A SunCam Online Continuing Education Course

Electromagnetic Theory

Electric & Magnetic Theory & Application

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by

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Nomenclature[1]

a	acceleration	m/s^2
a	unit vector	-
A, **A**	area	m^2
B, **B**	magnetic flux density	T (Wb/m^2)
c	speed of light, 2.9979×10^8	m/s
C	capacitance	F
d	distance	m
D, **D**	electric flux density	C/m^2
e^-	charge of an electron, -1.6022×10^{-19}	C
E, **E**	electric field strength	V/m
F, **F**	force	N
G	conductance	S (A/V)
G	universal gravitational constant, 6.672×10^{-11}	$N \cdot m^2 / kg^2$
h	Planck's constant, 6.6256×10^{-34}	$J \cdot s$
H, **H**	magnetic field strength	A/m
i	variable current	A
I, **I**	constant or rms current	A
J, **J**	current density	A/m^2
k	electrostatic proportionality constant, 8.987×10^9	$N \cdot m^2 / C^2$
l, **l**	length	m
ℓ	azimuthal quantum number	-
L	inductance	H
m	mass	kg
m	spatial quantum number	-
m_s, *s*	spin quantum number	-
M, **M**	magnetization	A/m
M	mutual induction	H
n	principle quantum number	-
N	number of turns	-
p	pole strength	Wb (N/T)
p^+	charge on a proton, 1.6022×10^{-19}	C

[1] Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related and may be found when reviewing the references listed for further information. Further, all the nomenclature, symbols, or subscripts will be found in of many electrical courses (on SunCam, PDH Academy, and also in many texts). For guidance on nomenclature, symbols, and electrical graphics: IEEE 280-2021. IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering. New York: IEEE; and IEEE 315-1975. Graphic Symbols for Electrical and Electronics Diagrams. New York: IEEE, approved 1975, reaffirmed 1993.

P	polarization	C/m^2
q, Q	charge	C
r, **r**	radius	m
R	ratio	-
R	resistance	Ω
$\mathcal{R}$ or $\mathfrak{R}$	reluctance	A/Wb
s, **s**	surface area	m^2
S	elastance	1/F
t	thickness	m
t	time	s
u	energy density	J/m^3
U	energy	J
υ or v	variable voltage	V_{volume}
V	constant or rms voltage	V
v, **v**	velocity*	m/s
V_{volume}	volume	m^3

*Velocity is often used in tests where it would be more technically correct to us "speed". Velocity is a vector with speed as its magnitude.

Symbols

Γ	reciprocal inductance	1/H
ε	permittivity	F/m $\left(C^2 / N \cdot m^2\right)$
ε_0	permittivity of free-space, 8.854×10^{-12}	F/m $\left(C^2 / N \cdot m^2\right)$
θ	angle	degrees
Λ	flux linkage	Wb
μ	mobility	$m^2 / V \cdot s$
μ	permeability	H/m $\left(N / A^2\right)$
μ_0	free-space permeability, 1.2566×10^{-6}	H/m $\left(N / A^2\right)$
ρ	charge density	C/m^3
σ	conductivity	$S \cdot m$
ϕ, Φ	magnetic flux	Wb
χ	susceptibility	-
ψ, Ψ	electric flux	C
ω	angular velocity	rad/s

Subscripts

1–2	From 1 to 2 Or effect on 2 caused by the field on 1	-
ave	average	-
c	conduction	-
ci	cast iron	-
d	displacement drift	-
e	electric	-
l	line	-
m	magnetic	-
m	mean	-
r	relative	-

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COURSE INTRODUCTION

The theoretical information is primarily from the author's books, Refs. [A] and [B]. The NESC Ref. [C] and NEC Ref. [D] though not covered in this course are useful sources for electrical engineers. Information useful in many aspects of electric engineering may be found in [E] and [F]. Reference [G] has detailed descriptions of analysis techniques. Reference [H] covers many terms in EE with excellent definitions and explanations. The appendices cover information useful in many engineering tasks with App. H focused on this course and provides a side by side comparison of electric and magnetic equations.

PART I: FUNDAMENTALS

As a general rule, electrical effects emanate from static charges, while magnetic effects emanate from charges in motion. To be more precise, electricity is any manifestation of energy conversion of charge carriers that results in forces in the direction of motion of those charge carriers.[2] Magnetism is any manifestation of the kinetic energy of charge carriers that arises from forces or produces forces in the direction perpendicular to the motion of those charge carriers. The two were long thought to be separate phenomena, but were united by Maxwell's equations.[3] Electromagnetism deals with control of the average movement of charges, in generally linear elements, and usually at considerable power levels.

The charges of interest are the electron and the proton, designated as e^- and p^+. The mass, charge, and charge-to-mass ratios for the proton and electron are given in following table. The SI unit of charge is the coulomb (C). The charge of one electron is sometimes referred to as one electrostatic unit (esu). One C is approximately equal to 6.24×10^{18} esu. One mole of electrons is referred to as a faraday. One faraday is approximately equal to 96,500 C.

[2] The forces produce displacement, velocity, or acceleration.

[3] Special relativity also confirms the link between electricity and magnetism in that it shows that electric and magnetic fields appear or vanish depending on the motion of the observer. Further, it was the Lorentz transformation that required invariance in Maxwell's equations; that is, electric and magnetic fields must remain in the same form for stationary and moving systems. This requirement is necessary because without it electrical and optical phenomena would differ depending on one's motion. It is important to understand that, while the experimental results do differ depending on the motion of the observer, the fundamental phenomena remain unchanged. For example, Maxwell's equations describing electromagnetism remain in the same form. The Lorentz transformation was the genesis of Einstein's special theory of relativity.

Table 1: Properties of the Electron and Proton

	electron	proton
mass at rest, kg	9.1096×10^{-31}	1.6726×10^{-27}
charge, C	-1.6022×10^{-19}	$+1.6022 \times 10^{-19}$
charge-to-mass ratio, C/kg	1.7588×10^{11}	9.5791×10^{7}

The *extranuclear structure* of an atom, that is, its electrical or electronic structure, determines the defining characteristics of that atom. Orbiting electrons are arranged in shells designated *K, L, M, N, O, P,* and *Q*. The inner shell electrons are tightly bound and interact only with high-energy particles, such as gamma rays. The outer shells in complex atoms—those with numerous neutrons and protons, and thus numerous electrons—tend to be loosely bound. It is these outer shells that determine the electrical and chemical properties of the elements. The energy of the orbiting electron is determined by four quantum numbers: the principle quantum number (n), the azimuthal quantum number (ℓ), the spatial quantum number (m), and the spin quantum number (m_s or s).[4,5]

When electrons are in close proximity, such as in a crystalline solid, nearby atoms affect their behavior and the electron's energy is no longer uniquely determined. Indeed, the single energy level of an electron in a free atom is spread into a band, or range, of energy levels.[6] The *conduction band* is the range of energy states in a solid in which electrons can move freely (that is, they can effect transitions between energy levels). In other words, the band is not full of conduction electrons. When every energy level is full, the material is known as an insulator or a dielectric.

PART 2: ELECTRIC AND MAGNETIC PHENOMENA

Electromagnetic Effects

Electromagnetic effects are caused by the dynamic behavior of elementary particles, that is, the electron and the proton, due to their mass and charge. These effects can differ greatly from free-space effects, depending on the material in which the particles find themselves. There are three major classes of electromagnetic materials.

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[4] The azimuthal quantum number, ℓ, specifies the angular orbital momentum. Electrons whose value of is 0, 1, 2, and 3 are referred to as the *s, p, d,* and *f* electrons (for historical reasons related to the "picture" these electrons produced during experiments).

[5] The spin has two values, ±1/2, and is equal to $h/2\pi$, where h is Planck's constant.

[6] This band of energy levels allows what is referred to in some texts as an electron cloud to exist. The theory is by no means universal. Consult electrical conduction theory texts for more information.

- *Conductors or semiconductors* are materials through which charges flow more or less easily. Some common conductors are given in the next table. The properties of widely used semiconductors are given in the table following. Good conductors have a conductivity range of 1×10^3 S·m to 6×10^3 S·m.
- *Dielectrics or insulators* are materials that inhibit the passages of charges. Some common insulators are given in the table following. Insulators have a conductivity range of 10^{-18} S·m up to approximately 10^{-4} S·m. The determining factors for the conductivity of insulators are the application and the voltage stress (that is, the magnitude of the voltage being insulated).
- *Magnetic materials* are materials in which the motion of the charges produces perpendicular effects, that is, enhanced transverse effects. A very small portion of the elements in the periodic table exhibit such effects. These materials are given in the following table.

Table 2: Common Conductor

material	gauge[b]	diameter (in)	area[c] (circular mils)	nominal DC resistance (ohms/1000 ft at 68°F (20°C))
aluminum[d]	AWG 8	0.1285	16,510	1.030
copper[e]	AWG 8	p.1285	16,510	0.6533
steel[f]	BWG 8	0.165	27,239[g]	3.28[h]

[a]The English Engineering System is used in this table, as it represents the system most commonly used in this area of electrical engineering.
[b]AWG stands for American Wire Gauge, the usual standard for nonferrous wires, rods, and plates. BWG stands for Birmingham Wire Gauge, the usual standard for galvanized iron and steel wire.
[c]One circular mil is a unit having the area corresponding to the area of a circle with a diameter of 0.001 in.
[d]This information is for bare, solid, all-aluminum hard-drawn wire and is taken from ASTM International's Standard, ASTM B230/B230M.
[e]This value is considered a trade maximum. It depends on the specific process used when manufacturing the wire. ASTM requirements do exist for the resistance of copper for various processes.
[f]The type of steel referenced here is used for telephone and telegraph wire.
[g]The area in circular mils varies slightly depending on the construction, that is, the number of strands and arrangement.
[h]This number is referenced to 1000 ft here for convenience in comparison with the other conductors. Transmission cables and telephone or telegraph wires are often referenced to longer lengths, as in ohms/mile.

Table 3: Common Semiconductors

semiconductor	symbol	periodic table group
silicon	Si	IV
germanium	Ge	IV
gallium arsenide	GaAs	III and V
diamond and graphite	–	–
selenium	Se	VI
silicon carbide	SiC	IV
indium antimonide	InSb	III and V

Table 4: Common Insulators

insulating material
polyvinyl chloride (PVC)
butyl rubber
neoprene
plastics
ceramics
insulating gasses and liquids

Table 5: Magnetic Materials

magnetic material	symbol	atomic number
iron	Fe	26
cobalt	Co	27
nickel	Ni	28

Conduction Effects

Conduction effects occur in systems having mobile charges when an electric force is applied. The charges move in the direction of the applied force. Conduction effects result from the loosely bound outer shell electrons of atoms. In very strong conductors, electrons can be released and move under the application of small chemical, thermal, or other forces.

Dielectric Phenomena

When a system contains bound charges, the electrons still move under the application of an electric force, but only to a limited extent. This process is called the *dielectric phenomenon*. It results in a current termed the *displacement current*, which is related to the *electric flux density*, **D**, in the dielectric material. In turn, the electric flux density, **D**, is proportional to the *electric field strength* or *intensity*, **E**. Heat losses occur in dielectrics due to hysteresis effects at very high frequencies. Hysteresis effects occur in magnetic materials at much lower frequencies.

Dielectrics can sustain only a certain field strength before they break down and conduction occurs. This maximum field strength is called the *dielectric strength*, and its unit of measure is volts per meter of thickness of the material tested. The results vary, depending on thickness and the environmental conditions of the test. For the same material, thin dielectrics break down at lower field strengths than thick dielectrics.

Magnetic Phenomena

Magnetic phenomena are caused by the directed motion of charges. The effects occur perpendicular to this motion. In magnetic materials, the phenomena are associated with the orientation of the orbiting electrons and, to some extent, the spin of the electrons. Magnetic phenomena occur in a relatively small number of materials whose outer shell orbits fill prior to the inner shells. This occurs because the outer shell orbit configuration is actually a lower energy configuration than the inner shell.

Several types of magnetism exist.

- *Ferromagnetism* is produced by the exchange of forces between atomic moments. This type of magnetism produces strongly magnetic materials with high permeability. Large clusters of atoms group together to form *magnetic domains*, each of which has the same atomic moment alignment within the domain. Ferromagnetism, named for iron, is the common magnetism one associates with horseshoes or toy magnets. See Fig. 1 for an example of a crystalline structure with domains and Fig. 2 for an example of magnetization curves. Figure 3 shows the spin arrangements of several types of magnetism.

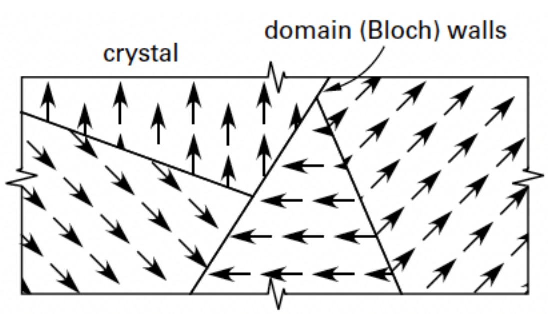


Figure 1: Magnetic Domains

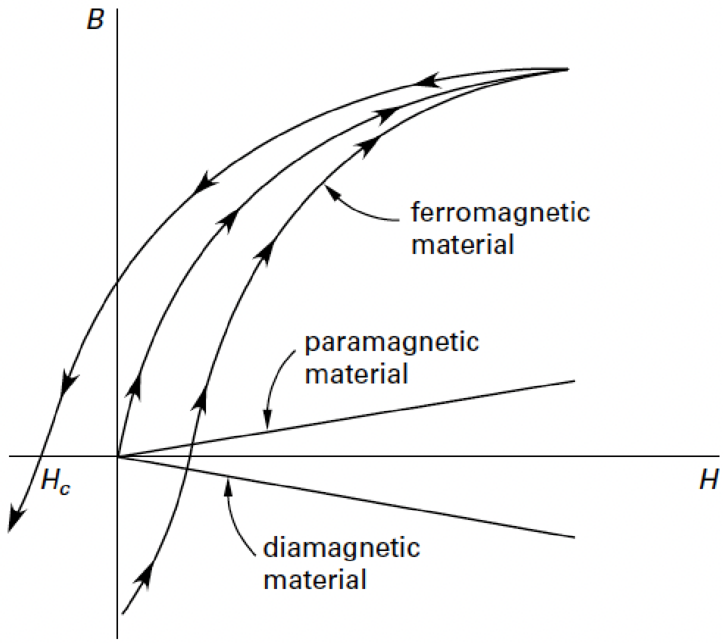


Figure 2: Magnetization Curves

- *Paramagnetism* is produced by the orbital or spin moments of the electrons or both. The atomic alignment is minimal, that is, there is little or no domain formation, and as a result, paramagnetic materials are not strongly magnetic.

- *Antiferromagnetism* is produced by the exchange of forces between atomic moments. Antiferromagnetic materials have an antiparallel arrangement of equal spins. Their magnetism is of a similar strength to that of paramagnetic materials.

- *Diamagnetism* is produced by electron spins in antiparallel pairs in closed electron shells. Diamagnetic materials are weakly repulsive to external magnetic fields. These materials possess an internal magnetic field that opposes any externally applied magnetic field.

- *Ferrimagnetism* is produced by the moment that results from the combination of two antiferromagnetic lattices. Ferrimagnetic materials contain two kinds of magnetic ions, arranged antiparallel but with unequal spins.

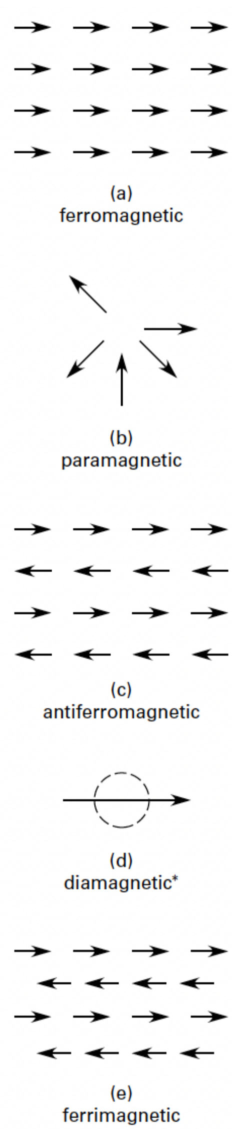


Figure 3: Magnetic Spin Arrangements
*The circle represents the internal magnetic field.

Thermoelectric Phenomena

There are three significant relations between electricity and heat of importance to the electrical engineer. These are the Seebeck effect, the Thomson effect, and the Peltier effect. The important point is that heat is capable of providing the energy to release and transport electrons.

PART 3: ELECTROSTATICS

As the name implies, electrostatics is the field of electrical engineering that deals with the electrical effects of stationary charges. An important concept in this field is that all electrical phenomena are expressible in terms of the masses and charges of the elementary particles involved.

Electric Charges

An electrically neutral material has equal numbers of positive and negative charge carriers. Equation 1 describes bodies in this unelectrified state.

Equation 1: Electrically Neutral

$$\begin{aligned} Q^+ &= Q^- \\ &= 0 \end{aligned}$$

Only integer numbers of charge carriers exist. Therefore, the transport of an integer number n charge carriers to a body from another body means the following.

Equation 2: Charge Carriers

$$Q^+ = np^+ \text{ and } Q^- = ne^-$$

Like charges exhibit an electric repulsion. Unlike charges exhibit an electric attraction. An attraction also exists between the masses of any two charged particles. The force associated with the gravitational attraction is very small in comparison to the force of the electrical attraction. Therefore, the gravitational force can be ignored except when the charges are accelerated.

Static electricity formed by positive charges, such as that produced by rubbing silk on a glass rod, is called *vitreous static electricity*. When the static electricity is formed by negative charges, such as that produced by rubbing fur on a rubber, amber, or plastic rod, it is termed *resinous static electricity*.

Example 1

What is the gravitational force of attraction between two electrons located 1 cm apart?

Solution

Use Newton's law of gravitation.

$$F = \frac{Gm_1m_2}{r^2}$$

Newton's universal constant or *gravitational constant*, G, is equal to 6.672×10^{-11} N·m²/kg². The mass of the electrons (taken from Table 1) and the 1 cm distance between the electrons are substituted into the equation, giving the following.

$$F = \frac{Gm_1m_2}{r^2} = \frac{Gm_e^2}{r^2}$$
$$= \frac{\left(6.672 \times 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\left(9.1096 \times 10^{-31}\ \text{kg}\right)^2}{\left(1\ \text{cm}\right)^2 \left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2}$$
$$= 5.537 \times 10^{-67}\ \text{N}$$

To state the obvious, the gravitational impact is small.

Coulomb's Law

Coulomb's law states that the force between charges at rest in an isotropic medium is proportional to the product of the charges and inversely proportional to both the square of the distance between them and the dielectric coefficient of the medium. The force acts in a straight line between the centers of the charges.[7,8,9] Equation 3 describes Coulomb's law in mathematical terms.

[7] In Eq. 3, the unit vector, and thus the force, is *from 1 to 2*. The force direction is the direction charge 2 will move due to the electric field of charge 1. See Eq 4. The notation often varies.

[8] The unit vector $\mathbf{a}_{r_{1-2}}$ (see Eq. 3) or a_{1-2} (see Eq. 5) is sometimes written $\mathbf{a}_{\mathbf{r}}$, $\mathbf{a}_{12}$, or even $\mathbf{a}_{21}$. The $\mathbf{a}_{21}$ notation is used as a reminder that, although the vector is *from 1 to 2*, it is determined mathematically by subtracting the position elements of 2 from 1. For example, $x_2 - x_1$ or $y_2 - y_1$, and $z_2 - z_1$. (The unit vector is then determined by dividing the distance between the two points.)

[9] At times, the notation 1– 2 merely indicates the connection between the points and one must determine the correct direction from other information.

Equation 3: Coulomb's Law

$$\mathbf{F}_{1-2} = \frac{Q_1 Q_2}{4\pi\varepsilon r_{1-2}^2}\mathbf{a}_{r_{1-2}}$$

The vector $\mathbf{a}_{r_{1-2}}$ is a unit vector having a direction along a straight line connecting Q_1 and Q_2. The term ϵ represents the *dielectric coefficient*, more commonly called the *permittivity*. The 4π term occurs in many electric field calculations and, for this reason, is included in this *rationalized* form of Coulomb's law in order to cancel terms in integrals over spheres and make calculations easier. This is perhaps more readily seen when Coulomb's law is written in terms of the electric field, **E**.

Equation 4: Coulomb's Law, Electric Field Version

$$\mathbf{F}_{1-2} = \mathbf{F}_2 = Q_2\mathbf{E}_1$$

Coulomb's law is also stated in its Gaussian SI form as follows.

Equation 5: Coulomb's Law, Gaussian SI Form

$$\mathbf{F}_{1-2} = \frac{kQ_1Q_2}{\varepsilon_r r^2}\mathbf{a}_{1-2}$$

The term ϵ_r represents the *relative permittivity* of the dielectric. (ϵ_r = 1.0 for a vacuum.) The constant k has an approximate value of 8.987×10^9 $\text{N·m}^2/\text{C}^2$. In Eq. 3 through Eq. 5 the force is measured in newtons, the distance in meters, and the charge in coulombs.

Example 2

What is the magnitude of the electrostatic force between two electrons 1 cm apart in a vacuum?

Solution

Take the magnitude of Eq. 5 and substitute the values for the relative permittivity and k given in this section. The relative permittivity has no units. As the term "relative" suggests, it is a comparison. This is discussed more fully later.

Thus,

$$\mathbf{F}_{1-2} = \frac{kQ_1Q_2}{\varepsilon_r r^2}\mathbf{a}_{1-2}$$

$$\left|\mathbf{F}_{1-2}\right| = F = \frac{kQ_1Q_2}{\varepsilon_r r^2}$$

$$= \frac{\left(8.987\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(-1.6022\times10^{-19}\ \text{C}\right)^2}{(1)(1\ \text{cm})^2\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2}$$

$$= 2.307\times10^{-24}\ \text{N}$$

Example 3

What is the ratio of the electrostatic force to the gravitational force between two electrons 1 cm apart in a vacuum?

Solution

As found in Ex. 2, the electrostatic repulsion is 2.307×10^{-24} N. As found in Ex. 1, the gravitational attraction is 5.537×10^{-67} N. The ratio of electrostatic force to gravitational force is as follows.

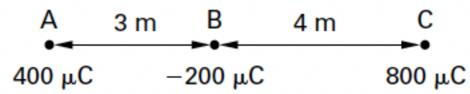
$$R = \frac{2.307\times10^{-24}\ \text{N}}{5.537\times10^{-67}\ \text{N}} = 4.166\times10^{42}$$

The electrostatic force that these elementary particles experience is 42 powers of ten greater than the gravitational force of attraction between them. Clearly, the gravitational attraction between such particles can be ignored in all but the most exacting calculations.

Coulomb's law implies that the principle of superposition applies for electric charges. This means that a system of charges is a linear system. The total resultant effect at any point within the system can be determined from the vector sum of the individual components.

Example 4

Three point charges in a vacuum are arranged in a straight line. Find the force on point charge A.



Solution

Because all three charges are in line, vector analysis is not needed. The force on point charge A is the sum of the individual forces from the other two point charges. Use the Gaussian form of Coulomb's law.

From Ex. 5,

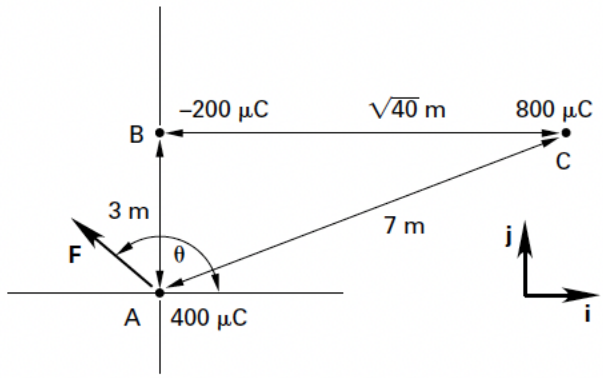
$$\mathbf{F}_{1-2} = \frac{kQ_1Q_2}{\varepsilon_r r^2}\mathbf{a}_{1-2}$$

$$\begin{aligned}
F_{A-B\&C} &= F_{B\&C-A} = F_{A-B} = F_{A-B} + F_{A-C} \\
&= \frac{kQ_AQ_B}{r_{A-B}^2} + \frac{kQ_AQ_C}{r_{A-C}^2} \\
&= \left(8.987\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{(400\times10^{-6}\ \text{C})(-200\times10^{-6}\ \text{C})}{(3\ \text{m})^2} + \frac{(400\times10^{-6}\ \text{C})(800\times10^{-6}\ \text{C})}{(7\ \text{m})^2}\right] \\
&= -79.9\ \text{N} + 58.7\ \text{N} \\
&= -21.2\ \text{N}
\end{aligned}$$

Because the sign is negative, the force on A is toward B and C.

Example 5

Determine the magnitude and direction of the force on point charge A caused by point charges B and C.



Solution

The charges and distances are the same as in Ex. 4.

$$F_{\text{B-A}} = F_{\text{A-B}} = -79.9\text{ N} \quad [\text{attractive}]$$
$$\text{F}_{\text{C-A}} = F_{\text{A-C}} = -58.7\text{N} \quad [\text{repulsive}]$$

Determine the unit vectors for these forces. Let point A correspond to the origin. The unit vectors ($a_{B\text{-}A}$ and $a_{C\text{-}A}$) from B and C to A are determined as follows.

$$\begin{aligned}\mathbf{a}_{\text{B-A}} &= \frac{(x_{\text{A}} - x_{\text{B}})\mathbf{i} + (y_{\text{A}} - y_{\text{B}})\mathbf{j}}{\sqrt{(x_{\text{A}} - x_{\text{B}})^2 + (y_{\text{A}} - y_{\text{B}})^2}} \\ &= \frac{(0-0)\mathbf{i} + (0-3)\mathbf{j}}{\sqrt{(0-0)^2 + (0-3)^2}} \\ &= -\mathbf{j} \\ \mathbf{a}_{\text{C-A}} &= \frac{(0-\sqrt{40})\mathbf{i} + (0-3)\mathbf{j}}{\sqrt{(0-\sqrt{40})^2 + (0-3)^2}} \\ &= -0.904\mathbf{i} - 0.429\mathbf{j}\end{aligned}$$

The total force is as follows.

$$\begin{aligned} &F_{\text{B-A}}\mathbf{a}_{\text{B-A}} + F_{\text{C-A}}\mathbf{a}_{\text{C-A}} \\ &\qquad = (-79.9\ \text{N})(-\mathbf{j}) + (58.7\ \text{N})(-0.904\mathbf{i} - 0.429\mathbf{j}) \\ &\qquad = -53.1\mathbf{i}\ \ \text{N} + 54.7\mathbf{j}\ \text{N} \end{aligned}$$

The magnitude of the force is as follows.

$$|\text{F}_{\text{A-B\&C}}| = \sqrt{(-53.1\ \text{N})^2 + (54.7\ \text{N})^2} = 76.2\ \text{N}$$

The direction (counter-clockwise angle with respect to the horizontal) is

$$\begin{aligned} \theta &= 180° - \arctan\frac{54.7\ \text{N}}{53.1\ \text{N}} = 180° - 45.9° \\ &= 134.1° \end{aligned}$$

Electric Fields

The electric field, E, with units of V/m, is the region in which an electric charge exerts a measurable force on another charge, above and beyond the gravitational force that the two charges exert on each other. It is a *vector force field* because a test charge placed in the field will experience a force in a given direction and with a specific magnitude. The direction of the field is along flux lines. These lines, ideally, represent the direction of the force on an infinitely small, isolated positive test charge.[10] The actual direction is represented by the unit vector **a**. Equation 6 represents the *electric field strength*, or *electric field intensity*, in a substance with permittivity ϵ at a distance r from a point charge Q.

Equation 6: Electric Field Strength

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon r^2}\mathbf{a}$$

Coulomb forces obey the principle of linear superposition in free space and in many dielectrics. Therefore, the electric field strength or intensity acting on a charge Q located at point 1 as a result of charges Q1, Q2, … ,Q_i at distances r_{i1} from point 1 can be represented as the following sum. The vector $\mathbf{a}_{r,i1}$ is the unit vector with direction from Q_i to point 1.

[10] The positive test charge is arbitrary but universal. The charge would ideally have to be infinitely small in order to prevent its own electric field from distorting the original field.

Equation 7: Electric Field Strength/Intensity using Coulomb Force

$$\mathbf{E}_1 = \sum_{i=1}^{i} \frac{Q_i}{4\pi\varepsilon r_{i1}^2}\mathbf{a}_{r,i1}$$

Additionally, the electric field intensity at any point p produced by charges Q_1, Q_2, … ,Q_n at distances r_1, r_2, … ,r_n from the point is the vector sum of the field strengths produced by the charges individually. If one is dealing with charges distributed uniformly throughout space, the electric field strength at a distance r that is large when compared to dV_{volume} is given by Eq. 8.

Equation 8: Electric Field Strength/Intensity by Volume

$$\mathbf{E}_1 = \int_{V_{\text{volume}}} \frac{\rho dV_{\text{volume}}}{4\pi\varepsilon r_{i1}^2}\mathbf{a}_{r,i1}$$

The unit vector $\mathbf{a}_{r,i1}$ is directed from dV to point 1, at which **E** is evaluated. The unit vector, the distance, r, and the charge density, ρ, are usually functions of the variable of integration. The total charge, Q, in the volume V_{volume} that has the charge density ρ is

Equation 9: Total Charge in a Volume

$$Q = \int_{V_{\text{volume}}} \rho dV_{\text{volume}}$$

The electric fields discussed to this point have been radial in nature. Not all fields are radial. As seen in Eq. 8, the electric field depends on the orientation of the surface or volume that contains the charge. A uniform electric field, commonly used in sensing instruments and other important applications, can be created between two flat plates, as shown in Fig. 4.

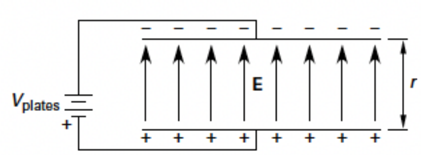


Figure 4: Uniform Electric Field

With a potential difference of V volts between the flat plates separated by distance r, the electric field strength is

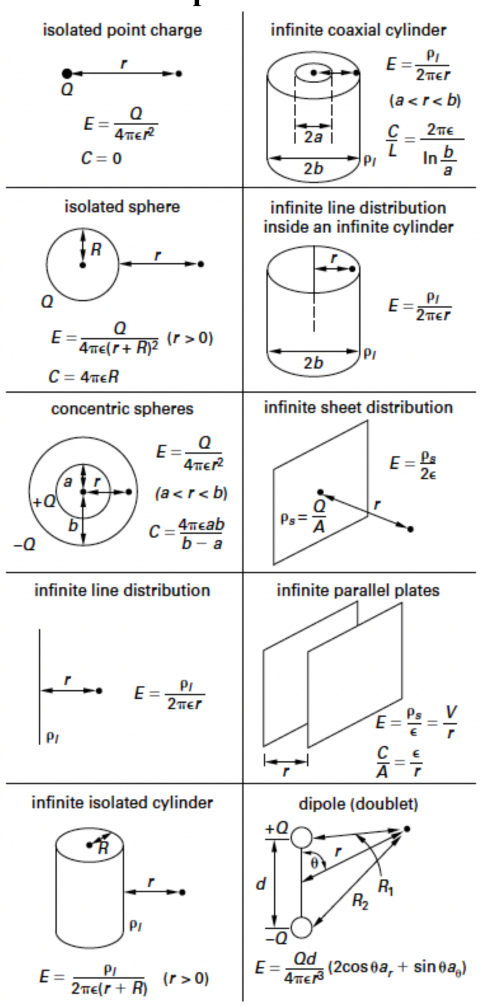
Equation 10: Uniform Electric Field

$$E_{\text{uniform}} = \frac{V_{\text{plates}}}{r}$$

The field strengths, or intensities, for several configurations are given in Table 6.

An electric field is an abstraction that allows one to visualize how charges in different locations within the field will experience a force. A *line of electric force* is a curve drawn so that it has the direction of the force acting on the test charge, (i.e., it is in the direction of the electric field). A *tube of force* is created by drawing lines of force through the boundary of any closed curve. The lines form a tubular surface not cut by any other lines of force.

Table 6: Electric Fields and Capacitances for Various Configurations



Permittivity and Susceptibility

The forces between electric charges depend on the environment in which they are located. The maximum coulombic force occurs in free space. In all other materials, the coulombic force is reduced. Another way of viewing this phenomenon is to say that the electric flux does not pass equally through all materials.[11] In fact, it cannot pass through conductive material at all and is diminished to varying degrees by various dielectrics.

The *permittivity of free space*, that is, a vacuum, ϵ_0, may be considered to be representative of the dielectric properties of free space. In actuality, it is a factor that provides consistent units for Coulomb's law when units of force, charge, and distance are arbitrary. The product of the permittivity of free space and the *relative permittivity* of a medium gives the *permittivity* of that medium.

Equation 11: Permittivity—Vacuum, Relative

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

In general, the relative permittivity is a complex number. Except for very high frequencies, the imaginary component, or quadrature component, can be ignored. The real component is often termed the *dielectric constant*. Because the value of ϵ_r depends on frequency as well as other factors, the term *dielectric coefficient* is more appropriate. The relative permittivity can also be viewed in terms of capacitance, as shown in Eq. 12.

Equation 12: Relative Permittivity

$$\varepsilon_r = \frac{C_{\text{with dielectric}}}{C_{\text{vacuum}}}$$

Typical relative permittivities are given in the table that follows.

The permittivity can also be expressed as follows.

Equation 13: Permittivity—Vacuum, Susceptibility

$$\varepsilon = \varepsilon_0 \left(1 + \chi_e\right)$$

[11] The concept of flux is another abstraction used to help one visualize electric interactions. Discussed later.

Table 7: Typical Relative Permittivities (20°C and 1 atm)

[Source*: Core Engineering Concepts for Students and Professionals*]

material	ϵ_r	material	ϵ_r
acetone	21.3	mineral oil	2.24
air	1.00059	mylar	2.8–3.5
alcohol	16–31	olive oil	3.11
amber	2.9	paper	2.0–2.6
asbestos paper	2.7	paper (kraft)	3.5
asphalt	2.7	paraffin	1.9–2.5
bakelite	3.5–10	polyethylene	2.25
benzene	2.284	polystyrene	2.6
carbon dioxide	1.001	porcelain	5.7–6.8
carbon		quartz	5
tetrachloride	2.238	rock	≈ 5
castor oil	4.7	rubber	2.3–5.0
diamond	16.5	shellac	2.7–3.7
glass	5–10	silicon oil	2.2–2.7
glycerine	56.2	slate	6.6–7.4
hydrogen	1.003	sulfur	3.6–4.2
lucite	3.4	teflon	2.0–2.2
marble	8.3	vacuum	1.000
methanol	22	water	80.37
mica	2.5–8	wood	2.5–7.7

The term χ_e represents the electric susceptibility of a given dielectric. It is a numeric measure of the polarization or displacement of electrons in the atoms or molecules of the dielectric.

The units for permittivity are $C_2/N{\cdot}m^2$ or F/m. The permittivity of free space, or vacuum, ϵ_0, in SI units is 8.854×10^{-12} $C^2/N{\cdot}m^2$ (F/m).

Example 6

A certain capacitor is constructed of two square parallel plates with air as the dielectric. If a teflon insert is used in place of the air, by what factor does the capacitance increase?

Solution

The capacitance is directly proportional to the permittivity and is given by the following formula.

$$C = \frac{\varepsilon A}{r} = \frac{\varepsilon_0 \varepsilon_r A}{r}$$

Because the example asks for the factor by which the capacitance increases, not the actual value of the capacitance, one need merely compare the permittivity of teflon to that of air. Using data from Table 7,

$$\frac{\varepsilon_{r,\text{teflon}}}{\varepsilon_{r,\text{air}}} = \frac{2.0}{1.00059} = 2.0$$

One could use the upper bound of the permittivity for the teflon, in which case the factor by which the capacitance increases would be approximately 2.2.

Electric Flux

Flux is defined as the electric or magnetic lines of force in a region. The *electric flux*, **Ψ**, is a scalar field representing these imaginary lines of force. Flux lines leave or enter any surface at right angles to that surface, and the orientations of the electric field lines and the electric flux lines always coincide. By convention, the flux lines are directed outward from the positive charge and inward toward the negative charge, or to infinity if no negative charge exists in the area, as shown in Fig. 5. In Fig. 5, only a few representative lines of flux are shown. Also, if the charge on a hollow sphere is all the same sign, the flux outside this sphere is the same as the flux that would exist from a point charge at the center. Therefore, for the purposes of drawing flux lines and calculating forces, a hollow charged sphere can be replaced by a point charge at the center.

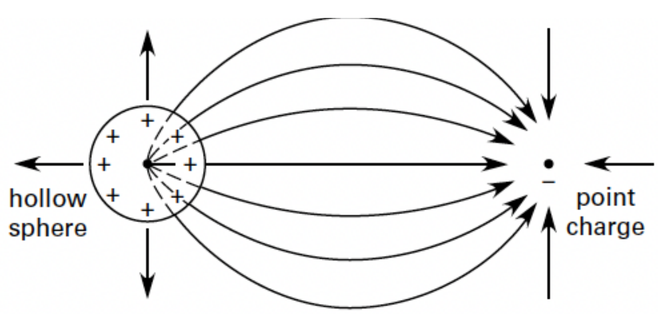


Figure 5: Flux Lines

The flux, **Ψ**, is numerically equal to the charge. That is, by definition, one coulomb of electric charge gives rise to one coulomb of electric flux.

Equation 14: Flux and Charge

$$\Psi = Q$$

The electric flux is also called the *displacement flux*. This is because the flux is a quantity associated with the amount of bound charge, Q, displaced in a dielectric material that is subject to an electric field. This terminology can be understood by referring to Fig. 6.

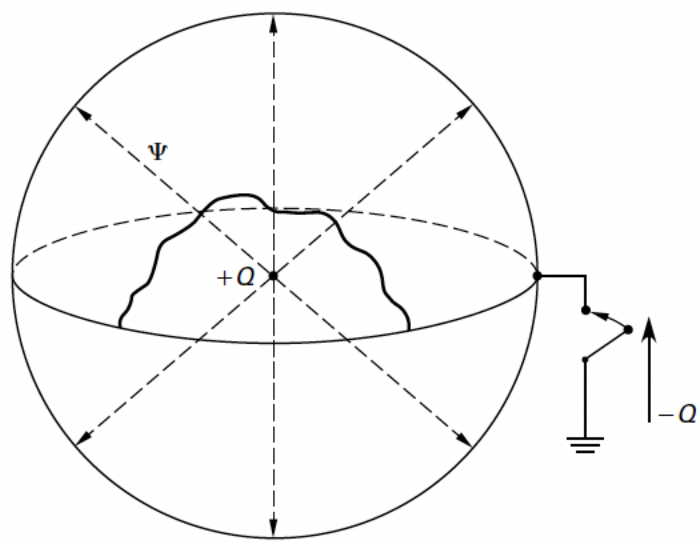


Figure 6: Displacement Flux Experiment

Early experimenters enclosed a fixed positive charge inside a spherical conducting shell as shown in Fig. 6. When the switch was closed, a charge of $-Q$, equal in magnitude to the enclosed positive charge, was found on the shell and was believed to be caused by the transient flow of negative charge induced by the flux from the $+Q$. In other words, the positive charge *displaced* the $-Q$ charge and forced it onto the surface. The scalar electric flux field, **Ψ**, is not directly measurable as is the electric field, **E**. Instead it is inferred from such experiments.

Electric Flux Density

The *electric flux density*, **D**, is also referred to as the *displacement density* or simply the *displacement*. It is also loosely referred to as the *vector flux density*. It is a vector quantity that, like the electric flux, Ψ, cannot be directly measured. It is given in Eq. 15. Unlike the electric field strength, **E**, the electric flux density or displacement, *D*, is independent of the medium.[12]

Equation 15: Electric Flux Density / Displacement

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_0\varepsilon_r\mathbf{E}$$

The magnitude of **D**, or *D*, is also referred to as the *flux density*. It is the number of flux lines per unit area perpendicular to the flux. It is a scalar quantity with units of C/m^2.

Equation 16: Flux Density

$$D = |\mathbf{D}| = \frac{\Psi}{A} = \frac{Q}{A} = \sigma$$

When the medium is nonisotropic, the permittivity, ϵ, is a tensor and the displacement, **D**, and electric field strength, **E**, are not necessarily in the same direction.

Many terms with differing units are used to refer to "flux density," such as ρ_l in units of C/m and ρ_V in units of C/m^3. This confusion is avoided in texts by referring to such terms as ρ_l and ρ_V as charge densities.[13] Care must be exercised when interpreting the phrase "flux density."

Gauss' Law for Electrostatics

Gauss' law for electrostatics is a necessary consequence of Coulomb's law. Gauss' law states that the amount of electric flux, **Ψ**, passing through any closed surface is proportional to (and in our rationalized system of SI units, equal to) the total charge, *Q*, contained within the surface. The surface is called a *Gaussian surface*. Gauss' law in a variety of forms is given in Eq. 17, Eq. 18, and Eq. 19. The term *d***s** is an element of the surface area with magnitude *ds* and direction normal to *d***s**, and *θ* is the angle between the normal to the surface area and **D**.

Equation 17: Gaussian Surface, Parallel to D

$$\Psi = \oiint D\,dA = \oiint \sigma\,dA = Q \quad [d\mathbf{A} \text{ parallel to } \mathbf{D}]$$

[12] This is because **E** is multiplied by the permittivity. See Eq. 6 by way of explanation, noting the permittivity in the denominator.

[13] This is consistent with the electric flux, **Ψ**, being numerically equal to the charge. Another way of considering this is to realize that a rationalized set of units (the SI system) that makes the flux equal to the charge is being used. If the system were not rationalized, the flux would merely be proportional to the charge.

Equation 18: Gaussian Surface, Arbitrary Surface

$$\Psi = \oint\oint D \cdot d\mathbf{s} = \oint\oint D\cos\theta\, ds = Q \quad \text{[arbitrary surface]}$$

Equation 19: Gaussian Surface, Arbitrary Volume

$$\Psi = \iiint \rho\, dv = Q \quad \text{[arbitrary volume]}$$

Because the electric field strength or intensity, **E**, is related to the displacement or electric flux density, **D**, by the permittivity, Gauss' law as given in Eq. 17 can also be stated mathematically as

Equation 20: Gaussian Surface, Parallel to E

$$\Psi = \oint\oint \varepsilon E\, dA = Q \quad [d\mathbf{A} \text{ parallel to } \mathbf{E}]$$

Special Gaussian surfaces are used in highly symmetrical charge configurations to simplify calculations with Gauss' law. Such surfaces meet the following criteria.

1. The surface is closed.

2. At each point on the chosen surface, D, is either normal or tangential to the surface.

3. The flux density, or displacement, magnitude, D, is sectionally constant over the portion of the surface where D is normal.

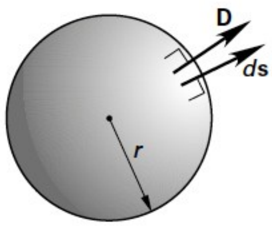
Requirement (1) is a general requirement of Gauss' law. Requirement (2) completely eliminates integrals and makes the calculation simple. Requirement (3) allows D to be removed from the integral.

Example 7

Given a point charge of constant value, Q, what is the flux density, D, at a distance r from the charge?

Solution

First, arbitrarily construct a sphere around the point charge as shown



A sphere surrounding a point charge meets all the requirements of a special Gaussian surface. Using Eq. 18 gives the following.

$$\Psi = \oint\!\!\oint \mathbf{D} \cdot d\mathbf{s} = \oint\!\!\oint D\cos\theta\, ds = Q$$

Because the lines of flux emanate radially outward from the point charge, **D** and *ds* are parallel and cos θ = 1. Additionally, the magnitude of the flux density, *D*, is constant at a given *r*. This results in the following manipulations.

$$\begin{aligned} Q &= \oint D\cos\theta\, ds \\ &= \oint D(1)\, ds \\ &= D\oint ds \end{aligned}$$

The surface area of a sphere is given by $4\pi r^2$.

$$Q = D\oint ds = D4\pi r^2$$

$$D = \frac{Q}{4\pi r^2}$$

Using the same logic as in Ex. 7 on a uniform line charge of density ρ_l of length *l* at a distance of *r* gives the following equation for displacement magnitude.

Equation 21: Line Charge Displacement Magnitude

$$D = \frac{\rho_l}{2\pi r}$$

Capacitance and Elastance

If charges Q+ and Q− exist on conductors separated by free space or by a dielectric, the mutual attraction "stores" the charges, holding them in place. The charges produce a stress in the dielectric measured by the electric field strength, **E**, and a potential difference exists between the plates. *Capacitance* is the property of a system of conductors and dielectrics that permits this storage of charges.

If a potential difference is applied across two conductors, charges will build up, creating an electric field between the conductors. The amount of the charge, Q, is proportional to the applied voltage. The proportionality constant is called the *capacitance* and has units of farads, F. This is shown in Eq. 22. A farad is a large capacitance and very few capacitors are constructed with a value of even 1.0 F. Most circuit elements have capacitances in the range of μF (10^{-6}) or pF (10^{-12}).

Equation 22: SI and U.S. Capacitance

$$Q = CV \quad \text{[SI]}$$

$$C = \frac{Q}{V} \quad \text{[U.S.]}$$

The *elastance*, S, is the reciprocal of the capacitance.

Equation 23: Elastance

$$S = \frac{1}{C} = \frac{V}{Q} = \frac{U/Q}{Q}$$

As shown in Eq. 23, the elastance can also be defined as the amount of energy, U, required per charge, Q, to transfer a unit charge between two conductors separated by a dielectric.

Capacitors

A *capacitor* (once called a *condenser*) is a device that stores electric charge. It consists of conductors separated by a dielectric. A common type of capacitor consists of two parallel plates of equal area A separated by a distance r. This type of capacitor is called a parallel plate capacitor and its capacitance is determined by Eq. 24.

Equation 24: Parallel Plate Capacitor

$$C = \frac{\varepsilon A}{r}$$

Equation 24 ignores the effects of fringing, as do many calculations involving capacitors. An example of fringing is shown in Fig. 7. The formula for the capacitance of other configurations is given in Table 6.

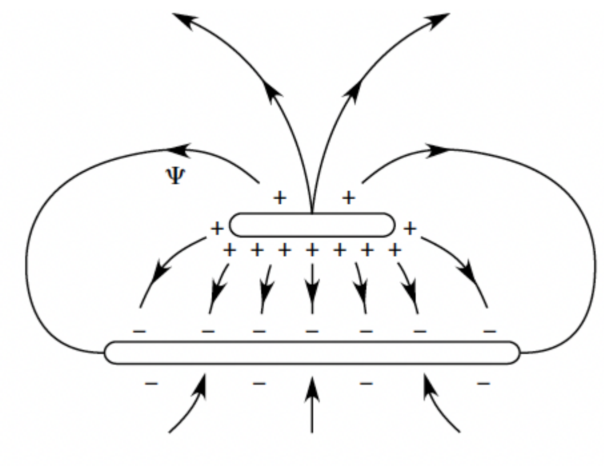


Figure 7: Fringing

Assuming the plates in a capacitor begin with the same potential ($V = 0$), the total energy, U (in J), in the electric field of a capacitor with capacitance C charged to a potential V is given by Eq. 25.

Equation 25: Capacitor Energy

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}VQ = \tfrac{1}{2}\frac{Q^2}{C}$$

For n capacitors in parallel, the total capacitance is

Equation 26: Capacitance in Parallel

$$C_{\text{total}} = C_1 + C_2 + C_3 + \cdots + C_n$$

For n capacitors in series, the total capacitance is

Equation 27: Capacitance in Series

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \frac{1}{C_n}$$

Energy Density in an Electric Field

In a rectangular dielectric solid of surface area A and thickness t with a potential difference of V applied to either surface, the average energy (in J) is given by

Equation 28: Capacitor Energy

$$\begin{aligned} U_{\text{ave}} &= \tfrac{1}{2} V_{\text{voltage}} Q = \tfrac{1}{2}(Et)(DA) = \tfrac{1}{2} EDV_{\text{volume}} \\ &= \tfrac{1}{2} \varepsilon E^2 V_{\text{volume}} \\ &= \tfrac{1}{2} \frac{D^2}{\varepsilon} V_{\text{volume}} \end{aligned}$$

Equation 28 in all its forms assumes that **D** and **E** are in the same direction. If not, the dot product must be used to determine the energy.

The average energy density at any point in this electric field is given by Eq. 29.

Equation 29: Average Capacitance Energy

$$u_{\text{ave}} = \frac{U_{\text{ave}}}{V_{\text{volume}}} = \tfrac{1}{2} \varepsilon E^2 = \tfrac{1}{2} \frac{D^2}{\varepsilon}$$

PART 4: ELECTROKINETICS

Speed and Mobility of Charge Carriers

A positive free charge moving in an electric field through a vacuum is shown in Fig. 8. Such a particle would experience a force given by the following.[14]

Equation 30: Force on Charge Carriers

$$\mathbf{F} = Q\mathbf{E} = m\mathbf{a}$$

[14] This is identical to Eq. 4 without the subscripts. Nominally, subscripts need only be used to avoid confusion when multiple fields or charges are involved

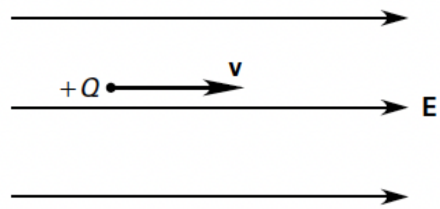


Figure 8: Free Charge in the Electric Field of a Vacuum

The force the particle experiences is unopposed and results in the constant acceleration of the particle as long as it remains in the electric field.

In metals, according to *electron-gas theory*, the electrons instead reach an average *drift velocity*, $\mathbf{v}_d$, because they undergo collisions as they move through the electric field. These collisions are between the electrons and the atoms within the crystalline structure of the metal and are caused by the thermal vibrations of the individual atoms.[15] The average drift velocity is

Equation 31: Drift Velocity Average

$$\mathbf{v}_d = \int \mathbf{a}\, dt = \int \frac{\mathbf{F}}{m} dt = \frac{Q}{m} t_m \mathbf{E} = \mu \mathbf{E}$$

The term t_m is the mean free time between collisions and m is the mass of the particle.[16] The mobility of the moving charge is given by μ. Because the particles in the metal undergo an increase in vibratory motion as the temperature increases, the mobility varies with the temperature—and with the crystalline structure of the material. As the temperature increases, the mobility decreases, resulting in a lower drift velocity and a lower current. In circuit analysis this phenomenon is called *resistivity*. The resistivity is the reciprocal of the conductivity.

Current[17]

An *electric current* is the movement of charges past a particular reference point or through a specified surface. The symbol I is generally used for constant currents while i is used for time-varying currents. Current is measured in *ampères* (A), commonly called amps. 1 A = 1 C/s. By convention, the current moves in the direction a positive charge would move (from the positive

[15] The thermal vibrations can be treated in a quantum mechanical fashion as discrete particles called *phonons*.

[16] The variation in the overall field within the metal is caused by the periodic variations established by the lattice atoms. The field variations can result in collisions, accounted for in some formulas by an *effective electron mass* , m_e*. Only the thermal vibrations must be determined when such a mass is used.

[17] Compare an electric field's induced current to a magnetic field's induced voltage.

terminal to the negative terminal), that is, opposite to the flow of electrons. Ohm's law relates current to voltage and resistance as shown in Eq. 32.

Equation 32: Ohm's Law

$$V = IR$$

For simple DC circuits, this relationship is adequate and the current is defined by Eq. 33.[18]

Equation 33: Constant Current

$$I = \frac{dQ}{dt}$$

However, when the charges exist in a liquid or a gas, as in the field of electronic engineering, or when the charges are both positive and negative and have different characteristics, as in semiconductors, Eq. 33 is inadequate and one must be more specific in defining current. As a result, the current density vector, **J**, with units of A/m^2 is more frequently utilized in electromagnetic theory.

The *total current* is given by the sum of the convection current, the displacement current, and the conduction current. The convection and conduction currents are considered true currents and the displacement current a virtual current. Unless otherwise stated, the "current" will normally be the conduction current, but care must be used when determining what "current" is being referenced.

Convection Current

In a material containing a volume V of mobile charges with a charge density of ρ moving across a surface area A with an average speed of $\mathrm{v}_{\mathrm{ave}}$, the convection current will be given by Eq. 34.

Equation 34: Convection Current

$$I = \rho A \mathrm{v}_{\mathrm{ave}} = \left(\frac{Q}{V}\right) A \mathrm{v}_{\mathrm{ave}} = \left(\frac{Q}{Al}\right) A \left(\frac{l}{t}\right) = \frac{Q}{t}$$

In terms of the current density vector, the convection current is given by Eq. 35 and can be visualized as shown in Fig.9.

[18] The use of the capital I indicates that the speed of the charges referred to is constant. If the speed is not constant then the current varies with time and i would be used as the appropriate symbol. In practical terms, this means that currents in DC circuits (unidirectional current flow) in which the current magnitude varies could be labeled i.

Equation 35: Current Density Vector

$$\mathbf{J} = \rho \mathbf{v}_d$$

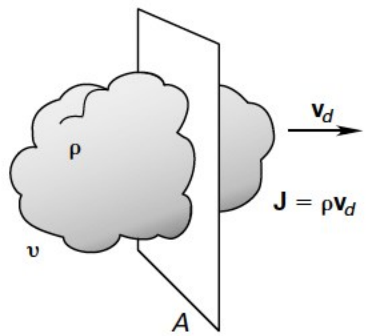


Figure 9: Convection Current

Convection, as the name implies, involves the diffusion of charges in a medium, as opposed to the movement of charges under the influence of an electric field. Although not of general use in electrical engineering, the concept of a convection current is sometimes useful in electromagnetic field theory.

Displacement Current[19]

When the electric field varies with time, a current can be made to flow in a dielectric material. That is, charges can be displaced within a dielectric by the electric fields of the charges moving outside the borders of the dielectric. This is because the electric fields of the charged particles extend well beyond the dimensions of the charges themselves. The current so created is called the *displacement current*, i_d, and is given by Eq. 36.

Equation 36: Displacement Current

$$i_d = \frac{d\psi}{dt}$$

The displacement current in terms of the current density vector, **J**, is given by Eq. 37.

[19] Compare displacement current to the displacement magnetic effect.

Equation 37: Displacement Current, Current Density Vector

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t}$$

The displacement current, i_d, through any given surface can be obtained by integration of the normal component of the displacement current density vector, $\mathbf{J}_d$, over the surface as in Eq. 38.

Equation 38: Displacement Current, over any Surface

$$i_d = \frac{d}{dt}\int_A \mathbf{D} \cdot d\mathbf{A}$$

The displacement current is equal to zero if the electric flux is constant with respect to time. That is, to cause a current to "flow" through a dielectric without the benefit of a direct connection to a conductor (as in the case of a capacitor), the electric flux must be changing with respect to time.[20]

Conduction Current[21]

A *conduction current* occurs in the presence of an electric field within a conductor. The magnitude of this current is given by the number of mobile charges transferred per unit time within the conductor. The charges may be electrons (in metals and semiconductors), holes (in semiconductors), or ions (in gases and electrolytes). Conduction current is given by Eq. 39.

Equation 39: Conduction Current

$$I_c = \sigma\left(\frac{A}{l}\right)V = GV$$

The symbol σ indicates the conductivity of the material and is a property of a unit volume of the material. The area of the conductor is given by A and the potential difference by V. The length represented by l is the length over which the potential is applied. The quantity G is the conductance of the specified conductor and depends on the dimensions of the material used.

The conduction current in terms of the conduction current density vector, $\mathbf{J}_c$, is given by Eq. 40.

[20] The charges do not flow through the dielectric but are instead displaced within the dielectric. Also, this displacement ceases if the electric flux within the dielectric no longer changes.

[21] Compare conduction current with the principles of the magnetic circuit.

Equation 40: Conduction Current, Current Density Vector

$$\mathbf{J}_c = \rho \mathbf{v}_d$$

This is identical to Eq. 35. Applying Eq. 31, specifically $\mathbf{v}_d = \mu \mathbf{E}$, puts Eq. 40 in the more widely recognized form shown in Eq. 41.

Equation 41: Conduction Current, Point Form of Ohm's Law

$$\mathbf{J} = \sigma \mathbf{E}$$

SUMMARY

This course is meant to provide an overview of the items of concern. I hope the task is accomplished. All the best.

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NOTE

Electrical refers to something related to electricity while "electric" refers to a device or machine that runs on electricity. Nevertheless, the NEC is sometimes referred to as the National Electric Code.

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Appendix A: Equivalent Units Of Derived And Common SI Units

Symbol	Equivalent Units			
A	C/s	W/V	V/Ω	J/(s·V)
C	A·s	J/V	(N·m)/V	V·F
F	C/V	$\mathrm{C^2/J}$	s/Ω	(A·s)/V
F/m	C/(V·m)	$\mathrm{C^2/(J \cdot m)}$	$\mathrm{C^2/(N \cdot m^2)}$	s/(Ω·m)
H	W/A	(V·s)/A	Ω·s	$\mathrm{(T \cdot m^2)/A}$
Hz	1/s	$\mathrm{s^{-1}}$	cycles/s	radians/(2π·s)
J	N·m	V·C	W·s	$\mathrm{(kg \cdot m^2)/s^2}$
$\mathrm{m^2/s^2}$	J/kg	(N·m)/kg	(V·C)/kg	$\mathrm{(C \cdot m^2)/(A \cdot s^3)}$
N	J/m	(V·C)/m	(W·C)/(A·m)	$\mathrm{(kg \cdot m)/s^2}$
$\mathrm{N/A^2}$	$\mathrm{Wb/(N \cdot m^2)}$	$\mathrm{(V \cdot s)/(N \cdot m^2)}$	T/N	1/(A·m)
Pa	$\mathrm{N/m^2}$	$\mathrm{J/m^3}$	$\mathrm{(W \cdot s)/m^3}$	$\mathrm{kg/(m \cdot s^2)}$
Ω	V/A	$\mathrm{W/A^2}$	$\mathrm{V^2/W}$	$\mathrm{(kg \cdot m^2)/(A^2 \cdot s^3)}$
S	A/V	1/Ω	$\mathrm{A^2/W}$	$\mathrm{(A^2 \cdot s^3)/(kg \cdot m^2)}$
T	$\mathrm{Wb/m^2}$	N/(A·m)	(N·s)/(C·m)	$\mathrm{kg/(A \cdot s^2)}$
V	J/C	W/A	C/F	$\mathrm{(kg \cdot m^2)/(A \cdot s^3)}$
V/m	N/C	W/(A·m)	J/(A·m·s)	$\mathrm{(kg \cdot m)/(A \cdot s^3)}$
W	J/s	V·A	V^2/Ω	$\mathrm{(kg \cdot m^2)/s^3}$
Wb	V·s	H·A	$\mathrm{T/m^2}$	$\mathrm{(kg \cdot m^2)/(A \cdot s^2)}$

Appendix B: Physical Constants

Table Note 1

Quantity	Symbol	US Customary	SI Units
Charge			
electron	e		$-1.6022\square10^{-19}$ C
proton	p		$+1.6022\square10^{-19}$ C
Density			
air [STP][32°F, (0°C)]		0.0805 lbm/ft^3	1.29 kg/m^3
air [70°F, (20°C), 1 atm]		0.0749 lbm/ft^3	1.20 kg/m^3
sea water		64 lbm/ft^3	1025 kg/m^3
water [mean]		62.4 lbm/ft^3	1000 kg/m^3
Distance			
Earth radius[2]	⊕	$2.09\square10^{7}$ ft	$6.370\square10^{6}$ m
Earth-Moon separation[2]	⊕☾	$1.26\square10^{9}$ ft	$3.84\square10^{8}$ m
Earth-Sun separtion[2]	⊕⊙	$4.89\square10^{11}$ ft	$1.49\square10^{11}$ m
Moon radius[2]	☾	$5.71\square10^{6}$ ft	$1.74\square10^{6}$ m
Sun radius[2]	⊙	$2.28\square10^{9}$ ft	$6.96\square10^{8}$ m
first Bohr radius	a_0	$1.736\square10^{-10}$ ft	$5.292\square10^{-11}$ m
Gravitational Acceleration			
Earth [mean]	g	32.174 (32.2) ft/sec^2	9.8067 (9.81) m/s^2
Mass			
atomic mass unit	μ or m_μ $\frac{1}{12}m\left({}^{12}C\right)$	$3.66\square10^{-27}$ lbm	$1.6606\square10^{27}$ kg or 10^{-3} kg mol^{-1} / N_A
Earth[2]	⊕	$4.11\square10^{23}$ slugs	$6.00\square10^{24}$ kg
Earth [customary U.S.][2]	⊕	$1.32\square10^{25}$ lbm	-
Moon[2]	☾	$1.623\square10^{23}$ lbm	$7.36\square10^{22}$ kg
Sun[2]	⊙	$4.387\square10^{30}$ lbm	$1.99\square10^{30}$ kg
electron rest mass	m_e	$2.008\square10^{-30}$ lbm	$9.109\square10^{-31}$ kg
neutron rest mass	m_n	$3.693\square10^{-27}$ lbm	$1.675\square10^{-27}$ kg
proton rest mass	m_p	$3.688\square10^{-27}$ lbm	$1.672\square10^{-27}$ kg
Pressure			
atmospheric		14.696 (14.7) lbf/in^2	$1.0133\square10^{5}$ Pa

Temperature			
standard		32˚F (492˚R)	0˚C (273 K)
absolute zero		–459.67˚F (0˚R)	–273.16˚C (0 K)
Velocity[3]			
Earth escape		3.67□10^4 ft/sec	1.12□10^4 m/s
light (vacuum)	c, c_0	9.84□10^8 ft/sec	2.9979 (3.00) □10^8 m/s
sound [air, STP]	a	1090 ft/sec	331 m/s
sound [air, 70°F, (20°C), 1 atm]		1130 ft/sec	344 ft/s
Volume			
Volume: molal ideal gas (STP)[4]		359 ft^3 / lbmol	22.41 m^3/kmol

Table 1 Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.
2. Symbols shown for the solar system are those used by NASA. See https://science.nasa.gov/resource/solar-system-symbols/.
3. Velocity technically is a vector. It has direction.
4. The unit "lbmol" is an actual unit, not a misspelling.

Appendix C: Fundamental Constants

Quantity	Symbols	US Customary	SI Units
Avogadro's number	N_A, L		6.022×10^{23} mol^{-1}
Bohr magneton	$\propto_B$		9.2732×10^{-24} J/T
Boltzmann constant	κ	5.65×10^{-24} ft-lbf/ R	1.3805×10^{-23} J/T
electron volt: $\left(\frac{e}{C}\right)$ J	eV		1.602×10^{-19} J
Faraday constant, $N_A e$	F		96485 C/mol
fine structure constant, inverse α^{-1}	α α^{-1}		7.297×10^{-3} (≈1/137) 137.035
gravitational constant	g_c	32.174 lbm-ft/lbf-sec^2	
Newtonian gravitational constant	G	3.44×10^{-8} ft^4 / lbf-sec^4	6.672×10^{-11} N$\cdot$m^2 / kg^2
nuclear magneton	$\propto_N$		5.050×10^{-27} J/T
permeability of a vacuum	$\propto_0$		1.2566×10^{-6} N/A^2 (H/m)
permittivity of a vacuum, electric constant $1/\mu_0 c^2$	ε_0		8.854×10^{-12} C^2 / N$\cdot$m^2 (F/m)
Planck's constant	h		6.6256×10^{-34} J$\cdot$s
Planck's constant: h/2π			1.0546×10^{-34} J$\cdot$s
Rydberg constant	$R_\times$		1.097×10^{7} m^{-1}
specific gas constant, air	R	53.3 ft-lbf/lbm- R	287 J/kg$\cdot$K
Stefan-Boltzmann constant		1.71×10^{-9} BTU/ft^2-hr-°R^4	5.670×10^{-8} W/m^2$\cdot$K^4
triple point, water		32.02 F, 0.0888 psia	0.01109 C, 0.6123 kPa
universal gas constant	R*	1545 ft-lbf/lbmol- R 1.986 BTU/lbmol- R	8314 J/kmol$\cdot$K

Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/. The unit in Volume of "lbmol" is an actual unit, not a misspelling.

Appendix D: Mathematical Constants

Quantity	Symbol	Value
Archimedes' constant (pi)	π	3.1415926536
base of natural logs	e	2.7182818285
Euler's constant	C or τ	0.5772156649

Appendix E: The Greek Alphabet

A	α	alpha	N	υ	nu
B	β	beta	Ξ	ξ	xi
Γ	γ	gamma	O	o	omicron
Δ	δ	delta	Π	π	pi
E	ε	epsilon	P	ρ	rho
Z	ζ	zeta	Σ	σ	sigma
H	η	eta	T	τ	tau
Θ	θ	theta	ϒ	υ	upsilon
I	ι	iota	Φ	ϕ	phi
K	κ	kappa	X	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
M	μ	mu	Ω	ω	omega

Appendix F: SI Prefixes

symbol	prefix	value
a	atto	10^{-18}
f	femto	10^{-15}
p	pico	10^{-12}
n	nano	10^{-9}
μ	micro	10^{-6}
m	milli	10^{-3}
c	centi	10^{-2}
d	deci	10^{-1}
da	deka	10
h	hecto	10^{2}
k	kilo	10^{3}
M	mega	10^{6}
G	giga	10^{9}
T	tera	10^{12}
P	peta	10^{15}
E	exa	10^{18}

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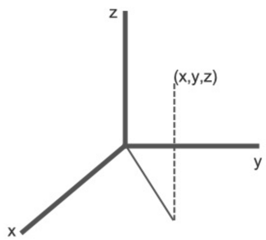
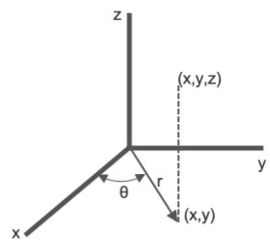
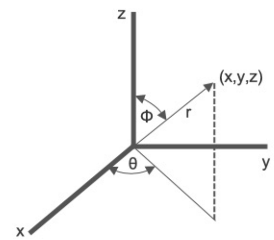
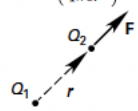
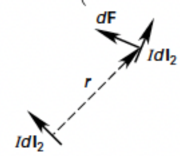
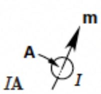
Appendix G: Coordinate Systems & Related Operations

Mathematical Operations	Rectangular Coordinates	Cylindrical Coordinates	Spherical Coordinates
Conversion to Rectangular Coordinants	 $x = x$ $y = y$ $z = z$	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	 $x = r\sin\phi\cos\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\phi$
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\boldsymbol{\theta} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\boldsymbol{\phi} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \theta}\boldsymbol{\theta}$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial (A_\phi \sin\phi)}{\partial \phi} + \frac{1}{r\sin\phi}\frac{\partial A_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{r} & \boldsymbol{\theta} & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & A_\theta & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r^2\sin\theta}\mathbf{r} & \frac{1}{r^2\sin\theta}\boldsymbol{\phi} & \frac{1}{r}\boldsymbol{\theta} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ A_r & rA_\phi & rA\theta A_\phi \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial r}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2 f}{\partial \theta^2}\right)$

Appendix H: Comparison of Electric & Magnetic Equations

equation description	electric version	magnetic version	remarks
experimental force law	Coulomb's law $\mathbf{F} = \left(\frac{Q_1 Q_2}{4\pi\epsilon r^2}\right)\mathbf{r}$ 	force between two current elements $d\mathbf{F} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I_2 d\mathbf{l}_2 \times (I_1 d\mathbf{l}_1 \times r)}{r^2}\right)$ 	The term Idl in the magnetic column is the equivalent of a "magnetic charge" q_m. The I or the dl can be the vector. The r is a unit vector pointing from 1 to 2.
field definitions from force law	$\mathbf{F} = Q\mathbf{E}$	$d\mathbf{F} = \mathbf{I}\times\mathbf{B}\,dl$ current element $d\mathbf{F} = \mathbf{J}\times\mathbf{B}\,dV$ distributed current element $d\mathbf{F} = q\,\mathbf{v}\times\mathbf{B}$ moving charge	The V used in this row represents volume, not voltage. The v is the velocity.
general force law	$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$ $d\mathbf{F} = (\rho\mathbf{E} + \mathbf{J}\times\mathbf{B})\,dV$ where $dQ = \rho\,dV$		The V in this row represents the volume, not voltage. The v is the velocity.
definition of scalar and vector potential	$E = -\nabla V$	$\mathbf{B} = \nabla \times \mathbf{A}$	**A** is the magnetic vector potential.
Poisson's equation for the potential function	$\nabla^2 V = -\frac{\rho}{\epsilon}$	$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$	From a knowledge of the charge distribution, the potential can be found and then the **E** and **B** fields determined.
Gauss's law enclosing charge and Ampère's law enclosing current	$\iint \mathbf{D}\cdot d\mathbf{A} = \iiint \rho\,dV = Q$ $\nabla\cdot\mathbf{D} = \rho$	$\oint H\cdot d\mathbf{l} = I$ $\nabla \times \mathbf{H} = \mathbf{J}$	The V in this row represents volume.
constitutive relations	$\mathbf{D} = \epsilon\mathbf{E}$ $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$	$\mathbf{B} = \mu\mathbf{H}$ $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$	The second set of equations is always valid. The first set assumes the medium is linear and isotropic.
definitions of relative permittivity and permeability	$\epsilon_r = \frac{\epsilon}{\epsilon_0}$ $\epsilon_0 = 8.854 \times 10^{-12}$ F/m	$\mu_r = \frac{\mu}{\mu_0}$ $\mu_0 = 4\pi \times 10^{-7}$ H/m	

equation description	electric version	magnetic version	remarks
capacitance and inductance of a field cell	$\epsilon_0 = \frac{C}{l}$	$\mu_0 = \frac{L}{l}$	Field cells are a construct designed to represent free space in terms of a parallel plate capacitor and an inductor. This capacitance and inductance exist regardless of the presence of an electric or magnetic field.
capacitance and inductance	$C = \frac{Q}{V}$	$L = \frac{\Lambda}{I}$	Λ is the flux linkage.
energy density of a field	$U = \frac{1}{2}\epsilon E^2$	$U = \frac{1}{2}\mu H^2$	Both energy and momentum are carried by a field.
energy stored by capacitance and inductance	$W = \frac{1}{2}CV^2$	$W = \frac{1}{2}LI^2$	
electromotive and magnetomotive force with sources present	$\oint \mathcal{E} \cdot d\mathbf{l} = \mathcal{E} = V$	$\oint \mathbf{H} \cdot d\mathbf{l} = NI = F_m = V_m$	The $\mathcal{E}$ is the emf, not the permittivity. Without sources present, both line integrals are equal to zero.
dipole moments	 $\mathbf{p} = q\mathbf{d}$	 $\mathbf{m} = I\mathbf{A}$	
dipole torque	$\mathbf{T} = \mathbf{p} \times \mathbf{E}$	$\mathbf{T} = \mathbf{m} \times \mathbf{B}$	This torque occurs due to the dipole being immersed in an external $\mathbf{E}$ or $\mathbf{B}$ field.
dipole potential energy	$W = -\mathbf{p} \cdot \mathbf{E}$	$W = -\mathbf{m} \cdot \mathbf{B}$	

