{aligned}$$

The current is limited in any one phase at 333.33 A. The secondary current through lines 1, 2, and 3 is restricted to this level. (See Fig. 16(d).) Given this restriction, the maximum capacity is

$$\begin{aligned} S &= \sqrt{3}VI \\ &= \frac{\sqrt{3}(600\ \text{V})(333.33\ \text{A})}{1000\ \frac{\text{V}}{\text{kVA}}} \\ &= 346.4\ \text{kVA} \end{aligned}$$

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The maximum capacity of the transformer, expressed as a percentage of the installed transformer rating is

$$\begin{aligned} C &= \frac{\text{maximum load}}{\text{installed transformer rating}} \times 100\% \\ &= \left(\frac{346.4 \text{ kVA}}{400 \text{ kVA}} \right) \times 100\% \\ &= 86.6\% \end{aligned}$$

ZIGZAG TRANSFORMERS

A *zigzag transformer* is a type of *grounding transformer* that provides a neutral point in an otherwise ungrounded system. The neutral point is then used for the return path for fault currents, for an earth reference point, and for harmonics mitigation (canceling triplet currents). The entire transformer can be used as an autotransformer. A zigzag connection is shown in Fig. 17(a), and the zigzag phasor diagram is shown in Fig. 17(b).

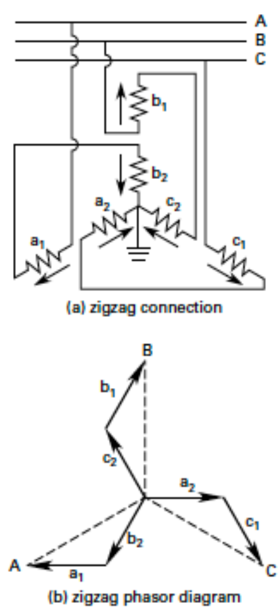

Figure 17: Zigzag Transformer

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Appendix A: Equivalent Units Of Derived And Common SI Units

Symbol	Equivalent Units			
A	C/s	W/V	V/Ω	$J/(s \cdot V)$
C	$A \cdot s$	J/V	$(N \cdot m)/V$	$V \cdot F$
F	C/V	C^2/J	s/Ω	$(A \cdot s)/V$
F/m	$C/(V \cdot m)$	$C^2/(J \cdot m)$	$C^2/(N \cdot m^2)$	$s/(\Omega \cdot m)$
H	W/A	$(V \cdot s)/A$	$\Omega \cdot s$	$(T \cdot m^2)/A$
Hz	1/s	s^{-1}	cycles/s	radians/$(2\pi \cdot s)$
J	$N \cdot m$	$V \cdot C$	$W \cdot s$	$(kg \cdot m^2)/s^2$
m^2/s^2	J/kg	$(N \cdot m)/kg$	$(V \cdot C)/kg$	$(C \cdot m^2)/(A \cdot s^3)$
N	J/m	$(V \cdot C)/m$	$(W \cdot C)/(A \cdot m)$	$(kg \cdot m)/s^2$
N/A^2	$Wb/(N \cdot m^2)$	$(V \cdot s)/(N \cdot m^2)$	T/N	$1/(A \cdot m)$
Pa	N/m^2	J/m^3	$(W \cdot s)/m^3$	$kg/(m \cdot s^2)$
Ω	V/A	W/A^2	V^2/W	$(kg \cdot m^2)/(A^2 \cdot s^3)$
S	A/V	1/Ω	A^2/W	$(A^2 \cdot s^3)/(kg \cdot m^2)$
T	Wb/m^2	$N/(A \cdot m)$	$(N \cdot s)/(C \cdot m)$	$kg/(A \cdot s^2)$
V	J/C	W/A	C/F	$(kg \cdot m^2)/(A \cdot s^3)$
V/m	N/C	$W/(A \cdot m)$	$J/(A \cdot m \cdot s)$	$(kg \cdot m)/(A \cdot s^3)$
W	J/s	$V \cdot A$	V^2/Ω	$(kg \cdot m^2)/s^3$
Wb	$V \cdot s$	$H \cdot A$	T/m^2	$(kg \cdot m^2)/(A \cdot s^2)$

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Appendix B: Physical Constants[1]

Quantity	Symbol	US Customary	SI Units
Charge			
electron	e		-1.6022×10^{-19} C
proton	p		$+1.6022 \times 10^{-19}$ C
Density			
air [STP][32°F, (0°C)]		0.0805 lbm/ft^3	1.29 kg/m^3
air [70°F, (20°C), 1 atm]		0.0749 lbm/ft^3	1.20 kg/m^3
sea water		64 lbm/ft^3	1025 kg/m^3
water [mean]		62.4 lbm/ft^3	1000 kg/m^3
Distance			
Earth radius[2]	⊕	2.09×10^7 ft	6.370×10^6 m
Earth-Moon separation[2]	⊕☾	1.26×10^9 ft	3.84×10^8 m
Earth-Sun separtion[2]	⊕⊙	4.89×10^{11} ft	1.49×10^{11} m
Moon radius[2]	☾	5.71×10^6 ft	1.74×10^6 m
Sun radius[2]	⊙	2.28×10^9 ft	6.96×10^8 m
first Bohr radius	a_0	1.736×10^{-10} ft	5.292×10^{-11} m
Gravitational Acceleration			
Earth [mean]	g	32.174 (32.2) ft/sec^2	9.8067 (9.81) m/s^2
Mass			
atomic mass unit	μ or m_μ $\frac{1}{12}m\left({}^{12}C\right)$	3.66×10^{-27} lbm	1.6606×10^{27} kg or 10^{-3} kg mol^{-1} / N_A
Earth[2]	⊕	4.11×10^{23} slugs	6.00×10^{24} kg
Earth [customary U.S.][2]	⊕	1.32×10^{25} lbm	-
Moon[2]	☾	1.623×10^{23} lbm	7.36×10^{22} kg
Sun[2]	⊙	4.387×10^{30} lbm	1.99×10^{30} kg
electron rest mass	m_e	2.008×10^{-30} lbm	9.109×10^{-31} kg
neutron rest mass	m_n	3.693×10^{-27} lbm	1.675×10^{-27} kg
proton rest mass	m_p	3.688×10^{-27} lbm	1.672×10^{-27} kg
Pressure			
atmospheric		14.696 (14.7) lbf/in^2	1.0133×10^5 Pa
Temperature			
standard		32°F (492°R)	0°C (273 K)
absolute zero		–459.67°F (0°R)	–273.16°C (0 K)
Velocity[3]			
Earth escape		3.67×10^4 ft/sec	1.12×10^4 m/s
light (vacuum)	c, c_0	9.84×10^8 ft/sec	2.9979 (3.00) $\times 10^8$ m/s
sound [air, STP]	a	1090 ft/sec	331 m/s

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Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.
2. Symbols shown for the solar system are those used by NASA. See https://science.nasa.gov/resource/solar-system-symbols/.
3. Velocity technically is a vector. It has direction.

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Appendix C: Fundamental Constants

Quantity	Symbols	US Customary	SI Units
Avogadro's number	N_A, L		6.022×10^{23} mol^{-1}
Bohr magneton	μ_B		9.2732×10^{-24} J/T
Boltzmann constant	κ	5.65×10^{-24} ft-lbf/°R	1.3805×10^{-23} J/T
electron volt: $\left(\frac{e}{C}\right)$ J	eV		1.602×10^{-19} J
Faraday constant, $N_A e$	F		96485 C/mol
fine structure constant, inverse α^{-1}	α α^{-1}		7.297×10^{-3} (≈1/137) 137.035
gravitational constant	g_c	32.174 lbm-ft/lbf-sec^2	
Newtonian gravitational constant	G	3.44×10^{-8} ft^4 / lbf-sec^4	6.672×10^{-11} N·m^2 / kg^2
nuclear magneton	μ_N		5.050×10^{-27} J/T
permeability of a vacuum	μ_0		1.2566×10^{-6} N/A^2 (H/m)
permittivity of a vacuum, electric constant $1/\mu_0 c^2$	ε_0		8.854×10^{-12} C^2 / N·m^2 (F/m)
Planck's constant	h		6.6256×10^{-34} J·s
Planck's constant: h/2π	ħ		1.0546×10^{-34} J·s
Rydberg constant	R_∞		1.097×10^{7} m^{-1}
specific gas constant, air	R	53.3 ft-lbf/lbm-°R	287 J/kg·K
Stefan-Boltzmann constant		1.71×10^{-9} BTU/ft^2-hr-°R^4	5.670×10^{-8} W/m^2·K^4
triple point, water		32.02°F, 0.0888 psia	0.01109°C, 0.6123 kPa
universal gas constant	R*	1545 ft-lbf/lbmol-°R 1.986 BTU/lbmol-°R	8314 J/kmol·K

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Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/. The unit in Volume of "lbmol" is an actual unit, not a misspelling.

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Appendix D: Mathematical Constants

Quantity	Symbol	Value
Archimedes' constant (pi)	π	3.1415926536
base of natural logs	e	2.7182818285
Euler's constant	C or τ	0.5772156649

Appendix E: The Greek Alphabet

Α	α	alpha	Ν	υ	nu
Β	β	beta	Ξ	ξ	xi
Γ	γ	gamma	Ο	o	omicron
Δ	δ	delta	Π	π	pi
Ε	ε	epsilon	Ρ	ρ	rho
Ζ	ζ	zeta	Σ	σ	sigma
Η	η	eta	Τ	τ	tau
Θ	θ	theta	ϒ	υ	upsilon
Ι	ι	iota	Φ	ϕ	phi
Κ	κ	kappa	Χ	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
Μ	μ	mu	Ω	ω	omega

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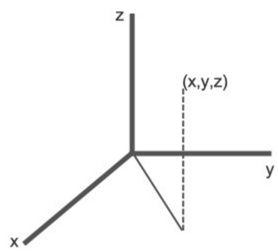
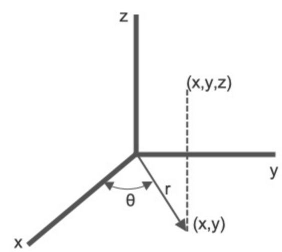
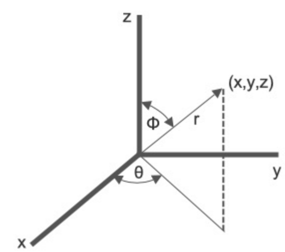
Appendix F: Coordinate Systems & Related Operations

Mathematical Operations	Rectangular Coordinates	Cylindrical Coordinates	Spherical Coordinates
Conversion to Rectangular Coordinants	 $x = x$ $y = y$ $z = z$	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	 $x = r\sin\phi\cos\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\phi$
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\boldsymbol{\theta} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\boldsymbol{\phi} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \theta}\boldsymbol{\theta}$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial (A_\phi \sin\phi)}{\partial \phi} + \frac{1}{r\sin\phi}\frac{\partial A_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{r} & \boldsymbol{\theta} & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & A_\theta & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r^2\sin\theta}\mathbf{r} & \frac{1}{r^2\sin\theta}\boldsymbol{\phi} & \frac{1}{r}\theta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ A_r & rA_\phi & rA\theta A_\phi \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial r}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2 f}{\partial \theta^2}\right)$

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