A SunCam Online Continuing Education Course

Nuclear Power

Basics

Basic Information / Atomic & Nuclear Physics / Radiation / Shielding

by

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Future Courses
[Based on Interest / Contact IDriveSubs@gmail.com]
Nuclear Power Systems
Nuclear Reactor Theory

Nomenclature[1]

a	abundance or atom	%
a	acceleration	m/s^2
a	unit vector	-
A, **A**	area	m^2
A	number of nucleons (protons & neutrons) in nucleus [atomic mass number][Z + N = A]	#
B, **B**	magnetic flux density	T (Wb/m^2)
BE, BE	Binding Energy	eV, MeV
c, *c*	speed of light, 2.9979×10^{8}	m/s
C	capacitance	F
d	distance	m
D	absorbed dose	rad or Gy
D, **D**	electric flux density	C/m^2
e^{-}	charge of an electron, -1.6022×10^{-19}	C
esu	electrostatic unit	1 R or 3.33×10^{-10} C
E, **E**	electric field strength	V/m
E	energy	J
F, **F**	force	N
G	conductance	S (A/V)
G	universal gravitational constant, 6.672×10^{-11}	$N \cdot m^2 / kg^2$
h	Planck's constant, 6.6256×10^{-34}	$J \cdot s$
H, **H**	magnetic field strength	A/m
H	dose equivalent [formerly, RBE Dose]	rem or Sv
i	variable current	A
I, **I**	constant or rms current	A
IR	Infrared	-
J, **J**	current density	A/m^2
k	electrostatic proportionality constant, 8.987×10^{9}	$N \cdot m^2 / C^2$
KE	kinetic energy	
l, **l**	length	m

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[1] Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related and may be found when reviewing the references listed for further information. Further, all the nomenclature, symbols, or subscripts will be found in of many electrical courses (on SunCam, PDH Academy, and also in many texts). For guidance on nomenclature, symbols, and electrical graphics: IEEE 280-2021. IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering. New York: IEEE; and IEEE 315-1975. Graphic Symbols for Electrical and Electronics Diagrams. New York: IEEE, approved 1975, reaffirmed 1993.

ℓ	azimuthal quantum number	-
L	inductance	H
m	mass	kg
m	spatial quantum number	-
m_s, s	spin quantum number	-
M, $\mathbf{M}$	magnetization	A/m
M	mutual induction	H
$m(^{A}Z)$	mass of neutral element Z	#
$M(^{A}Z)$	atomic weight of element Z compared to neutral ^{12}C	#
$n(t)$	concentration, number of radioactive atoms	#
n	principle quantum number	-
N	atom density	atoms/cm^3
N	number of neutrons in nucleus	#
p	pole strength	Wb (N/T)
p^+	charge on a proton, $1.6022\Box 10^{-19}$	C
$\mathbf{P}$	polarization	C/m^2
q, Q	charge	C
Q	Quality Factor	#
Q-value	energy of a reaction	MeV
r, $\mathbf{r}$, R	radius	cm
R	ratio	-
R	resistance	Ω
$\mathcal{R}$ or $\mathfrak{R}$	reluctance	A/Wb
s, $\mathbf{s}$	surface area	m^2
S	elastance	1/F
SI	Systéme International d'Unités / System International	-
t	thickness	m
t	time	s
$\bar{t}$	mean-life [activity falls to 1/*e*]	s
$T_{1/2}$	half-life	s
u	energy density	J/m^3
U	energy	J
υ or v	variable voltage	V
V	constant or rms voltage	V
v, $\mathbf{v}$	velocity*	m/s
V_{volume}	volume	m^3
X	exposure	C/kg
Z	number of protons in nucleus [atomic number]	#

*Velocity is often used in tests where it would be more technically correct to us "speed". Velocity is a vector with speed as its magnitude.

Symbols

α	activity	Bq (becquerel) [1 disintegration/s]
Γ	reciprocal inductance	1/H
ε	permittivity	$\text{F/m}\ \left(\text{C}^2/\text{N}\cdot\text{m}^2\right)$
ε_0	permittivity of free-space, 8.854×10^{-12}	$\text{F/m}\ \left(\text{C}^2/\text{N}\cdot\text{m}^2\right)$
θ	angle	degrees
Λ	flux linkage	Wb
λ	decay constant	s^{-1} [decay probability/sec]
λ	wavelength	m
μ	attenuation coefficient	cm^{-1}
μ	mobility	$\text{m}^2/\text{V}\cdot\text{s}$
μ	permeability	$\text{H/m}\ \left(\text{N}/\text{A}^2\right)$
μ_0	free-space permeability, 1.2566×10^{-6}	$\text{H/m}\ \left(\text{N}/\text{A}^2\right)$
$\mu_0 x$	thickness of shield in mean free paths [of radiation] evaluated at energy E_0	$\left(\frac{1}{\text{cm}}\right)$ (cm)
ν	neutrino	-
ρ	charge density	C/m^3
σ	cross section	cm^2 $10^{-24}\ \text{cm}^2 = 1\text{barn}$
ϕ, Φ	magnetic flux	Wb
χ	susceptibility	-
ψ, Ψ	electric flux	C
ω	angular velocity	rad/s

Subscripts

0	original, origin, zero value, at-rest value, evaluated value	-
1–2	From 1 to 2 Or effect on 2 caused by the field on 1	-
a	shield thickness	cm
ave	average	-
c	Compton scattering, conduction	-
ci	cast iron	-
d	displacement drift	-
D	deposited	-
e	electric, electron	-
l	line	-
m	magnetic	-
m	mean	-
m	monodirectional	-
n	neutron, nucleon	-
p	proton	-
pe	photoelectric effect	-
pp	pair production	-
r	relative	-
s, S	separation	-
x,X	light/heavy reactant	-
y,Y	light/heavy product	-

TABLE OF CONTENTS

Nomenclature....2
Symbols....4
Subscripts....5
List of Figures....7
List of Tables....7
List of Equations....7
COURSE INTRODUCTION....9
BASIC INFORMATION....10
Overview....10
History....10
Utilization of Nuclear Technology....11
Dimensions, Constants, & Symbols....12
ATOMIC AND NUCLEAR PHYSICS....13
Fundamental Particles....13
Atomic & Nuclear Structure—Symbology....14
Atomic & Nuclear Radii....17
Binding Energy....18
Mass and Energy....21
RADIATION, EFFECTS & PROTECTION, SHIELDING....26
Radioactivity Principles....26
Radiation Effects....28
Biological Effects....31
Radiation Protection....33
Shielding....35
Summary....40
FUTURE PROSPECTS....40
REFERENCES....42
Appendix A: Equivalent Units Of Derived And Common SI Units....44
Appendix B: Physical Constants....45
Appendix C: Fundamental Constants....47
Appendix D: Mathematical Constants....48
Appendix E: The Greek Alphabet....48
Appendix F: SI Prefixes....48
Appendix G: Coordinate Systems & Related Operations....49
Appendix H: Comparison of Electric & Magnetic Equations....50

List of Figures

Figure 1: References for Legal and Governmental Guidance 9
Figure 2: SI Base Units 13
Figure 3: Atomic Symbology 14
Figure 4: Binding Energy per Nucleon 19
Figure 5: Half-Life vs Mean-Life 28
Figure 6: Gamma Interaction with Shielding 35
Figure 7: Gamma Spectrum Before & After Shield 38
Figure 8: Nuclear Power Generations 41

List of Tables

Table 1: Oxygen Isotopes 15
Table 2: Q-Values 19
Table 3: Binding Energy Properties 20
Table 4: Electron, Neutron, Proton Masses 25
Table 5: Activity Units 28
Table 6: Absorbed Dose Units 30
Table 7: LET Q-Values 31
Table 8: Energy vs. Weighting Factor 32
Table 9: Radiation Limits 34
Table 10: Exposure Buildup Factors for Monodirectional Source 39

List of Equations

Equation 1: Photon Energy 14
Equation 2: Atomic Mass Number 14
Equation 3: Atomic Weight Defined 15
Equation 4: Atomic Weight with Isotopes 15
Equation 5: Avogadro’s Number 16
Equation 6: Atomic Mass Unit 17
Equation 7: Atomic Radii 17
Equation 8: Nucleus Radii 17
Equation 9: Two-Product Mass Energy Balance 18
Equation 10: Two-Product Q Value 18
Equation 11: Q-Value as Binding Energy 18
Equation 12: Mass as Energy 21
Equation 13: Einstein’s Equation 23
Equation 14: Relativistic Mass 23
Equation 15: Kinetic Energy 23
Equation 16: Classical Kinetic Energy 23
Equation 17: Neutron Velocity vs Energy 25
Equation 18: Activity via Concentration 26
Equation 19: Activity via Initial Activity 26

Equation 20: Half-Life 27
Equation 21: Activity via Half-Lives 28
Equation 22: Exposure 29
Equation 23: Imparted Energy 29
Equation 24: Absorbed Dose 30
Equation 25: Dose Equivalent 31
Equation 26: Sievert to rem 31
Equation 27: Total Gamma Cross Section 38
Equation 28: Gamma Attenuation Coefficient 38
Equation 29: Exposure Rate 39

List of Examples

Example 1....15
Example 2....16
Example 3....19
Example 4....22
Example 5....24
Example 6....24
Example 7....25
Example 8....27
Example 9....36
Example 10....39

COURSE INTRODUCTION

The theoretical information is primarily from the author's books, Refs. [A] and [B]. The NESC Ref. [C] and NEC Ref. [D] though not covered in this course are useful sources for electrical engineers. Information useful in many aspects of electric engineering may be found in [E] and [F]. Reference [G] has detailed descriptions of analysis techniques. Reference [H] covers many terms in EE with excellent definitions and explanations. References [I] and [J] provide indepth information on magnetics, though one should use the latest versions or similar references. The appendices cover information useful in many engineering tasks with App. H, which provides a side by side comparison of electric and magnetic equations. The strictly nuclear aspects of this course, are provided in Refs. [K] through [R]. While older texts the information is valid and Refs. [K] and [P] are very instructive and the best I've found. Reference [S] contains the latest Nuclear Examination requirements, which inform the overall structure of this course. Reference [T] is the author's book of nuclear problems to help prepare for the PE examination and solve issues for those working directly in the field. Reference [U] is a glossary of terms specifically focused on nuclear engineering. For those working in the Health Physics field, Ref [V] is an excellent reference. Later editions of this, and the other nuclear texts, are available. Use these texts or their counterparts for indepth information. References in bold are highly recommended.

This course will focus on basics, that rarely change, and provide the basis for all other knowledge. The list provide in Fig. 1 shows the very latest in guidance on areas requiring further study and may change more rapidly as science and engineering advances.

CODE/STANDARD	TITLE
—	Chart of the Nuclides
10 CFR 20	Standards for Protection Against Radiation
10 CFR 21	Reporting of Defects and Noncompliance
10 CFR 50	Domestic Licensing of Production and Utilization Facilities
10 CFR 51	Environmental Protection Regulations for Domestic Licensing and Related Regulatory Functions
10 CFR 52	Licenses, Certifications, and Approvals for Nuclear Power Plants
10 CFR 54	Requirements for Renewal of Operating Licenses for Nuclear Power Plants
10 CFR 61.55	Subpart D, Waste Classification
10 CFR 70	Domestic Licensing of Special Nuclear Material
10 CFR 71	Packaging and Transportation of Radioactive Material
10 CFR 72	Licensing Requirements for the Independent Storage of Spent Nuclear Fuel, High-Level Radioactive Waste, and Reactor-Related Greater than Class C Waste
10 CFR 835	Appendix D, Surface Contamination Values
10 CFR 835	Appendix E, Values for Establishing Sealed Radioactive Source Accountability and Radioactive Material Posting and Labeling Requirements
40 CFR 261	Subpart C, Characteristics of Hazardous Waste
NUREG 1571	Information Handbook on Independent Spent Fuel Storage Installations
Reg. Guide 1.3	Assumptions Used for Evaluating the Potential Radiological Consequences of a Loss of Coolant Accident for Boiling Water Reactors
Reg. Guide 1.7	Control of Combustible Gas Concentrations in Containment
Reg. Guide 1.28	Quality Assurance Program Criteria (Design and Construction)
Reg. Guide 1.105	Setpoints for Safety-Related Instrumentation
Reg. Guide 1.174	An Approach for Using Probabilistic Risk Assessment in Risk-Informed Decisions on Plant-Specific Changes to the Licensing Basis
Reg. Guide 1.183	Alternative Radiological Source Terms for Evaluating Design Basis Accidents at Nuclear Power Reactors
Reg. Guide 1.203	Transient and Accident Analysis Method

Figure 1: References for Legal and Governmental Guidance

[Source: *NCEES Principles & Practice Examination Requirements*—Effective 2021—Red Indicates Change]

The NUREG is an acronym for the Nuclear Regulatory Commission, who is responsible for the Regulatory Guides (Reg. Guide) shown. CFR listings are the Code of Federal Regulations.

BASIC INFORMATION

Overview

So, to commence, where/when can one say nuclear science started. In my view, like so much of western civilization, it started with the Greeks. Specifically, Democritus (ca. 5th century BCE) proposed the concept of an atom as the smallest bit of material that can exists. He considered it indivisible, eternal, and indestructible. Further, different combinations of these atoms exist in an infinite number and various shapes and sizes comprising all of the universe.[2] All in all, not a bad first assumption. It would be approximately 2000 years before "modern" atomic theory began though the term "atom" was retained to honor the ancients.[3]

History

It was the early Greek philosophers who first determined for western civilization that our planet was comprised of a very few elements or basic substances. Empedocles of Akrargas, about 430 BC, decided these elements were earth, air, water and fire. A century later Aristotle added the "aether" of the heavens.

About 450 BC, in an argument about whether matter could be subdivided indefinitely, Democritus decided it could not and named the smallest particles "atoms," meaning "nondivisible."

Alchemists of the medieval ages attached properties to various substances in an effort to categorize elements. Many, however, were sidetracked in the effort to change base metal into gold; so much so, in fact, that the name alchemist was abandoned in favor of chemist and alchemy became chemistry.

In 1661, Boyle laid down the modern criterion for an element: a basic substance that cannot be broken down to any simpler substance after it is isolated from a compound. Following this, much effort was expended to identify such substances. Cavendish showed that water was in fact hydrogen and oxygen, while Lavoisier proved that air was oxygen and nitrogen. The Greek "elements" were no more.

Boyle, Newton, and Dalton were all convinced that atoms did indeed exist. An Italian chemist named Amedeo Avogadro applied the atomic theory to gases and determined that equal volumes contained equal numbers of atoms. In the nineteenth century, atoms moved from the abstract to the tangible as atomic weights were worked out (first using oxygen as the standard) and then Brownian motion explained. Mendeleev placed order on the growing information regarding atoms with his periodic table. In the twentieth century we have at last "seen" the atom. A field ion

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[2] Atom comes from the Greek word *atomos* meaning undivided.

[3] And, as those reading this course likely aware, the term "atomic" should be "nuclear" since nuclear refers to the nucleus; however incorrect, the terms are used interchangeably.

microscope was used to strip positively charged ions from the tip of a fine needle and shoot them at a fluorescent screen, producing a five million-fold magnification that made the atoms visible as bright dots. In 1970 a scanning electron microscope was used to detect individual uranium and thorium atoms.

It is the twentieth century from which most would date "nuclear history." For though many still call nuclear energy "atomic," it is actually the interior of the atom, the nucleus, which is manipulated to release energy.

Early in the century, J. J. Thompson "discovered" the electron. Following this, the shell structure of the atom was defined and all of chemistry and the periodic table had a solid theoretical foundation. It is from this foundation that an understanding of the interior of the atom grew.

In the early 1900s radioactivity was discovered. Not all the "rays" identified were electrons, however. Some of these rays were positively charged and must have originated elsewhere. Rutherford determined the basic structure of the atom by firing alpha particles at a thin foil of metal. When some were scattered directly back and some went completely through, he surmised the atom must consist of a small central core surrounded by electrons. From here, things progressed rapidly, with Bohr developing his "liquid drop" model of the nucleus in 1936.

To explore the nucleus, Fermi used neutrons because they were not repulsed by the protons' positive charged. In one of his experiments, he found new radioactive substances with properties he could not explain. The answer came from Austrian physicist Meitner working in Berlin: the uranium had undergone fission. The information was carried by Bohr to a conference in Washington. A letter was written by Einstein, at the insistence of Wigner and Teller, to President Franklin Delano Roosevelt, and the Manhattan project was born.

The first reactor was built at the University of Chicago in a "pile" under the football stadium and went critical on December 2, 1942. In 1954, the submarine USS Nautilus was launched, showing the usefulness of nuclear energy. The first US commercial plant began operation in 1958 in Shippingport, Pennsylvania. In 1965, a satellite powered by a small reactor was launched. And, the nuclear renaissance has just begun.

Utilization of Nuclear Technology

Nuclear technology is more often than not associated with power production.[4] The Nuclear Energy Institute estimates that approximately 18% of all US electricity comes from nuclear power plants with 45% of the nation's emission-free power coming from the same. And, nuclear generation electricity costs are the lowest since 2002. The United States is the largest producer of nuclear

---

[4] …and nuclear weapons. But contrary to popular belief, power plants don't "explode". (There can be steam explosions due to overheating and overpressurization, but not a nuclear explosion.) And, meltdowns don't burn holes to China. They spread out, the chain reaction stops, and they cool down (admittedly, causing other problems.)

power with France having the largest share of electricity produced by nuclear power (70%). The total lifetime emissions of carbon is lowest for nuclear power with only one exception, on-shore wind power.[5]

Using recycling[6] and breeder reactors uranium fuel could power the world for some 3,000 years or more.[7]

Non-power applications of nuclear technology includes *radiotracer techniques*. This is the use of radioactive material as tags or tracers in everything from piston ring wear, underground tank volumes and leakage, and many medical uses. *Industrial uses* include the smoke detectors in your home, sterilization of medical products, preservation of foodstuffs, radiography of welds, nuclear gaging, and numerous others. *Medical applications* include photon emission tomography (PET), computer aided tomography (CAT) using x-rays, and of course x-rays themselves to take "pictures" of the body's internals.

Dimensions, Constants, & Symbols

The use dimensions are primarily focused on the SI system of seven base units with multiple derived units. See both App. A and Fig. 2.

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[5] Even lower that solar power and hydroelectric power.

[6] Using recycling, which the military and several nations have been using for decades, nuclear waste would be reduced to some 10% of its current volume. And, even that may be able to be put to use instead of thrown away.

[7] The 3000 years is based on known uranium reserves, world electric power consumption, recycling—all at the time of writing the author's book on nuclear engineering problems, Ref [T]. Should fusion become a reality, this number reaches into billions of years!

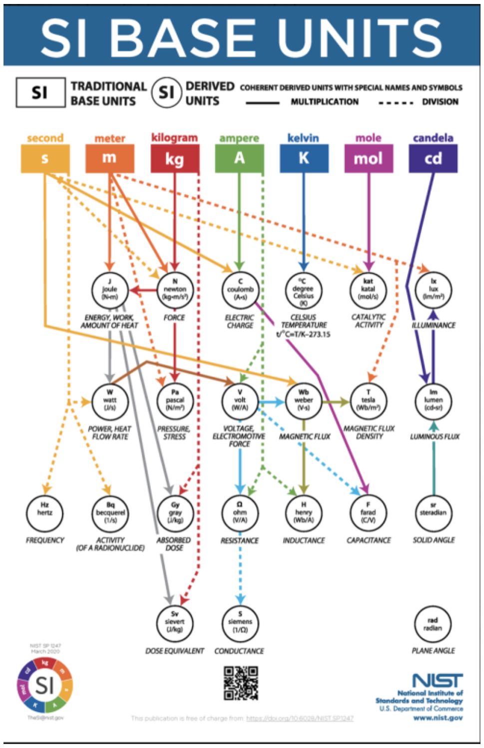

Figure 2: SI Base Units

[Credit: *E. Tiesinga, K. Dill, D. Newell/NIST*]

Physical and Fundamental Constants are in App. B and C, with Mathematical Constants in App. D. The Greek alphabet used as symbols in engineering and science is shown in App. D. Appendix F contains common SI prefixes along with their values.

For proper usage and definitions the National Institute of Standards and Technology offers pamphlets describing the history, proper symbology, accepted constant values, and more.[8]

ATOMIC AND NUCLEAR PHYSICS

Fundamental Particles

Though there are many particles, the following are those important in nuclear engineering. While the other particles are important to nuclear physics, they don't impact the power production focus of this course.

Electron: The electron rest mass, m_e, is 9.1095×10^{-31} kg. The electron charge, e, is -1.6022×10^{-19} C. (If the particle is a positron, the charge is $+1.6022\times10^{-19}$ C.[9])

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[8] See https://www.nist.gov for more information.

[9] See App. B for such values. Also, positrons are seen in Beta-plus (β+) decay of radioactive products.

Proton: The proton rest mass, m_p, is 1.6727×10^{-27} kg. The proton charge is equal and opposite to the electron charge, $+1.6022 \times 10^{-19}$ C.

Neutron: The mass of the neutron, m_n, is 1.6750×10^{-27} kg. It has no charge (electrically neutral). It is stable only inside the nucleus. Outside of the nucleous, it decays to a proton via β– decay.

Photon: This "wave" or "particle" has zero rest mass and zero charge, travels at the speed of light, and does have energy given by

Equation 1: Photon Energy

$$E = h\nu = \frac{hc}{\lambda}$$

Neutrinos: This "particle" has zero mass (or approximately so) and no electric charge. Of the six types, only the electron neutrino and electron antineutrino impact nuclear engineering and are both bunched together as neutrinos and given the symbol "nu", ν.

Atomic & Nuclear Structure—Symbology

The atomic symbology is shown in Fig. 3. The Z is the number of protons and is called the *atomic number*.[10] The symbol N, shown in Eq. 2, is the number of neutrons with Z in the nucleus giving the *atomic mass number, A*. The X is the element's symbol or symbols.

Equation 2: Atomic Mass Number

$$Z + N = A$$

$${}^{A}_{Z}X$$

Figure 3: Atomic Symbology

As an example, ${}^{1}_{1}H$ hydrogen, ${}^{2}_{1}H$ is deuterium, and ${}^{3}_{1}H$ is tritium—all of which have applications in nuclear engineering.

The term *isotope* indicates the same atomic number (Z) but different atomic "weight" or atomic mass number (A). Isotopes exist in nature and usually are listed by percentage of "abundance" or *atom percent*, given as ${}^{a}\!/_{0}$.[11]

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[10] The symbol Z originates from the German word "Zahl", which simply means "number". Initially, "Zahl" was used to indicate an element's position on the periodic table. When it was found to be the charge, the name change to "atomzahl" or atomic number, but the symbol Z was retained.

[11] One could also use "weight abundance" or ${}^{w}\!/_{0}$.

The atomic *weight* of an atom is defined as the *mass* of a neutral atom relative to the mass of a neutral carbon atom, ${}^{12}\mathrm{C}$. One caveat, the mass of the neutral carbon 12 atom is taken as exactly 12. This is represented by the following where M is the "weight" and m is the mass, both of neutral atoms.

Equation 3: Atomic Weight Defined

$$M\left({}^{A}Z\right)=12\cdot\frac{m\left({}^{A}Z\right)}{m\left({}^{12}C\right)}$$

Now, since isotopes exist in various abundances, the "weight" will be an average of those weights. Let γ_I be the isotopic abundance in atom percent of the *i*th isotope of atomic weight M_i, then the weight of the element, M, accounting for the isotopes is given by the following.

Equation 4: Atomic Weight with Isotopes

$$M=\sum_i\frac{\gamma_i M_i}{100}$$

Example 1

Consider the abundances of the following isotopes of oxygen given in the following table.

Table 1: Oxygen Isotopes

Isotope	Abundance $[{}^{a}/_{o}]$	Atomic Weight
${}^{16}\mathrm{O}$	99.759	15.99492
${}^{17}\mathrm{O}$	0.037	16.99913
${}^{18}\mathrm{O}$	0.204	17.99916

What is the atomic weight of naturally occurring oxygen?

Solution

Using Eqn. 4,

$$
\begin{aligned}
M &= \sum_i \frac{\gamma_i M_i}{100} \\
&= \frac{\gamma_{16}\left(M_{16}\right)+\gamma_{17}\left(M_{17}\right)+\gamma_{18}\left(M_{18}\right)}{100} \\
&= \frac{(99.759)(15.99492)+(0.037)(16.99913)+(0.204)(17.99916)}{100} \\
&= 15.99938
\end{aligned}
$$

It is this number that shows up on a Chart of the nuclides as the atomic weight of oxygen.[12]

Atomic and molecular weights are unitless as they are ratios. The gram atomic weight or gram molecular weight is the amount of substance having a mass, in grams, equal to the atomic or molecular weight—the *mole*. A mole, by definition, has the same number of atoms or molecules as any other mole, which is *Avogadro's Law*. That number is called Avogadro's Number and is given by the following.[13]

Equation 5: Avogadro's Number

$$\mathrm{N_A} = 6.02217 \times 10^{23}\ \frac{\text{atoms}}{\text{mole}} \text{ or mol}^{-1} = 0.602217 \times 10^{24}\ \text{mol}^{-1}$$

Example 2

What is the mass on one atom of $^{12}\mathrm{C}$?

Solution

Since one gram-mole of Carbon-12 has a mass of 12 g and contains N_A atoms,

$$
\begin{aligned}
m\left({}^{12}\mathrm{C}\right) = \frac{12\ \text{g}}{N_A} &= \frac{12\ \text{g/mol}}{0.602217 \times 10^{24}\,\text{atoms/mol}} \\
&= 1.99264 \times 10^{-23}\ \text{g}
\end{aligned}
$$

---

[12] A nuclide is a species of atom characterized by its mass number, atomic number, and nuclear energy state provided that state's mean life is long enough to be observable.

[13] The second value, using 0.6… and the exponent of 24th is used in nuclear engineering because the unit of barns is in the power of the 24th and shows up in many equations.

While the mass of a single atom in Ex. 2 is accurate, it is unwieldly. A more natural unit is the *atomic mass unit* (amu), which is defined as one-twelfth of the mass of a neutral ${}^{12}\mathrm{C}$.

Equation 6: Atomic Mass Unit

$$1\ \mathrm{amu} = {}^{1}\!/_{12} \cdot m\left({}^{12}\mathrm{C}\right)$$

This means that the mass of Carbon-12 can be represented as 12 amu. Using Eq. 3 and rearranging gives

$$\begin{aligned}
M\left({}^{A}Z\right) &= 12 \cdot \frac{m\left({}^{A}Z\right)}{m\left({}^{12}C\right)} \\
m\left({}^{A}Z\right) &= \frac{m\left({}^{12}C\right)}{12} \cdot M\left({}^{A}Z\right) = \frac{12\ \mathrm{amu}}{12} \cdot M\left({}^{A}Z\right) \\
m\left({}^{A}Z\right) &= M\left({}^{A}Z\right)
\end{aligned}$$

Meaning, *the mass of any atom, in amu, is equalt to the atomic weight of the atom in question.*

Atomic & Nuclear Radii

The size of an atom is difficult to discern. But, using the average distance of the electrons from the nucleus the average radii is given by the following.[14] Since the radius is relatively constant, the average electron density increases with the atomic number, *Z*.

Equation 7: Atomic Radii

$$R_{\mathrm{atomic}} = 2 \cdot 10^{-8}\ \mathrm{cm}$$

The nucleus also does not have a sharp boundary, but testing indicates it is approximately a sphere with a radius given by the following.

Equation 8: Nucleus Radii

$$R_{\mathrm{nucleus}} = 1.25 \cdot 10^{-13}\mathrm{A}^{1/3}$$

Equation 8 indicates, because the volume (*V*) is proportional to the cube of the radius[15], then the volume *V* is proportional to *A*, the atomic mass number. This means that the ratio *A/V*, nucleons

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[14] This applies to all but the lightest atoms.

[15] That is, the volume is $4/3\pi$ x R^3.

per unit volume, is constant and the nucleus can be modeled as a liquid-drop [because a liquid-drop has the same density whether large or small].[16]

The other major model of the nucleus is the shell model, similar to that for electrons.[17]

Binding Energy

The binding energy (BE) of a given particle in a given system is the net energy required to remove it from the system (also called the *separation energy*). For a system, the BE is the net energy to decompose it into its constituent particles (also called the *total binding energy*).[18] Or, considered another way, binding energy is the *minimum energy synthesis* for creation of the products.[19] When viewed in this manner, *the Q value, which is the is the difference between the total energy of the reactants [symbol X for heavy nuclides, and x for light nuclides] and the total energy of the products [symbol Y for heavy nuclides and y for light nuclides]*. The symbol m_j indicates the rest mass of a species.[20] And, *a mass difference indicates a kinetic energy change*. Thus, for a binary two product reaction, all participants (before and after) in a neutral state[21], isolated, and in the ground nuclear and atomic state[22], the following mass/energy balance occurs.

Equation 9: Two-Product Mass Energy Balance

$$KE_X + m_X \mathrm{c}^2 + KE_X + m_x c^2 = KE_Y + m_Y \mathrm{c}^2 + KE_y + m_y c^2$$

Equation 10: Two-Product Q Value

$$Q \equiv c^2(m_X + m_x - m_Y - m_y) = KE_Y + KE_y - KE_X - KE_x$$

The *Q*-value in terms of the binding energy can then be represented as follows.

Equation 11: Q-Value as Binding Energy

$$Q = BE_Y + BE_y - BE_X - BE_x$$

---

[16] The liquid-drop model accounts for many of the properties of the nucleus—**binding energy** and fission.

[17] The shell model accounts for **stability**, magic numbers, and certain nuclear reactions. Magic numbers represent the total nucleons in especially stable atoms (think Shell Model): 2, 6, 8, 14, 20, 28, 50, 82, 126.

[18] See Ref. [U].

[19] See Ref. [L] for a complete explanation.

[20] The subscript j is used in binding energy calculations and atomic energy levels to indicate the unique state of the electrons, accounting for both electron orbital angular momentum (*l*) and spin angular momentum (*s*).

[21] That is the number of electrons equal the number of protons.

[22] A ground atomic state indicate electrons are not excited (and equal to the number of protons). To account for this electron binding energy one will see m_H instead of m_p in the equations , thus accounting for neutrality as well as the electron binding energy.

The *Q*-value possibilities are shown in the following table.

Table 2: Q-Values

$Q > 0$	exothermal reaction	energy liberation [fission/fusion]
$Q = 0$	scattering reaction	no energy change no mass change
$Q < 0$	endothermal reaction	energy absorption

The binding energy in a fission reaction is calculated from the original particles (e.g., U-235 and a neutron) as compared to the reaction by-products. Meaning the intermediate processes (and nuclear excitation states) need not be considered.

When BE/A is calculated for the elements, the result is shown in the following figure.[23]

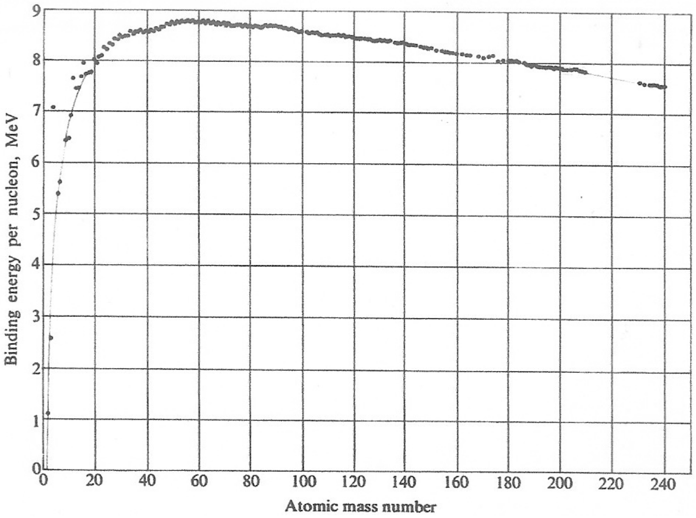


Figure 4: Binding Energy per Nucleon

[Source: *Introduction to Nuclear Engineering,* Ref. [K]]

Example 3

Calculate the binding energy of the last neutron (the least bound neutron) in ^{13}C. [This process removes the neutron without providing it with any kinetic energy.]
Solution

By removing the neutron from ^{13}C, the residual (remaining) nucleus is ^{12}C.

---

[23] The liquid drop model results are very consistent with the binding energy per nucleon shown.

In terms of symbology, the equation is the mass of the neutron plus the mass of the residual atom minus the mass of the resulting atom.[24]

$$E_S = \left[M_n + M\left({}^{A-1}Z \right) - M\left({}^{A}Z \right) \right] c^2$$

Looking up the value in a chart of nuclides gives The following.

$$M_n = 1.00866 \text{ amu and } \mathrm{M}\left({}^{12}\mathrm{C} \right) = 12.00000 \text{ amu}$$
$$M_n + \mathrm{M}\left({}^{12}\mathrm{C} \right) = 13.00866 \text{ amu}$$
$$\mathrm{M}\left({}^{13}\mathrm{C} \right) = 13.00335 \text{ amu}$$

Therefore,

$$E_S = \left[M_n + M\left({}^{A-1}Z \right) - M\left({}^{A}Z \right) \right] c^2 = \left[1.00866 + 12.00000 - 13.00335 \right] c^2 = 0.00531 \text{ amu} = 4.95 \text{ MeV}$$

The properties of binding energy and its curve are indicated in the following table.

Table 3: Binding Energy Properties

Property	**Notes**
BE > 0	By Definition
$\dfrac{d\mathrm{BE}}{dA} > 0$	In General [variations exists at low *A*]
$\dfrac{d\mathrm{BE}}{dA}$	Essentially a Stability Index. High Values indicates greater mass defect and greater BE. Peak at *A* = 4 for extremely stable ${}^4\mathrm{He}$
$2.2 \text{ MeV} \le \mathrm{BE} \le 1900 \text{ MeV}$	Range of Binding Energy
BE(MeV) = 931 MeV per amu	Conversion amu to MeV
A > 60	Weakly Decreasing / Smooth Function of *A* / Area of Fission
A < 20	Steeply Increasing / Large Variations in *A* / Area of Fusion

---

[24] The use of c^2 will be discussed in the next section. Also, the value of an amu in MeV is shown in App. B.

Mass and Energy

Mass and Energy are equivalent.[25]

By mass–energy equivalence, the electronvolt thus corresponds to a unit of mass. It is common in particle physics, where units of mass and energy are often interchanged, to express mass in units of eV/c^2, where c is the speed of light in vacuum (from $E = mc^2$). It is common to informally express mass in terms of energy using eV as a unit of mass, effectively using a system of natural units with c set to 1. The kilogram equivalent of 1 eV/c^2 is as follows [using conversions from App. A and the definition of eV—one electron across poential of 1 V].

Equation 12: Mass as Energy

$$
\begin{aligned}
\frac{E}{c^2} = \frac{E[\mathrm{eV}]}{c^2} &= m[kg] \\
&= \frac{1.602 \times 10^{-19}\ \mathrm{C}\ \times\ 1\ \mathrm{Volt}}{299{,}792{,}458\ \dfrac{\mathrm{m}^2}{\mathrm{s}^2}} \\
&= \frac{1.602 \times 10^{-19}\ \left[\dfrac{\mathrm{J}}{\mathrm{V}}\right] \times \left[1\ \mathrm{V}\right]}{299{,}792{,}458\ \dfrac{\mathrm{m}^2}{\mathrm{s}^2}} = \\
&\frac{1.602 \times 10^{-19}\ \mathrm{J}\left[\dfrac{\left(\mathrm{kg}\cdot\mathrm{m}^2\right)/\mathrm{s}^2}{\mathrm{J}}\right]}{299{,}792{,}458\ \dfrac{\mathrm{m}^2}{\mathrm{s}^2}} \\
&= 1.782 \times 10^{-36}\ \mathrm{kg} \\
&\therefore \\
1\ \mathrm{eV} &\equiv 1.782 \times 10^{-36}\ \mathrm{kg}
\end{aligned}
$$

Most texts will list the mass in energy terms simply as *E*. Some show it explicitly as E/c^2 but not mention they're using c = 1. Others show, as in the last section, $E = [\#\#\#]c^2$ making it formally correct but still describe the units in terms of energy, eV or MeV. When c = 1, the electron volt is 1 eV = 1.60219 x 10^{-19} J.

---

[25] Some scientist once remarked that mass is frozen energy. It does seem apropos.

In any nuclear reaction involving changing the mass into energy, the following items are conserved.[26]

- Conservation of Nucleons
    - protons and neutrons
- Conservation of Charge
    - sum of charges remains the same / included in this is *the number of electrons is conserved* (that is the total Lepton number is conserved[27])
- Conservation of Momentum
    - hence the need for the neutrinos
    - includes linear and angular momentum, and spin
- Conservation of Mass/Energy
    - the form may change but total mass/energy will be conserved

Example 4

Based on the conservation of nucleons, what is the missing element of the following reaction?

$$^{14}\mathrm{N} + \mathrm{n} \rightarrow ? + {}^{1}\mathrm{H}$$

Solution

Of note, a hydrogen atom could be written as ${}^{1}_{1}\mathrm{H}$ whereas a proton by itself would be ${}^{1}_{1}\mathrm{p}$ or ${}^{1}\mathrm{p}$. Rewriting the reaction using the same format for the neutron gives

$$^{14}_{7}\mathrm{N} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{14+1-1}_{7+0-1}\mathrm{X} + {}^{1}_{1}\mathrm{H}$$

Therefore, the element X must be of the form ${}^{14}_{6}\mathrm{X}$ which is ${}^{14}_{6}\mathrm{C}$.

Note: Carbon 14 is primarily known for its use in radiocarbon dating, a technique used to estimate the age of organic materials up to around 50,000 years old. This method relies on the fact that carbon-14 is constantly being produced in the atmosphere and incorporated into living organisms. When an organism dies, it stops taking in new carbon-14, and the existing carbon-14 begins to decay at a known *rate*, with a half-life of approximately 5,730 years. By measuring the

---

[26] This "conservation" theory goes deep having to do with symmetries in space-time and rotations/translations in the same, which in general a nuclear engineering need not be concerned with. For those interested, check out Noether's Theorem.

[27] Neutrinos change form so the Lepton family numbers are not conserved but the total Lepton number will be.

remaining carbon-14 in a sample, and comparing to stable carbon isotopes, scientists can estimate how long ago the organism died.

In summary, Einstein's equation is the theoretical basis for the mass defect in nuclear reactions and the resulting binding energy so calculated in the products.

Equation 13: Einstein's Equation

$$E_{\text{rest}} = m_0 \mathrm{c}^2$$

As the speed of a particle increases, its mass increases relative to an observer by the following.

Equation 14: Relativistic Mass

$$m = \frac{m_0}{\sqrt{1 - {}^{v^2}\!/_{c^2}}}$$

The kinetic energy is thus as follows.

Equation 15: Kinetic Energy

$$\begin{aligned} E_{KE} &= E_{\text{total}} - E_0 = m\mathrm{c}^2 - m_0\mathrm{c}^2 \\ &= m_0\mathrm{c}^2 \left[\frac{1}{\sqrt{1 - {}^{v^2}\!/_{c^2}}} - 1 \right] \end{aligned}$$

The radical in this form [Eq. 15] of the kinetic energy can be expanded in powers of $(\nu/\mathrm{c})^2$ a binomial series and then when $v \quad \mathrm{c}$, the classical kinetic energy formula results.

Equation 16: Classical Kinetic Energy

$$E_{KE} = {}^{1}\!/_{2}\, m_0 v^2$$

This means the classical form of KE [Eq. 13] can only be used when ${}^{1}\!/_{2}\, m_0 v^2 \quad m_0 v^2$. That is, when the KE is much less than the rest mass energy.

Per Ref. [K] a good thumb rule for accuracy involving speed particles is when the total energy is less than 20% of the rest energy, $E \le 0.02 E_{\text{rest}}$.

Example 5

At what amount of electron energy can one ignore the relativistic effects?
Solution

Using the thumb rule: $E \leq 0.02E_{\text{rest}}$, and taking the rest mass energy of an electron from App. B of 0.511 MeV, calculations for electron energies equal to or below the following may ignore relativistic effects.

$$\begin{aligned} E &\leq 0.02E_{\text{rest}} \\ &= (0.02)(0.511 \text{ MeV}) \\ &= 0.010 \text{ MeV} \quad (10 \text{ keV}) \end{aligned}$$

Since most electrons encountered in nuclear engineering have kinetic energies less than this, one may ignore relativistic effects.

Example 6

At what amount of neutron energy can one ignore the relativistic effects?

Solution

Using the thumb rule: $E \leq 0.02E_{\text{rest}}$, and taking the rest mass energy of an electron from App. B of 939.6 MeV [round to 1000 MeV], calculations for neutron energies equal to or below the following may ignore relativistic effects.

$$\begin{aligned} E &\leq 0.02E_{\text{rest}} \\ &\cong (0.02)(1000 \text{ MeV}) \\ &= 20 \text{ MeV} \end{aligned}$$

Since most neutrons encountered in nuclear engineering have kinetic energies less than this, one may ignore relativistic effects.

Using the results of Ex. 5, one may obtain this useful formula for neutron energy. Note that velocity v is in cm/s and energy E is in eV.

Equation 17: Neutron Velocity vs Energy

$$v = 1.383 \times 10^{6}\sqrt{E}$$

A table of equivalents mass/energy follows.

Table 4: Electron, Neutron, Proton Masses

Particle	Mass in kg	Mass in amu	Mass in Energy [MeV]
electron	9.109×10^{-31}	5.486×10^{-4}	0.511
neutron	1.675×10^{-27}	1.008	939.6
proton	1.672×10^{-27}	1.007	938.6

Example 7

Binding Energy for Fission

What is the amount of energy released in the following asymmetrical fission process?[28]

$$^{235}_{92}\mathrm{U} \rightarrow {}^{13}_{6}\mathrm{C} + {}^{222}_{86}\mathrm{Rn}$$

Solution

Note: The “amu” is technically not a “unit” but is shown for clarification.
Note: The c^2 term can be handled in numerous ways as discussed previously and shown below.

$$BE_{^{235}_{92}\mathrm{U}} = \mathrm{c}^2\left[92m_{\mathrm{H}} + 143m_{\mathrm{n}} - m_{^{235}_{92}\mathrm{U}}\right]$$

$$\frac{BE_{^{235}_{92}\mathrm{U}}}{\mathrm{c}^2} = \left(931.5\ \frac{\mathrm{MeV}}{\mathrm{amu}}\right)\left[(92)(1.007825\ \mathrm{amu}) + (143)(1.008665\ \mathrm{amu}) - 235.043943\ \mathrm{amu}\right]$$
$$= 1783.9\ \mathrm{MeV}$$

$$BE_{^{13}_{6}\mathrm{C}} = \mathrm{c}^2\left[6m_{\mathrm{H}} + 7m_{\mathrm{n}} - m_{^{13}_{6}\mathrm{C}}\right]$$
$$= (931)\left[(6)(1.007825) + (7)(1.008665) - 13.003354\right]$$
$$= 97.1\ \mathrm{MeV}$$

---

[28] Asymmetrical fission is when the products are not identical (which would be called symmetrical fission).

$$
\begin{aligned}
BE_{^{222}_{86}\text{Rn}} &= \text{c}^2\left[86m_\text{H}+136m_\text{n}-m_{^{222}_{86}\text{rn}}\right]\\
&=(931)\big[(86)(1.007825)+(136)(1.008665)-222.01761\big]\\
&=1708.2\ \text{MeV}
\end{aligned}
$$

Using these results,

$$\text{BE}_{^{13}_{6}\text{C}}+\text{BE}_{^{222}_{86}\text{Rn}}=97.1+1708.2=1805\ \text{MeV}$$

Finally, compare the BE of the products to the reactant.

$$
\begin{aligned}
\text{BE}_{^{13}_{6}\text{C}}+\text{BE}_{^{222}_{86}\text{Rn}}-\text{BE}_{^{235}_{92}\text{U}} &= 97.1\ \text{MeV}+1708.2\ \text{MeV}-1783.9\ \text{MeV}\\
E_\text{released} &= 21.4\ \text{MeV}
\end{aligned}
$$

The nuclear BE of the products is 21.4 MeV greater than the original nucleus. This is the energy than is release in the reaction, which can be called the "synthesis process".

RADIATION, EFFECTS & PROTECTION, SHIELDING

Radioactivity Principles

Radioactivity is governed by the *radioactive decay law* that describes how the number of unstable nuclei in a radioactive sample decreases exponentially over time. Or, put another way, the probability per unit time that a nucleus will decay is a constant, independent of time. The constant is called the *decay constant* with the symbol λ. The decay rate [decays per unit time], or α, depends upon the decay constant [probability of decay per unit time] and the initial concentration of radioactive material, $n(t)$ [number of atoms].

Equation 18: Activity via Concentration

$$\alpha(t)=\lambda n(t)$$

After some integration, the activity can be shown to be dependent upon the initial activity.

Equation 19: Activity via Initial Activity

$$\alpha(t)=\alpha_0 e^{-\lambda t}$$

The time it takes for activity to fall to ½ its original value is called the *half-life*, $T_{1/2}$.

Example 8

Derive the equation for the half-life.

Solution

First, the definition half-life is when activity is ½ its original value.

$$\alpha\left(T_{1/2}\right)=\alpha_0/2$$

Substitute into Eq. 19.

$$\begin{aligned}
\alpha\left(t\right)&=\alpha_0 e^{-\lambda t}\\
\frac{\alpha_0}{2}&=\alpha_0 e^{-\lambda T_{1/2}}\\
\ln\left(\frac{1}{2}\right)&=\ln\left(e^{-\lambda T_{1/2}}\right)\\
\ln\left(\frac{1}{2}\right)&=-\lambda T_{1/2}\\
T_{1/2}&=\frac{\ln\left(2^{-1}\right)}{-\lambda}=\frac{-\ln 2}{-\lambda}\\
T_{1/2}&=\frac{\ln 2}{\lambda}=\frac{0.693}{\lambda}
\end{aligned}$$

Equation 20: Half-Life

$$T_{1/2}=\frac{\ln 2}{\lambda}=\frac{0.693}{\lambda}$$

Since half-lives are more widely tabulated than decay constants, Eq. 19 can be rearranged substituting the result of Ex. 8 into it with the result being an equation using only half-lives. Figure 5 shows the relationship.

Equation 21: Activity via Half-Lives

$$\alpha\left(t\right)=\frac{\alpha_0}{2}e^{\frac{t}{T_{1/2}}}$$

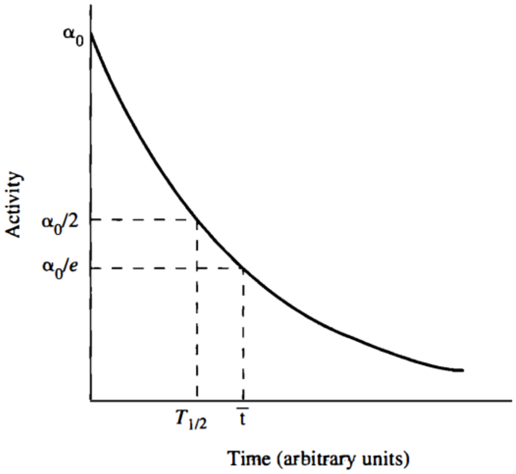


Figure 5: Half-Life vs Mean-Life
[Source: Lamarsh, Ref. [K]]

The official SI unit for radioactivity is the becquerel, Bq; though historically it was the curie, Ci. The following table shows the relationships.

Table 5: Activity Units

Unit of Activity	Curie	Becquerel
disintegrations/s	$3.7\,\square\,10^{10}$ disintegrations/s	1 disintegrations/s
s^{-1}	$3.7\,\square\,10^{10}$ Bq	$2.703\,\square\,10^{-11}$ Ci

Radiation Effects

Radiation and its impact on humans comes under the rubric of Health Physics (see Ref. [V]). Humans are surrounded by natural background radiation as well as exposure from man-made devices and activities: radionuclide devices, television sets, commercial jets that fly high in the atmosphere. The nuclear industry has contributed very little to radiation injury to its own personnel or the general public. In fact, most exposure to radiation comes from medical x-rays.

To measure the effects of radiation one must define the proper units. The *conventional system* exists and is widely used [see International Commission on Radiation Units and Measurement, ICRU Report 19] though this system is being replaced by the *SI system*[29] [ICRU Report 33].

The term *exposure* in the conventional system is used to describe the gamma ray (δ-ray) field incident upon as body at any given point.[30] They produce ions and electrons in air and ionization within the body—thus interfering with biological functions. Exposure is defined by the following.

Equation 22: Exposure

$$X = \frac{\Delta q}{\Delta m}$$

The exposure (X) is the amount of electrical charges (+ and −) produced in a mass of air (m) by photons stopped in the air. It was originally measured in roentgens (R) but is measured in the SI system by coulomb per kg (C/kg). One R = 2.58×10^{-4} C/kg = 1 esu.[31] This, as it states, represents the exposure, not the biological effects on the body. And, it doesn't account for other types of radiation: protons, neutrons, and electrons. For the biological effects one needs the amount of energy imparted to the body and the overall absorbed dose.

The biological effect of radiation is a function of how much energy is deposited in the body, E_D, and is called the *imparted energy*. The formula for the energy deposited follows.

Equation 23: Imparted Energy

$$E_D = E_{\text{in}} - E_{\text{out}} + Q$$

The energy in and out represents the sum of all the kinetic energies of all the particles—photons, neutrons, and charged particles (alpha, beta—electrons/positrons, protons) incident upon a given volume, ΔV, containing a given mass, Δm. The Q-value is present in the equation to account for any nuclear reactions.

The *absorbed dose* to a given mass, Δm, is denoted as D and defined as the imparted energy per unit mass.

---

[29] The SI system and technical information in general is explained well in the many publications of NIST (National Institute of Standards and Technology).

[30] The δ-ray field includes x-rays even though the gammas originate in the nucleus while x-rays originate in the orbital electrons.

[31] The esu is from an older cgs (centimeter-gram-second) system and represents the charge produced in 1 cm^3 of air at 1 atmosphere of pressure at 0°C.

Equation 24: Absorbed Dose

$$D = \frac{\Delta E_D}{\Delta m}$$

The conventional unit for the absorbed dose, *D*, is the rad, which is an acronym for *radiation absorbed dose.* The SI unit for *D* is the gray, Gy. Their relationships are shown in the next table.[32]

Table 6: Absorbed Dose Units

Unit	SI Unit Equivalent	Value in cgs System
1 rad	0.01 J/kg	100 ergs/g
1 Gy	1 J/kg	
Gy to rad	1 Gy = 100 rads	

Essential to shielding effectiveness [especially for fast neutrons] is a measurement closely related to absorbed dose, D, and that is the Kerma, *K*, which stands for Kinetic Energy Released per unit Mass. It is a measure of the energy transferred to charged particles by uncharged radiation [neutrons and photons].[33] It's essentially the initial kinetic energy of all the charged particles liberated by uncharged ionizing radiation in a specific mass of material. Kerma is a crucial concept in radiation physics, particularly in understanding the effects of radiation on matter.

While related, kerma and absorbed dose are not the same. Kerma is the initial energy transfer, while absorbed dose is the energy deposited in a specific volume of material. At lower radiation energies, kerma and absorbed dose are often similar, but at higher energies [again, think fast neutrons and intense gamma fields], the charged particles liberated by the radiation can travel further, leading to a difference between the location of *energy transfer* (kerma) and the location of *energy deposition* (absorbed dose).[34]

The discussion on radiation effects would be over except for the fact that biological effects are NOT directly related to the energy deposited. One needs to account for the *relative biological effectiveness* (RBE), and the radiations' associated *quality factor* (Q), leading to a *dose equivalent* (H) measured conventionally in *rem* (*roentgen equivalent man*) and in SI as a *sievert*.

---

[32] And, of course, if one is working in a radiation field, the dose rate needs to be known in order to limit exposures.

[33] Important because it is the charged particles created/released that damage the body.

[34] Energy may be deposited in a given volume, but because at high energies, the absorption takes place both inside this volume but can move well outside this initial volume—measured as *linear energy transfer* (LET). Thus, the initial absorptionz is given a quality factor, Q, in order to relate directly to biological effects.

Biological Effects

Biological effects of radiation depend not only on the energy deposited per gram or per cm^3 but also on how the energy is distributed along the path of the radiation. This deposition is called the *linear energy transfer* (LET). The impact that for the same absorbed dose, the biological damage is greater for high-LET radiation [α and neutrons] than for low-LET radiation [β and γ]. Thus, radiations of different types deposit energy in different ways, which is termed the *quality* of the radiation.[35] Each radiation, based on the type and energy is assigned a *Quality Factor* [Q].[36] The value of Q converts the absorbed dose, D, into the dose equivalent, H.

Equation 25: Dose Equivalent

$$H = D \cdot Q$$

The conventional unit for dose equivalent, H, is *rem* (Roentgen Equivalent Man)[37]. The SI unit is the *sievert*, Sv.

Equation 26: Sievert to rem

$$1 \text{ Sv} = 100 \text{ rem}$$

Of note, taking the derivative of the absorbed dose, $\dot{D}$, results in the dose equivalent rate, $\dot{H}$, in Sv/time.

Quality factor based on LET energy (Ref. [K]) is shown in the following table.

Table 7: LET Q-Values

LET [keV per micron]	Q
≤3.5	1
7	2
23	5
53	10
≥175	20

---

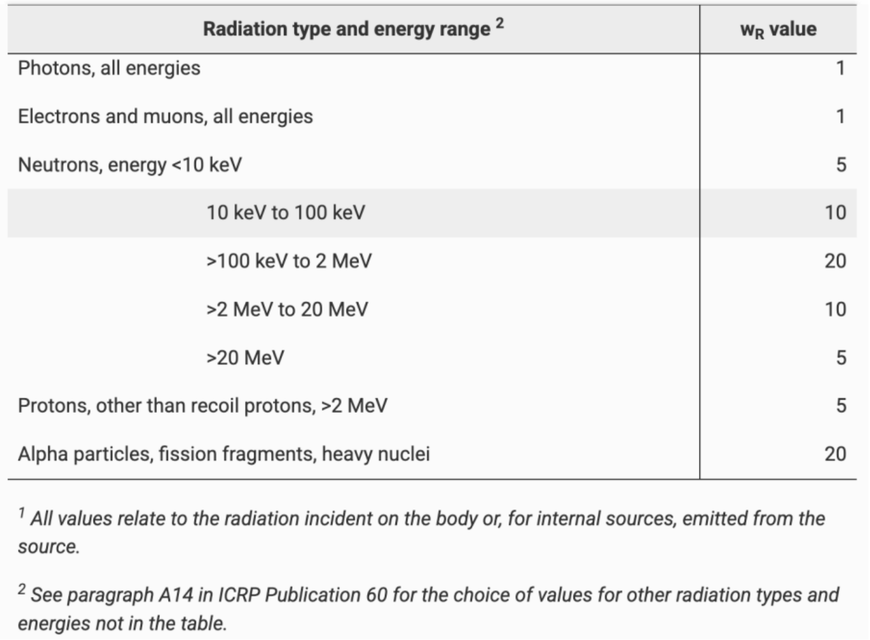
[35] Keep in mind, the higher the quality the worse the biological impact.

[36] The RBE is always determined by experiment. The Q is assigned, though based on the RBE, on theoretical information. Q is also called the *weighting factor*, w_R.

[37] Using this term (RBE) is now discouraged because roentgen is now a unit of exposure, not dose.

From 40CFR191 App. B, the quality factor *Q* energy levels (in this case, evaluated by the weighting factor [w_R]) for those energy levels is shown in the following table.

Table 8: Energy vs. Weighting Factor



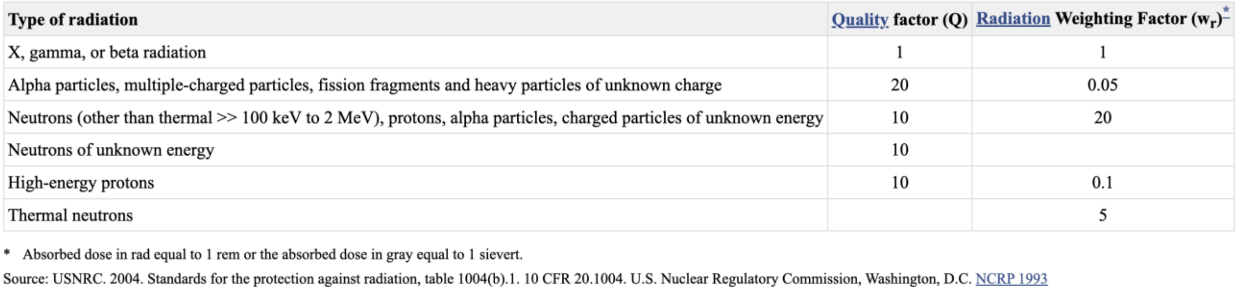
Radiation type and energy range [2]	w_R value
Photons, all energies	1
Electrons and muons, all energies	1
Neutrons, energy <10 keV	5
10 keV to 100 keV	10
>100 keV to 2 MeV	20
>2 MeV to 20 MeV	10
>20 MeV	5
Protons, other than recoil protons, >2 MeV	5
Alpha particles, fission fragments, heavy nuclei	20

[1] *All values relate to the radiation incident on the body or, for internal sources, emitted from the source.*

[2] *See paragraph A14 in ICRP Publication 60 for the choice of values for other radiation types and energies not in the table.*

Quality factor (Q) is a measure of the biological impact of a specific type of radiation at the point of energy transfer, directly linked to its linear energy transfer (LET). Weighting factor (w_R), on the other hand, is a *standardized value* assigned to different radiation types, *primarily used for calculating equivalent and effective doses in radiation protection*, *regardless of the specific LET* at the point of interest.

Consider the following table for different types of radiation from the Nuclear Regulatory Commission. Note that as an example of the difference between Q and w_R, alpha particles have a Q of 20 due to their high LET value but a weighting fact of 0.05 since they don't even have the ability to penetrate skin. [Also, such particles in this category with the alpha particles are contained in the cladding and thus are not a hazard unless released, hence the low weighting factor.]



Type of radiation	Quality factor (Q)	Radiation Weighting Factor (w_r)*
X, gamma, or beta radiation	1	1
Alpha particles, multiple-charged particles, fission fragments and heavy particles of unknown charge	20	0.05
Neutrons (other than thermal >> 100 keV to 2 MeV), protons, alpha particles, charged particles of unknown energy	10	20
Neutrons of unknown energy	10	
High-energy protons	10	0.1
Thermal neutrons		5

* Absorbed dose in rad equal to 1 rem or the absorbed dose in gray equal to 1 sievert.

Source: USNRC. 2004. Standards for the protection against radiation, table 1004(b).1. 10 CFR 20.1004. U.S. Nuclear Regulatory Commission, Washington, D.C. NCRP 1993

So, protection standards depend on the energy deposited (imparted), the location of the deposition, and the biological effects in the region of deposition.

Radiation Protection

The International Commission on Radiological Protection (ICRP) advances radiological protection information via established experts in the field. The National Council on Radiation Protection (NCRP) performs at the national level those functions of the ICRP. The NCRP states their mission as follows: The NCRP seeks to formulate and widely disseminate information, guidance and *recommendations* on radiation protection and measurements which represent the consensus of leading scientific thinking [taken from their website https://ncrponline.org].[38] Legally binding standards for exposure to radiation are issued by the Environmental Protection Agency (EPA) in *radiation protection guides* (RPGs). [The EPA takes this authorization from the Atomic Energy Act of 1954—as amended in 2024.]

The standard of the ICRP, NCRP, and EPA are not identical. However, these bodies agree on the following ground rules.

- Since any exposure may be harmful, no deliberate exposure is justified without some compensatory benefit will be realized.
- All exposure should be "as low as reasonably achievable" (ALARA).
- Radiation doses should not exceed certain recommended values. [The EPA uses those values in the RPGs.]

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[38] The NCRP is charted by congress under Title 36 of the United States Code. A **congressional charter** is a law passed by the United States Congress that *states the mission, authority, and activities of a group*. Congress has issued corporate charters since 1791 and the laws that issue them are codified in Title 36 of the United States Code. The NCRP is a scientific body and is not subject to government oversight.

No evidence exists as to the harm from low levels of exposure. Nevertheless, the standard setting bodies make the following assumptions.

- There is a linear does-effect relationship from high dose levels down to zero dose.
- No threshold dose limit exists above which an effect occurs, but below which it does not.
- Low organ doses are additive regardless of the rate or at what interval the doses are delivered.
- There is no biological recovery from radiation effects at low doses.

Having stated all that, there is now enough information based on comparisons of radiation workers and other trades that the risk of death (from cancer) for radiation workers is less than the risk of job-related deaths in the retail trades. And further, the use of calculations reviewed previously has now enable a quantitative basis for radiation standards.

Dose limiting recommendations for the whole body and various body parts are shown in the following table, based on the most recent standards (1977).

Table 9: Radiation Limits

Type of Exposure [Group]	EPA Limit	Source /Notes
Whole Body	5 rem/yr	40CFR190 Radiation Workers
Whole Body	100 mrem/yr	Public Dose Equivalent
Thyroid	75 mrem/yr	Most Radiation Sensitive Organ in Body
Public (any other Organ)	25 mrem/yr	
Pubic (all Sources)	100 mrem/yr	
Worker Emergency Exposure	25 rem/yr	Life Saving Operations

Shielding

When *electrons and gamma rays* interact with shielding material, the *energy transferred from the radiation to the material ultimately ends up as heat.*

- Ionization and Excitation: When electrons or gamma rays interact with atoms in the shielding material, they can knock electrons out of their orbits (ionization) or raise them to higher energy levels (excitation).
- Kinetic Energy Transfer: The ejected electrons and excited atoms will then transfer their kinetic energy to other atoms and molecules in the material through collisions and other interactions.
- Heat Generation: This transfer of energy increases the kinetic energy of the atoms and molecules in the material, which is perceived as an increase in temperature, or heat.

Essentially, *the energy that was initially carried by the electrons and gamma rays is converted into the thermal energy of the shielding material*. This is why thicker or denser materials are often used for shielding, as they provide more atoms for the radiation to interact with, thus absorbing more of the energy and dissipating it as heat. [Lighter materials are used to stop neutrons since a greater transfer of energy occurs when materials of similar weight collide.[39]]

The figure below represents the interactions gamma rays may undergo through shield. The shaded are is the nucleus and circles (approximate) are the electron shells.

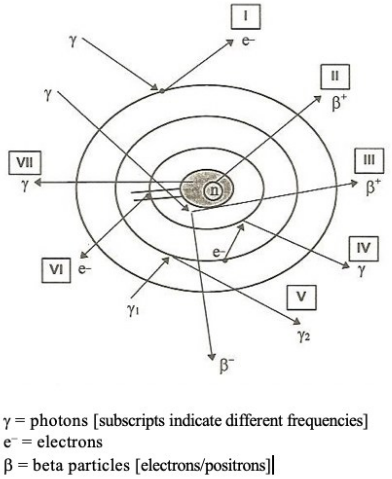

Figure 6: Gamma Interaction with Shielding

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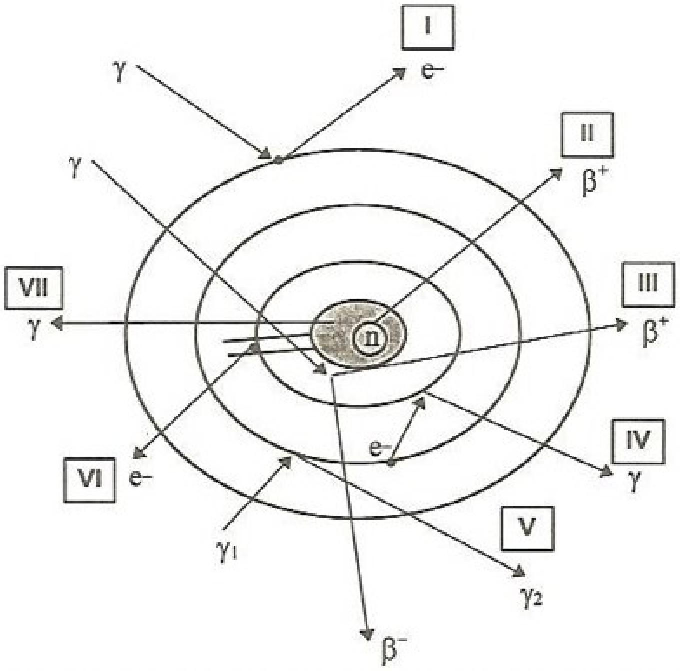
[39] Think of pool balls of similar weight, with one stationary before the collision.

Example 9

Consider the representation of a generic atom interaction with gamma radiation.



What roman numeral represents Compton Scattering?

Solution

Process V represents Compton scattering and is sometimes called elastic scattering as both energy and momentum are conserved. In this process, the incoming photon is scattered through interaction with an orbital electron, usually a loosely bound electron, transferring some of its energy to the electron and emerging with a lower energy and thus a longer wavelength. The electron, termed a *recoil electron*, is not shown in the given illustration.

By way of explanation, Events II and VI do produce radiation but not from interactions of major concern in shielding. Event VII does occur with fission products.

Process II represents the β^+ (positron) decay of a proton.[40] This tends to occur in nuclides where too few neutrons exist for stability.

Process VI represents *internal conversion*. An excited nucleus can interact with one of its innermost electrons and in doing so transfer excitation energy to this electron which then manifests itself as kinetic energy. The electron is ejected from the atom with energy equal to the excitation energy (i.e., the nuclear transition energy) of the nucleus minus the ionization energy of that particular electron.

Process VII represents *isomeric transition* (IT). Most nuclei in excited states decay essentially immediately. If, due to the arrangement of their internal structure, an excited state's decay is delayed, the state appears as semistable or metastable. Such states are called *isomeric states*. When the decay occurs a photon is emitted from the nucleus and the entire process is called an isomeric transition.[41]

Given all that has been stated, prompt fission neutrons and gammas from various sources are the greatest impacting sources. Therefore, what a shield needs to do is as follows.

- Slow fast neutrons down to thermal speeds, then absorb them.[42]
- Interaction with gammas lowering their energy out of the ionization region into the IR region.

To slow the neutrons, all lightweight shields contain water molecules. To lower the energy of the gammas, one must account for the various interactions mentioned in the example. Figure 7 shows the high energy gamma spectrum incident upon the shield as a spike. Passing through the shield additional gammas are produced but at a lower level of energy as shown on the right of Fig. 7. Thus, while the amount of radiation is built-up, it was attenuated to a lower (more safe—non-ionizing) level.

---

[40] In this context, these and the β^- are sometimes referred to as a beta rays.

[41] The photon emitted from the nucleus is termed a gamma ray.

[42] Thermal neutrons are those moving at 2200 m/s.

Thus, the shielding goal is to attenuate the radiation (or in the case of gammas, lower the level to that of "heat"). To accomplish this, one must account for the buildup of the radiation while accounting for the attenuation via the many interactions in the example.[43]

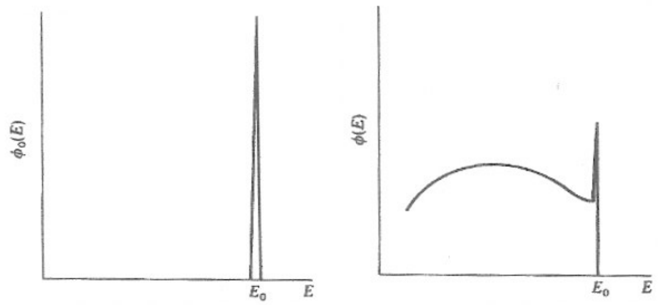


Figure 7: Gamma Spectrum Before & After Shield

The cross section for gamma interaction is the sum of the individual cross sections for photoelectric effect, pair production, and Compton scattering *for an atom*. The units will be cm^2, with a barn being 10^{-24} cm^2. Cross sections are determined experimentally and theoretically.

Equation 27: Total Gamma Cross Section

$$\sigma = \sigma_{pe} + \sigma_{pp} + \sigma_C$$

To obtain a *macroscopic cross section* for a given shield material one must multiply the atom cross section by the atom density, *N*, with units of atoms/cm^3. The result is macroscopic cross section, called an *attenuation coefficient*, μ.

Equation 28: Gamma Attenuation Coefficient

$$\mu = N\sigma = \mu_{pe} + \mu_{pp} + \mu_C$$

Now, this brings the ability to calculate the exposure rate accounting for both the buildup factor and the attenuation.[44]

---

[43] The *attenuation coefficients* are cross sections for interactions, much like neutron cross sections, but by tradition are named differently.

[44] One will see the "x" in this equation as an "a" because the buildup factor can be tabulated from a monodirectional (subscript m) source or a point source (μr) or other [infinite planar, finite planar--μ_p, or disc--μ_d] all of which use their own nomenclature. *When a subscript 0 is used it indicate the value of the exposure rate without a shield AND/OR the buildup factor value for a given energy ($\mu_0 x$) for E_0.*

Equation 29: Exposure Rate

$$\dot{X} = \dot{X}_0 B_m(\mu x) e^{-\mu x}$$

A sampling of buildup factors for various materials and energies is shown in the following table.

Table 10: Exposure Buildup Factors for Monodirectional Source

Material	***E_0*** **MeV**	**$\mu_0 x$** **[Shield Thickness in Mean Free Paths at Energy *E_0*]**	
		1	10
Water	0.5	2.63	35.9
Water	4.0	1.58	6.19
Lead	0.5	1.24	2.08
Lead	4.0	1.28	4.69
Uranium	0.5	1.17	1.73
Uranium	4.0	1.25	4.06

Example 10

The γ exposure rate inside a pressurized water reactor at the mid-plane [general area] of the primary coolant piping is 50 rem/hr. The lead shielding contemplated for this area is 1 or 10 mean free paths for 4.0 MeV gammas.

What is the exposure rate outside the shield at a similar planar location for 1 and 10 mean free paths?

Solution

Using the exposure rate formula and the buildup factor for lead for 4.0 MeV gamma and 1 mean free path gives the following result.

$$\begin{aligned}\dot{X} &= \dot{X}_0 B_m(\mu x) e^{-\mu x}\\ &= \left(50\ \frac{\text{rem}}{\text{hr}}\right)(1.28)e^{-1} = \left(64\ \frac{\text{rem}}{\text{hr}}\right)e^{-1} = \left(64\ \frac{\text{rem}}{\text{hr}}\right)\left(367.879\times10^{-3}\right)\\ &= 23.5\frac{\text{rem}}{\text{hr}}\end{aligned}$$

Using the exposure rate formula and the buildup factor for lead for 4.0 MeV gamma and 10 mean free paths gives the following result.

$$\begin{aligned}\dot{X} &= \dot{X}_0 B_m(\mu x)e^{-\mu x} \\ &= \left(50\ \frac{\text{rem}}{\text{hr}}\right)(4.69)e^{-10} = \left(234.5\ \frac{\text{rem}}{\text{hr}}\right)e^{-10} = \left(234.5\ \frac{\text{rem}}{\text{hr}}\right)\left(45.4\times 10^{-6}\right) \\ &= 10.6\times 10^{-3}\ \frac{\text{rem}}{\text{hr}} \quad \left(10.6\ \frac{\text{mrem}}{\text{hr}}\right)\end{aligned}$$

So, it spite of a buildup factor nearly four times as large [4.69/1.28], using 10 mean free paths of shielding reduces the exposure by over two-thousand times [23.5/10.6 x 10^{-3}].

There are many more aspects to shielding to learn or know about—especially if one is reponsible for the design of said shielding. Once built, operators monitor their exposures and perform maintenance checks on shield integrity to ensure proper operation.

SUMMARY

Considerably more information must be known by nuclear designers and nuclear plant operators. But, having been the latter, I hope this information helps a bit with the "why" of a design or operation. More indepth information is obviously in many of the references. Additional problem practice well beyond the topics of this course may be found in the author's Ref. [T]. The best overview is in the latest edition of Ref. [K]. The best book, I've found, for radiation detection and measurement is Ref. [P].

FUTURE PROSPECTS

The amount of power available from nuclear sources; the cleanliness of the sources from a climate perspective; the ability to recycle (without proliferation possibilities) and reuse waste thus limiting its impact to well below the environmental impact of oil, gas, and solar; the inherent safety of the next generation of reactors; and the massive need for power in the AI economy bodes well for nuclear power in the future. Below lists some of the possibilities.

Small Modular Reactors

Large AP1000 Reactors: known for its advanced passive safety systems and modular construction, making it a Generation III+ reactor.

Spacecraft Nuclear Electric and Thermal Propulsion

Space Nuclear Power on the Moon and Mars

Space Nuclear Radioactive Decay Power Sources—think Voyager 1 and 2.

Nuclear Waste Batteries

Generation IV Reactors than minimize waste, reduce proliferation concerns, and are passively safe, generate additional fuel during operation, are more efficient, and less expensive to construct and maintain. See Fig. 3.

Fusion Reactors

From the seas to space, this environmentally clean source of some 3000+ years' worth of energy is the key to the future.

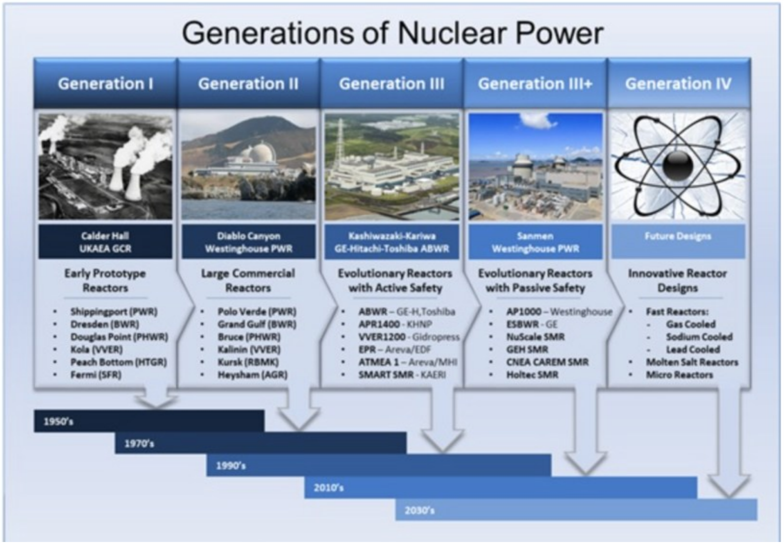


Figure 8: Nuclear Power Generations

[Source: *https://nuclearforclimate.com.au/plant-technologies*]

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NOTE

Electrical refers to something related to electricity while "electric" refers to a device or machine that runs on electricity. Nevertheless, the NEC is sometimes referred to as the National Electric Code.

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Note

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Appendix A: Equivalent Units Of Derived And Common SI Units

Symbol	Equivalent Units			
A	C/s	W/V	V/Ω	$\mathrm{J/(s\cdot V)}$
C	$\mathrm{A\cdot s}$	J/V	$\mathrm{(N\cdot m)/V}$	$\mathrm{V\cdot F}$
F	C/V	$\mathrm{C^2/J}$	s/Ω	$\mathrm{(A\cdot s)/V}$
F/m	$\mathrm{C/(V\cdot m)}$	$\mathrm{C^2/(J\cdot m)}$	$\mathrm{C^2/(N\cdot m^2)}$	$\mathrm{s/(\Omega\cdot m)}$
H	W/A	$\mathrm{(V\cdot s)/A}$	$\mathrm{\Omega\cdot s}$	$\mathrm{(T\cdot m^2)/A}$
Hz	1/s	$\mathrm{s^{-1}}$	cycles/s	$\mathrm{radians/(2\pi\cdot s)}$
J	$\mathrm{N\cdot m}$	$\mathrm{V\cdot C}$	$\mathrm{W\cdot s}$	$\mathrm{(kg\cdot m^2)/s^2}$
$\mathrm{m^2/s^2}$	J/kg	$\mathrm{(N\cdot m)/kg}$	$\mathrm{(V\cdot C)/kg}$	$\mathrm{(C\cdot m^2)/(A\cdot s^3)}$
N	J/m	$\mathrm{(V\cdot C)/m}$	$\mathrm{(W\cdot C)/(A\cdot m)}$	$\mathrm{(kg\cdot m)/s^2}$
$\mathrm{N/A^2}$	$\mathrm{Wb/(N\cdot m^2)}$	$\mathrm{(V\cdot s)/(N\cdot m^2)}$	T/N	$\mathrm{1/(A\cdot m)}$
Pa	$\mathrm{N/m^2}$	$\mathrm{J/m^3}$	$\mathrm{(W\cdot s)/m^3}$	$\mathrm{kg/(m\cdot s^2)}$
Ω	V/A	$\mathrm{W/A^2}$	$\mathrm{V^2/W}$	$\mathrm{(kg\cdot m^2)/(A^2\cdot s^3)}$
S	A/V	1/Ω	$\mathrm{A^2/W}$	$\mathrm{(A^2\cdot s^3)/(kg\cdot m^2)}$
T	$\mathrm{Wb/m^2}$	$\mathrm{N/(A\cdot m)}$	$\mathrm{(N\cdot s)/(C\cdot m)}$	$\mathrm{kg/(A\cdot s^2)}$
V	J/C	W/A	C/F	$\mathrm{(kg\cdot m^2)/(A\cdot s^3)}$
V/m	N/C	$\mathrm{W/(A\cdot m)}$	$\mathrm{J/(A\cdot m\cdot s)}$	$\mathrm{(kg\cdot m)/(A\cdot s^3)}$
W	J/s	$\mathrm{V\cdot A}$	V^2/Ω	$\mathrm{(kg\cdot m^2)/s^3}$
Wb	$\mathrm{V\cdot s}$	$\mathrm{H\cdot A}$	$\mathrm{T/m^2}$	$\mathrm{(kg\cdot m^2)/(A\cdot s^2)}$

Appendix B: Physical Constants

Table Note 1

Quantity	Symbol	US Customary	SI Units
Charge			
electron	e		-1.6022×10^{-19} C
proton	p		$+1.6022\times10^{-19}$ C
Density			
air [STP][32°F, (0°C)]		0.0805 lbm/ft^3	1.29 kg/m^3
air [70°F, (20°C), 1 atm]		0.0749 lbm/ft^3	1.20 kg/m^3
sea water		64 lbm/ft^3	1025 kg/m^3
water [mean]		62.4 lbm/ft^3	1000 kg/m^3
Distance			
Earth radius[2]	⊕	2.09×10^{7} ft	6.370×10^{6} m
Earth-Moon separation[2]	⊕☾	1.26×10^{9} ft	3.84×10^{8} m
Earth-Sun separtion[2]	⊕⊙	4.89×10^{11} ft	1.49×10^{11} m
Moon radius[2]	☾	5.71×10^{6} ft	1.74×10^{6} m
Sun radius[2]	⊙	2.28×10^{9} ft	6.96×10^{8} m
first Bohr radius	a_0	1.736×10^{-10} ft	5.292×10^{-11} m
Gravitational Acceleration			
Earth [mean]	g	32.174 (32.2) ft/sec^2	9.8067 (9.81) m/s^2
Mass			
atomic mass unit	μ or m_μ $\frac{1}{12}m\left({}^{12}C\right)$	3.66×10^{-27} lbm	1.6606×10^{-27} kg or 10^{-3} kg mol^{-1} / N$_A$ or 931.481 MeV
Earth[2]	⊕	4.11×10^{23} slugs	6.00×10^{24} kg
Earth [customary U.S.][2]	⊕	1.32×10^{25} lbm	-
Moon[2]	☾	1.623×10^{23} lbm	7.36×10^{22} kg
Sun[2]	⊙	4.387×10^{30} lbm	1.99×10^{30} kg
electron rest mass	m_e	2.008×10^{-30} lbm	9.109×10^{-31} kg [0.511 MeV]
neutron rest mass	m_n	3.693×10^{-27} lbm	1.675×10^{-27} kg [939.6 MeV]
proton rest mass	m_p	3.688×10^{-27} lbm	1.672×10^{-27} kg [938.2 MeV]

Pressure			
atmospheric		14.696 (14.7) lbf/in^2	1.0133×10^5 Pa
Temperature			
standard		32˚F (492˚R)	0˚C (273 K)
absolute zero		–459.67˚F (0˚R)	–273.16˚C (0 K)
Velocity[3]			
Earth escape		3.67×10^4 ft/sec	1.12×10^4 m/s
light (vacuum)	c, c_0	9.84×10^8 ft/sec	2.9979 (3.00) $\times 10^8$ m/s
sound [air, STP]	a	1090 ft/sec	331 m/s
sound [air, 70°F, (20°C), 1 atm]		1130 ft/sec	344 ft/s
Volume			
Volume: molal ideal gas (STP)[4]		359 ft^3 / lbmol	22.41 m^3/kmol

Table 1 Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.
2. Symbols shown for the solar system are those used by NASA. See https://science.nasa.gov/resource/solar-system-symbols/.
3. Velocity technically is a vector. It has direction.
4. The unit “lbmol” is an actual unit, not a misspelling.

Appendix C: Fundamental Constants

Quantity	Symbols	US Customary	SI Units
Avogadro's number	N_A, L		6.022×10^{23} mol^{-1}
Bohr magneton	μ_B		9.2732×10^{-24} J/T
Boltzmann constant	κ	5.65×10^{-24} ft-lbf/ R	1.3805×10^{-23} J/T
electron volt: $\left(\frac{e}{C}\right)$ J	eV		1.602×10^{-19} J
Faraday constant, $N_A e$	F		96485 C/mol
fine structure constant, inverse α^{-1}	α α^{-1}		7.297×10^{-3} (≈1/137) 137.035
gravitational constant	g_c	32.174 lbm-ft/lbf-sec^2	
Newtonian gravitational constant	G	3.44×10^{-8} ft^4 / lbf-sec^4	6.672×10^{-11} N$\cdot$m^2 / kg^2
nuclear magneton	μ_N		5.050×10^{-27} J/T
permeability of a vacuum	μ_0		1.2566×10^{-6} N/A^2 (H/m)
permittivity of a vacuum, electric constant $1/\mu_0 c^2$	ε_0		8.854×10^{-12} C^2 / N$\cdot$m^2 (F/m)
Planck's constant	h		6.6256×10^{-34} J$\cdot$s
Planck's constant: h/2π			1.0546×10^{-34} J$\cdot$s
Rydberg constant	R_∞		1.097×10^{7} m^{-1}
specific gas constant, air	R	53.3 ft-lbf/lbm- R	287 J/kg$\cdot$K
Stefan-Boltzmann constant		1.71×10^{-9} BTU/ft^2-hr-°R^4	5.670×10^{-8} W/m^2$\cdot$K^4
triple point, water		32.02 F, 0.0888 psia	0.01109 C, 0.6123 kPa
universal gas constant	R*	1545 ft-lbf/lbmol- R 1.986 BTU/lbmol- R	8314 J/kmol$\cdot$K

Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/. The unit in Volume of "lbmol" is an actual unit, not a misspelling.

Appendix D: Mathematical Constants

Quantity	Symbol	Value
Archimedes’ constant (pi)	π	3.1415926536
base of natural logs	*e*	2.7182818285
Euler’s constant	*C* or τ	0.5772156649

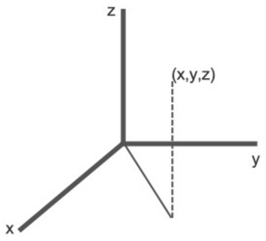
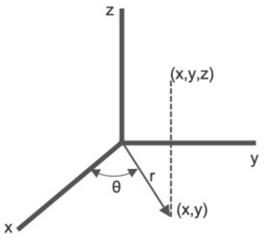
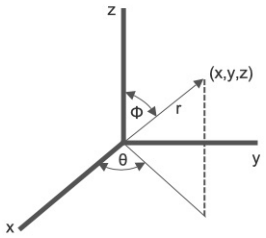
Appendix E: The Greek Alphabet

Α	α	alpha	Ν	υ	nu
Β	β	beta	Ξ	ξ	xi
Γ	γ	gamma	Ο	ο	omicron
Δ	δ	delta	Π	π	pi
Ε	ε	epsilon	Ρ	ρ	rho
Ζ	ζ	zeta	Σ	σ	sigma
Η	η	eta	Τ	τ	tau
Θ	θ	theta	ϒ	υ	upsilon
Ι	ι	iota	Φ	ϕ	phi
Κ	κ	kappa	Χ	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
Μ	μ	mu	Ω	ω	omega

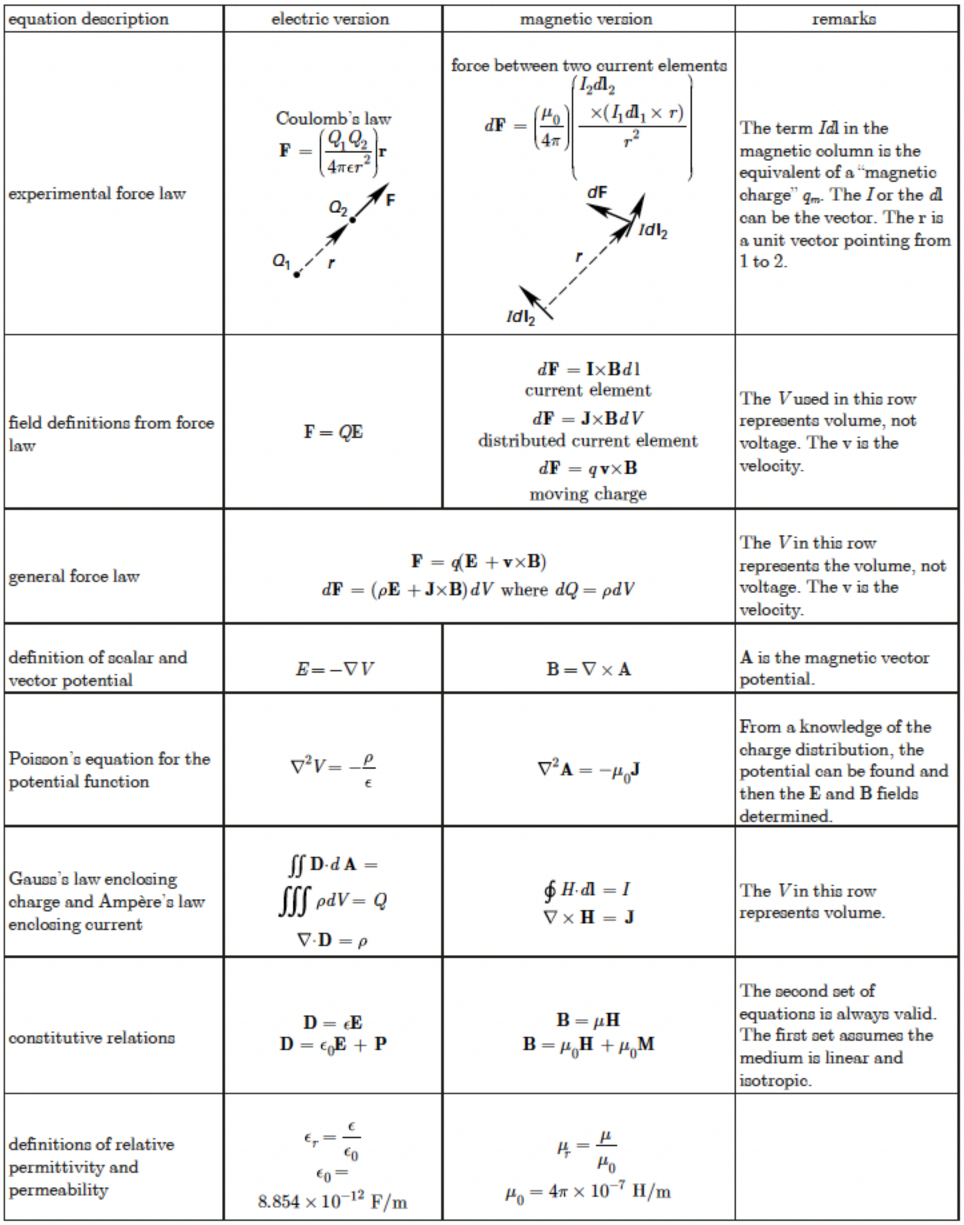
Appendix F: SI Prefixes

symbol	prefix	value
a	atto	10^{-18}
f	femto	10^{-15}
p	pico	10^{-12}
n	nano	10^{-9}
μ	micro	10^{-6}
m	milli	10^{-3}
c	centi	10^{-2}
d	deci	10^{-1}
da	deka	10
h	hecto	10^{2}
k	kilo	10^{3}
M	mega	10^{6}
G	giga	10^{9}
T	tera	10^{12}
P	peta	10^{15}
E	exa	10^{18}

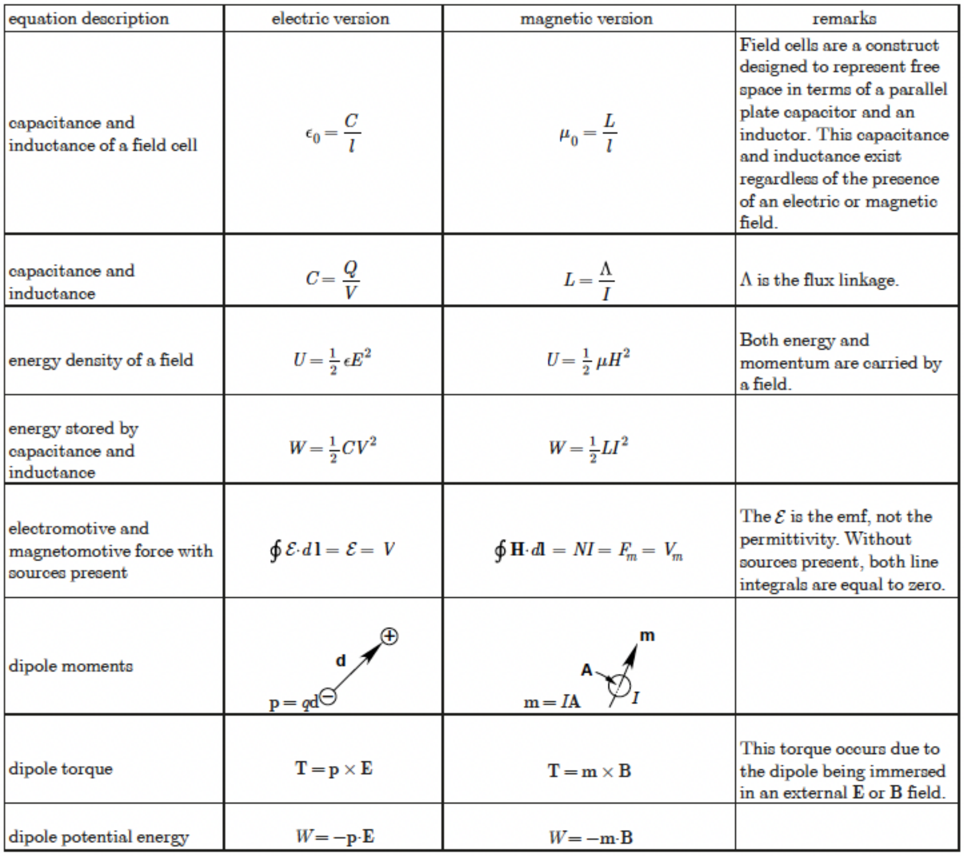
Appendix G: Coordinate Systems & Related Operations

Mathematical Operations	Rectangular Coordinates	Cylindrical Coordinates	Spherical Coordinates
Conversion to Rectangular Coordinants	 $x = x$ $y = y$ $z = z$	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	 $x = r\sin\phi\cos\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\phi$
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\boldsymbol{\theta} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\boldsymbol{\phi} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \theta}\boldsymbol{\theta}$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial (A_\phi \sin\phi)}{\partial \phi} + \frac{1}{r\sin\phi}\frac{\partial A_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{r} & \boldsymbol{\theta} & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & A_\theta & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r^2\sin\theta}\mathbf{r} & \frac{1}{r^2\sin\theta}\boldsymbol{\phi} & \frac{1}{r}\boldsymbol{\theta} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ A_r & rA_\phi & rA\theta A_\phi \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial r}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2 f}{\partial \theta^2}\right)$

Appendix H: Comparison of Electric & Magnetic Equations



equation description	electric version	magnetic version	remarks
experimental force law	Coulomb's law $\mathbf{F} = \left(\frac{Q_1 Q_2}{4\pi\epsilon r^2}\right)\mathbf{r}$	force between two current elements $d\mathbf{F} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I_2 d\mathbf{l}_2 \times (I_1 d\mathbf{l}_1 \times r)}{r^2}\right)$	The term Idl in the magnetic column is the equivalent of a "magnetic charge" q_m. The I or the dl can be the vector. The r is a unit vector pointing from 1 to 2.
field definitions from force law	$\mathbf{F} = Q\mathbf{E}$	$d\mathbf{F} = \mathbf{I}\times\mathbf{B}\,dl$ current element $d\mathbf{F} = \mathbf{J}\times\mathbf{B}\,dV$ distributed current element $d\mathbf{F} = q\,\mathbf{v}\times\mathbf{B}$ moving charge	The V used in this row represents volume, not voltage. The v is the velocity.
general force law	$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$ $d\mathbf{F} = (\rho\mathbf{E} + \mathbf{J}\times\mathbf{B})\,dV$ where $dQ = \rho\,dV$		The V in this row represents the volume, not voltage. The v is the velocity.
definition of scalar and vector potential	$E = -\nabla V$	$\mathbf{B} = \nabla \times \mathbf{A}$	**A** is the magnetic vector potential.
Poisson's equation for the potential function	$\nabla^2 V = -\frac{\rho}{\epsilon}$	$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$	From a knowledge of the charge distribution, the potential can be found and then the **E** and **B** fields determined.
Gauss's law enclosing charge and Ampère's law enclosing current	$\iint \mathbf{D}\cdot d\,\mathbf{A} =$ $\iiint \rho\,dV = Q$ $\nabla\cdot\mathbf{D} = \rho$	$\oint H\cdot d\mathbf{l} = I$ $\nabla \times \mathbf{H} = \mathbf{J}$	The V in this row represents volume.
constitutive relations	$\mathbf{D} = \epsilon\mathbf{E}$ $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$	$\mathbf{B} = \mu\mathbf{H}$ $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$	The second set of equations is always valid. The first set assumes the medium is linear and isotropic.
definitions of relative permittivity and permeability	$\epsilon_r = \frac{\epsilon}{\epsilon_0}$ $\epsilon_0 =$ 8.854×10^{-12} F/m	$\mu_r = \frac{\mu}{\mu_0}$ $\mu_0 = 4\pi \times 10^{-7}$ H/m	



equation description	electric version	magnetic version	remarks
capacitance and inductance of a field cell	$\epsilon_0 = \frac{C}{l}$	$\mu_0 = \frac{L}{l}$	Field cells are a construct designed to represent free space in terms of a parallel plate capacitor and an inductor. This capacitance and inductance exist regardless of the presence of an electric or magnetic field.
capacitance and inductance	$C = \frac{Q}{V}$	$L = \frac{\Lambda}{I}$	Λ is the flux linkage.
energy density of a field	$U = \frac{1}{2}\epsilon E^2$	$U = \frac{1}{2}\mu H^2$	Both energy and momentum are carried by a field.
energy stored by capacitance and inductance	$W = \frac{1}{2}CV^2$	$W = \frac{1}{2}LI^2$	
electromotive and magnetomotive force with sources present	$\oint \mathcal{E} \cdot d\mathbf{l} = \mathcal{E} = V$	$\oint \mathbf{H} \cdot d\mathbf{l} = NI = F_m = V_m$	The $\mathcal{E}$ is the emf, not the permittivity. Without sources present, both line integrals are equal to zero.
dipole moments	$\mathbf{p} = q\mathbf{d}$	$\mathbf{m} = I\mathbf{A}$	
dipole torque	$\mathbf{T} = \mathbf{p} \times \mathbf{E}$	$\mathbf{T} = \mathbf{m} \times \mathbf{B}$	This torque occurs due to the dipole being immersed in an external **E** or **B** field.
dipole potential energy	$W = -\mathbf{p} \cdot \mathbf{E}$	$W = -\mathbf{m} \cdot \mathbf{B}$	