A SunCam Online Continuing Education Course

Electronics

Course I Fundamentals

Course II Devices

Fundamentals / Semiconductor Materials Science / PN Junction Theory / Diodes / SCRs

by

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Future Courses
Electronics Course II Devices

Nomenclature[1]

A	constant	$K^{-8} \cdot m^{-8}$
A	gain	-
BW	bandwidth	Hz
C	capacitance	F
c, c	speed of light, 2.9979×10^{8}	m/s
$CMRR$	common-mode rejection ratio	_-
D	diffusion constant	m^2 / s
E	energy	J or eV
E	source voltage	V
E, **E**	electric field strength	V/m
f	frequency	Hz
F_m	figure of merit	rad/s
G	gain	-
GBW	gain bandwidth	rad/s
h	Planck's constant, 6.6256×10^{-34}	$J \cdot s$
H	high	V
h_{fe}	CE small-signal (AC) forward current transfer ratio or gain	-
h_{FE}	CE small-signal (DC) forward current transfer ratio or gain	-
i	instantaneous or variable current	A
I, **I**	constant or rms current or effective or DC current	A
I_{CBO}	reverse saturation current	A
I_{EBO}	DC emitter cutoff current	A
I_{ZK}	keep-alive current	A
n	electron concentration [negative charge concentration]	cm^{-3}
n, N	concentration	m^{-3}
N	fan-out (number of loads)	-
p	hole concentration [positive charge concentration]	cm^{-3}
p	pole strength	Wb (N/T)
P	polarization	C/m^2
P	power	W
p^+	charge on a proton, 1.6022×10^{-19}	C
q	electric unit charge; charge of an electron, -1.6022×10^{-19}	C

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[1] Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related and may be found when reviewing the references listed for further information. Further, all the nomenclature, symbols, or subscripts will be found in of many electrical courses (on SunCam, PDH Academy, and also in many texts). For guidance on nomenclature, symbols, and electrical graphics: IEEE 280-2021. IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering. New York: IEEE; and IEEE 315-1975. Graphic Symbols for Electrical and Electronics Diagrams. New York: IEEE, approved 1975, reaffirmed 1993.

q, Q	charge, electric unit charge 1.6022×10^{-19}	C
r, R	resistance	Ω
S_R	slew rate	V/s
T	temperature	K
t	time	s
T	temperature	K
U	energy	J
υ or v	instantaneous or variable voltage	V
V	constant or rms voltage or effective or DC voltage	V
VR	voltage regulation	%
V_T	voltage equivalent of temperature	V
Z	impedance	Ω

Symbols

β	CE forward current ratio or gain	-
ε	permittivity	F/m $\left(\mathrm{C}^2/\mathrm{N} \cdot \mathrm{m}^2\right)$
ε_0	permittivity of free-space, 8.854×10^{-12}	F/m $\left(\mathrm{C}^2/\mathrm{N} \cdot \mathrm{m}^2\right)$
κ	Boltzmann’s constant, 1.3807×10^{-23}	J/K
μ	mobility	$\mathrm{m}^2/\mathrm{V} \cdot \mathrm{s}$
μ	permeability	H/m $\left(\mathrm{N}/\mathrm{A}^2\right)$
μ_0	free-space permeability, 1.2566×10^{-6}	H/m $\left(\mathrm{N}/\mathrm{A}^2\right)$
σ	conductivity	S/m
α	CB forward current ratio or gain	-
δ	feedback factor	-
ω	angular frequency	rad/s

Subscripts

0	original, origin, zero value, at-rest value, evaluated value, at 0K, barrier value, resonance	-
A	acceptor	-
A,B	input A, input B	-
ac	ac current or small signal	=
amp	amplifier	-
ave	average	-
B	DC or biased base	-
b, B	base	-
BB	base supply or base biasing	-
be, BE	base to emitter	-
BO	breaker over	-
BQ	base or quiescent point	-
c	conduction band or AC or small-signal collector	-
C	collector	-
C	DC or biased collector	-
c, C	collector or control	-
CB	collector to base	-
CC	collector supply	-
CE	collector to emitter	-
cl	current limiting	-
cm	common mode	-
co, CO	cutoff	-
d	diffusion	-
D	donor, diode, or drain	-
d	delay	-
D	diode	-
DC	direct current	-
DD	drain supply	-
dm	differential mode	-
e	AC or small-signal emitter	-
E	DC or biased emitter	-
e,E	emitter	-
EBO	emitter to base, collector open	-
EE	emitter supply	-
f	forward (AC or instantaneous)	-
F	forward (DC component) or total	-
f	fall, feedback, or forward (AC or instantaneous)	-
F	forward (DC component or total)	-
fl	full load	-

G	gap or gate	-
GS	gate to source	-
H	hold	-
i	intrinsic or input (AC or instantaneous)	-
I, I	current	-
L	load	-
m	maximum	-
m	merit	-
max	maximum	-
min	minimum	-
n	electrons or n-type	-
n	noise	-
nl	no load	-
p	holes or p-type	-
p	power	-
pn	p to n	-
r	reverse (AC or instantaneous)	-
R	reverse (DC component or total)	-
r	rise	-
R	resistance	-
ref	reference	-
RTD	resistance temperature detector	-
s	saturation	-
S	source	-
t	transition	-
T	thermal	-
T	thermal	-
u	unity	-
υ, ν, Υ	instantaneous voltage	-
v	valence band	-
V	constant or effective voltage	-
V	voltage	-
Z	zener	-
Z	zener	-
ZK	zener knee	-
ZM	zener at maximum rated current	-

TABLE OF CONTENTS

List of Figures

List of Equations

List of Examples

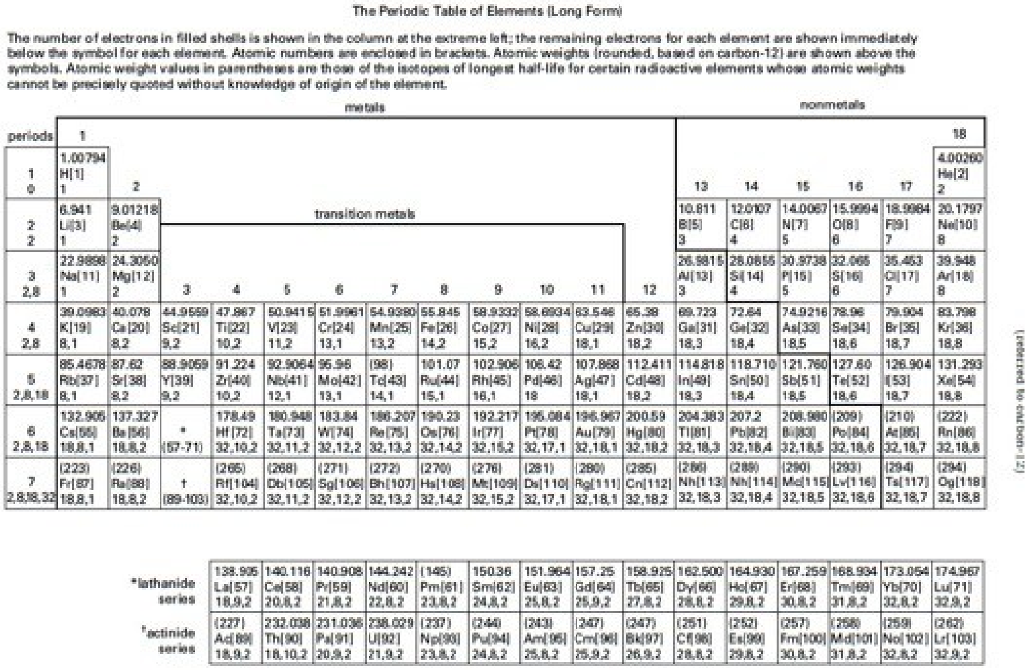
COURSE INTRODUCTION

The theoretical information is primarily from the author’s books, Refs. [A] and [B]. The NESC Ref. [C] and NEC Ref. [D] though not covered in this course are useful sources for electrical engineers. Information useful in many aspects of electric engineering may be found in [E] and [F]. Reference [G] has detailed descriptions of analysis techniques. Reference [H] covers many terms in EE with excellent definitions and explanations. The appendices cover information useful in many engineering tasks with App. G providing a side by side comparison of electric and magnetic equations. Use these texts or their counterparts for indepth information. References in bold are highly recommended.

This course will focus on basics, that rarely change, and provide the basis for all other knowledge.

OVERVIEW & PERIODIC TABLE

Electronics involves charge motion through materials other than metals, such as vacuums, gases, or semiconductors. The focus in this course will be on semiconductor materials. Because an understanding of the electron structure is vital to understanding electronics, a periodic table of the elements is shown below.



The Periodic Table of Elements (Long Form)

The number of electrons in filled shells is shown in the column at the extreme left; the remaining electrons for each element are shown immediately below the symbol for each element. Atomic numbers are enclosed in brackets. Atomic weights (rounded, based on carbon-12) are shown above the symbols. Atomic weight values in parentheses are those of the isotopes of longest half-life for certain radioactive elements whose atomic weights cannot be precisely quoted without knowledge of origin of the element.

metals

nonmetals

transition metals

periods	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 0	1.00794 H[1] 1																	4.00260 He[2] 2
2 2	6.941 Li[3] 1	9.01218 Be[4] 2											10.811 B[5] 3	12.0107 C[6] 4	14.0067 N[7] 5	15.9994 O[8] 6	18.9984 F[9] 7	20.1797 Ne[10] 8
3 2,8	22.9898 Na[11] 1	24.3050 Mg[12] 2											26.9815 Al[13] 3	28.0855 Si[14] 4	30.9738 P[15] 5	32.065 S[16] 6	35.453 Cl[17] 7	39.948 Ar[18] 8
4 2,8	39.0983 K[19] 8,1	40.078 Ca[20] 8,2	44.9559 Sc[21] 9,2	47.867 Ti[22] 10,2	50.9415 V[23] 11,2	51.9961 Cr[24] 13,1	54.9380 Mn[25] 13,2	55.845 Fe[26] 14,2	58.9332 Co[27] 15,2	58.6934 Ni[28] 16,2	63.546 Cu[29] 18,1	65.38 Zn[30] 18,2	69.723 Ga[31] 18,3	72.64 Ge[32] 18,4	74.9216 As[33] 18,5	78.96 Se[34] 18,6	79.904 Br[35] 18,7	83.798 Kr[36] 18,8
5 2,8,18	85.4678 Rb[37] 8,1	87.62 Sr[38] 8,2	88.9059 Y[39] 9,2	91.224 Zr[40] 10,2	92.9064 Nb[41] 12,1	95.96 Mo[42] 13,1	(98) Tc[43] 14,1	101.07 Ru[44] 15,1	102.906 Rh[45] 16,1	106.42 Pd[46] 18	107.868 Ag[47] 18,1	112.411 Cd[48] 18,2	114.818 In[49] 18,3	118.710 Sn[50] 18,4	121.760 Sb[51] 18,5	127.60 Te[52] 18,6	126.904 I[53] 18,7	131.293 Xe[54] 18,8
6 2,8,18	132.905 Cs[55] 18,8,1	137.327 Ba[56] 18,8,2	* (57-71)	178.49 Hf[72] 32,10,2	180.948 Ta[73] 32,11,2	183.84 W[74] 32,12,2	186.207 Re[75] 32,13,2	190.23 Os[76] 32,14,2	192.217 Ir[77] 32,15,2	195.084 Pt[78] 32,17,1	196.967 Au[79] 32,18,1	200.59 Hg[80] 32,18,2	204.383 Tl[81] 32,18,3	207.2 Pb[82] 32,18,4	208.980 Bi[83] 32,18,5	(209) Po[84] 32,18,6	(210) At[85] 32,18,7	(222) Rn[86] 32,18,8
7 2,8,18,32	(223) Fr[87] 18,8,1	(226) Ra[88] 18,8,2	† (89-103)	(265) Rf[104] 32,10,2	(268) Db[105] 32,11,2	(271) Sg[106] 32,12,2	(272) Bh[107] 32,13,2	(270) Hs[108] 32,14,2	(276) Mt[109] 32,15,2	(281) Ds[110] 32,17,1	(280) Rg[111] 32,18,1	(285) Cn[112] 32,18,2	(286) Nh[113] 32,18,3	(289) Nh[114] 32,18,4	(290) Mc[115] 32,18,5	(293) Lv[116] 32,18,6	(294) Ts[117] 32,18,7	(294) Og[118] 32,18,8

(referred to carbon-12)

*lanthanide series	138.905 La[57] 18,9,2	140.116 Ce[58] 20,8,2	140.908 Pr[59] 21,8,2	144.242 Nd[60] 22,8,2	(145) Pm[61] 23,8,2	150.36 Sm[62] 24,8,2	151.964 Eu[63] 25,8,2	157.25 Gd[64] 25,9,2	158.925 Tb[65] 27,8,2	162.500 Dy[66] 28,8,2	164.930 Ho[67] 29,8,2	167.259 Er[68] 30,8,2	168.934 Tm[69] 31,8,2	173.054 Yb[70] 32,8,2	174.967 Lu[71] 32,9,2
†actinide series	(227) Ac[89] 18,9,2	232.038 Th[90] 18,10,2	231.036 Pa[91] 20,9,2	238.029 U[92] 21,9,2	(237) Np[93] 23,8,2	(244) Pu[94] 24,8,2	(243) Am[95] 25,8,2	(247) Cm[96] 25,9,2	(247) Bk[97] 26,9,2	(251) Cf[98] 28,8,2	(252) Es[99] 29,8,2	(257) Fm[100] 30,8,2	(258) Md[101] 31,8,2	(259) No[102] 32,8,2	(262) Lr[103] 32,9,2

Figure 1: Periodic Table

An electronic component is one able to amplify, control, or switch voltages or currents without mechanical or other nonelectrical commands. The charge in metals is carried by the electron, with a charge of -1.6022×10^{-19} C. In semiconductor materials, the charge is carried both by the electron and by the absence of the electron in a covalent bond, which is referred to as a hole with a charge of $+1.6022 \times 10^{-19}$ C.[2] The concentration of electrons, n, and holes, p, is given by the *mass action law*, which follows.

Equation 1: Mass Action Law

$$n_i^2 = np$$

The term n_i is the concentration of carriers in a pure (intrinsic) semiconductor.[3] In intrinsic semiconductor materials, the number of electrons equals the number of holes, and the mass action law is stated as in Eq. 1. Nevertheless, the mass action law applies for intrinsic and extrinsic semiconductor materials. *Extrinsic* semiconductor materials are those that have had impurities deliberately added to modify their properties, normally their conductivity. For extrinsic semiconductors, the mass action law is probably better understood as indicating that the product np remains constant regardless of position in the semiconductor or doping level. Carriers, either n or p, in a semiconductor are constantly generated by thermal creation of electron-hole pairs and constantly eliminated by recombination of electron-hole pairs. *The carriers generated are caused to diffuse by concentration gradients in the semiconductor, a phenomenon that does not occur in metals.*

Semiconductor devices are inherently nonlinear. Nevertheless, they are commonly analyzed over ranges in which their behavior is approximately linear. Such an analysis is called *small-signal analysis*. The DC or effective value of the electrical parameters in the models used for analysis will be represented by uppercase letters. AC or instantaneous values will be represented by lowercase letters. Equivalent parameters in the models will use lowercase letters. For convenience, the lowercase letter t is omitted from functions of time. For example, $v(t)$ is written simply as v. The subscripts on *currents indicate the terminals into which current flows*. Subscripts on voltages indicate the terminals across which the voltage appears. Subscripts indicating biasing voltages are

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[2] The concept of holes explains the conduction of electricity without free electrons. The hole is considered to behave as a free positive charge—quantum mechanics justifies such an interpretation. (The Hall voltage is experimental confirmation.) The calculation of total charge motion in semiconductors is simplified as a result. The same equations used for electron movement can be used for hole movement with a change of sign and a change of values for some terms, such as mobility.

[3] Intrinsic means natural. Pure silicon is silicon in its natural state, with no doping, though it will have naturally occurring impurities. The terms pure and intrinsic are used interchangeably.

capitalized. For example, V_{BB} is the base biasing voltage while v_{be} is the instantaneous voltage signal applied to the base-emitter junction. A list of common designations is given in Ref. [H].

Electronic components are often connected in an array known as an *integrated circuit* (IC). An integrated circuit is a collection of active and passive components on a single semiconductor substrate (*chip*) that function as a complete electronic circuit. *Small-scale integration* (SSI) involves the use of less than 100 components per chip. *Medium-scale integration* (MSI) involves between 100 and 999 components per chip. *Large-scale integration* involves between 1000 and 9999 components per chip. *Very large-scale integration* involves more than 10,000 components per chip. *Ultra large-scale integration* (ULSI) is a term used to describe circuits with 1,000,000 or more components per chip.[4]

The nonlinearity of electronic devices makes them attractive for use as amplifiers. An *amplifier* is a device capable of increasing the amplitude or power of a physical quantity without distorting the wave shape of the quantity. The amplifying properties of transistors are covered in this course. Topics peculiar to amplifiers and amplifier types are covered in a follow-on course. The application of the electronic components to a variety of circuits is extremely broad. A few examples are provided in the end sections of this course. Additionally, some electronic circuits are used often enough to be designed into *standard modular devices*, that is, off-the-shelf ICs with all the necessary components to accomplish a given circuit task. An understanding of the basic electrical and electronic principles for the components used in such devices will enable an understanding of the overall circuit functions of the standard modular devices and electrical/electronic circuits in general.

Example 1

Silicon has an intrinsic carrier concentration of 1.6×10^{10} cm^{-3}. What is the concentration of holes?

Solution

Regardless of whether the material is intrinsic or extrinsic, the law of mass action applies. However, in intrinsic materials, the concentration of electrons and the concentration of holes are equal. Equation 1 can be used as follows.

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[4] The definition is shifting, this now refers to potentially "billions" of transistors of a chip.

$$n_i^2 = np = p^2$$

$$\therefore$$

$$n_i = p = 1.6 \times 10^{10} \text{ cm}^{-3}$$

SEMICONDUCTOR MATERIALS

The most common semiconductor materials are silicon (Si) and germanium (Ge). Both are in group 14 of the periodic table and contain four valence electrons. A *valence electron* is one in the outermost shell of an atom. Both materials use covalent bonding to fill the outer shell of eight when they create a crystal lattice. Semiconductor materials ($N \approx 10^{10}$ to 10^{13} electrons/m^3) are slightly more conductive than insulators ($N \approx 10^7$ electrons/ m^3) but less conductive than metals ($N \approx 10^{28}$ electrons/m^3). The conductivity, σ, of semiconductor materials can be made to vary from approximately 10^{-7} to 10^5 S/m.

The semiconductor crystal lattice has many defects (i.e., free electrons and their corresponding holes). The formation of free electrons and holes is driven by the temperature and is called *thermal carrier generation*. The density of these electron-hole pairs in intrinsic materials is given by

Equation 2: Thermal Carrier Generation

$$n_i^2 = A_0 T^3 e^{-E_{G0}/\kappa T}$$

The term A_0 is a constant independent of temperature and related to the density states at the bottom edge of the conduction band and the top edge of the valence band. T is the absolute temperature. E_{G0} is the energy gap at 0K, that is, the energy between the conduction band and the valence band. This is the energy required to break the covalent bond. The energy gap for silicon at 0K is approximately 1.21 eV. The energy gap for germanium is approximately 0.78 eV. The term κ is Boltzmann's constant, with values of 1.3807×10^{-23} J/K and 8.621×10^{-5} eV/K.[5] At a given temperature at thermal equilibrium, Eq. 2 becomes

Equation 3: Carrier Generation at Thermal Equilibrium

$$n_i^2 = N_c N_\upsilon e^{-E_{G0}/\kappa T}$$

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[5] The symbology for Boltzmann's constant often varies with the units of the energy gap. If eV is used as the energy unit, Boltzmann's constant may be seen as either κ or $\bar{\kappa}$ to distinguish between eV/K and J/K.

The terms N_c and N_υ are the *effective density states*[6] in the conduction band and valence band, respectively. The energy gap for silicon at room temperature (300K) is 1.12 eV. The energy gap for germanium at room temperature is 0.80 eV.

When minor amounts of impurities called *dopants* are added, the materials are termed *extrinsic semiconductors*.[7] If the impurities added are from group 13, with three valence electrons, an additional hole is created in the lattice. These dopants are called *acceptors*, and semiconductors with such impurities are called *p-types*. The majority carriers in *p*-type semiconductors are holes, and the *minority carriers* are electrons. The majority of the charge movement takes place in the valence band. Typical dopants are indium (In) and gallium (Ga). If the impurities added are from group 15, with five valence electrons, an additional electron is provided to the lattice.[8] These dopants are called *donors*, and semiconductors with such impurities are called *n-types*. The majority carriers in *n*-type semiconductors are electrons, and the minority carriers are holes. The majority of charge movement takes place in the conduction band. Typical dopants are phosphorus (P), arsenic (As), and antimony (Sb).

The conductivity of the semiconductor is determined by the carriers. The mobility of electrons is higher than that of holes.[9] The total conductivity in any semiconductor is a combination of the movement of the electrons and holes and is given by

Equation 4: Semiconductor Conductivity

$$\sigma = q\left(n\mu_n + p\mu_p\right)$$

The law of mass action, Eq. 1, applies to extrinsic semiconductors. With the addition of dopants, the *law of electrical neutrality* given by Eq. 5 also applies. The concentration of acceptor atoms is N_A, each contributing one positive charge to the lattice. (Do not confuse NA with Avogadro's number.) The concentration of donor atoms is N_D, each contributing one negative charge. Since neutrality is maintained, the result is[10]

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[6] The effective density of states is a simplified, integrated value that represents the number of states available to electrons at or near the band edges (conduction band minimum or valence band maximum).

[7] A minor amount of dopant material is on the order of 10 parts per billion.

[8] The additional electron exists because the outer shell octet is satisfied by the covalent bonding of the first four electrons in the impurity.

[9] The mobility of electrons is a factor of 2.5 higher in silicon and about 2.1 higher in germanium.

[10] Neutrality is maintained because each atom of an added impurity removes one intrinsic atom.

Equation 5: Law of Electrical Neutrality

$$N_A + n = N_D + p$$

In a *p*-type material, the concentration of donors is zero, ($N_D = 0$). Additionally, the concentration of holes is much greater than the number of electrons ($p \gg n$). For a *p*-type material, Eq. 5 can be rewritten as $N_A \approx p$. Using the law of mass action, Eq. 1, the concentration of electrons in a *p*-type material is

Equation 6: Electron Concentration in *p*-type Material

$$n = \frac{n_i^2}{N_A}$$

In a *n*-type material, the concentration of acceptors is zero, ($N_A = 0$). Additionally, the concentration of electrons is much greater than the number of holes ($n \gg p$). For an *n*-type material, Eq. 5 can be rewritten as $N_D \approx n$. Using the law of mass action, Eq. 1, the concentration of electrons in a *n*-type material is

Equation 7: Hole Concentration in *n*-type Material

$$p = \frac{n_i^2}{N_D}$$

Example 2

The energy gap for silicon at room temperature (300K) is 1.12 eV. The energy density states are $N_c = 2.8 \times 10^{19}\ \text{cm}^{-3}$ and $N_v = 1.02 \times 10^{19}\ \text{cm}^{-3}$. What is the intrinsic carrier concentration?

Solution

The intrinsic carrier concentration, assuming thermal equilibrium, is given by Eq. 3.

$$
\begin{aligned}
n_i^2 &= N_c N_v e^{-E_G/\kappa T} \\
&= (2.8 \times 10^{19}\ \text{cm}^{-3}) \\
&\qquad \times (1.02 \times 10^{19}\ \text{cm}^{-3})\, e^{-1.12\,\text{eV}/(8.621\times10^{-5}\text{eV/K})(300\text{K})} \\
&= 4.452 \times 10^{19}\ \text{cm}^{-6} \\
n_i &= \sqrt{4.452 \times 10^{19}\text{cm}^{-6}} = 6.67 \times 10^{9}\ \text{cm}^{-3} \\
&\approx 0.7 \times 10^{10}\ \text{cm}^{-3}
\end{aligned}
$$

This number differs slightly from the number determine experimentally and more exact calculations ($1.6\square10^{10}\ \text{cm}^{-3}$).

Device Performance Characteristics

Electronic components function on some variation of *pn* junction principles. Amplifiers function on some variation of transistor principles, that is, *pnp* or *npn* junction principles. Most electronic components can be modeled as two-port devices with two variables—current and voltage—for each port. The relationship between the variables depends on the type of device. The relationship can be expressed *mathematically*, as for MOSFETs (*metal-oxide semiconductor field-effect transistors*), *modeled* in equivalent circuits, as with transistor *h*-parameters, or described *graphically*, as with *BJT* characteristic curves.

Semiconductor devices are inherently nonlinear. The performance of such devices is analyzed in a linear fashion over small portions of the characteristic curve(s). Such analysis is called *small-signal analysis*. A small signal is one that is much less than the average, that is, steady-state, value for the device, which usually is the biasing value. The models used in each linear portion of the characteristic curve are termed *piecewise linear models*. If the operation is outside the linear region or if the input signal is large compared to the average value, the device distorts the input signal. Such operation is called *nonlinear operation*.

The characteristic curve for an ideal transistor operating as a current-amplifying device is shown in Fig. 2. (The shape of the curve is the same for a diode, that is, a *pn* junction.)

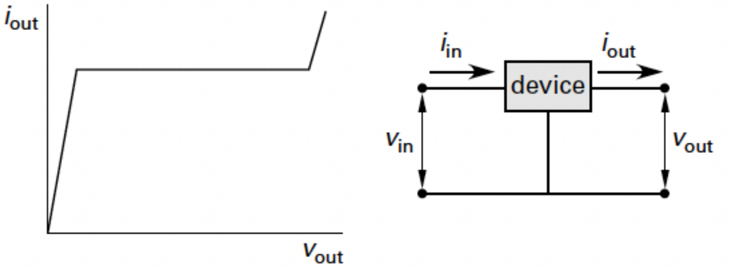

Figure 2: Typical Semiconductor Model

The voltage-current graph is divided into various regions known by names such as *saturation* (or *on*), *cutoff* (or *off*), *active*, *breakdown*, *avalanche*, and *pinchoff* regions. The locations of these regions depend on the type of transistor—for example, BJT or FET—and its polarity. Operation is normally in the linear active region, but applications for operation in other regions exist in digital and communications (radio frequency) applications.

Bias

Bias is a DC voltage applied to a semiconductor junction to establish the operating point, also called the *quiescent* (*no-signal*) *point*. *Biasing* establishes the operating point with no input signal.[11] Biasing, then, is the process of establishing the DC voltages and currents (the bias) at the device's terminals when the input signal is zero (or nearly so).

For *pn* junctions, *forward bias* (or *on condition*) is the application of a positive voltage to the *p*-type material or, equivalently, the flow of current from the *p*-type to the *n*-type material. In a small semiconductor device, forward bias results in current in the milliampere range.

Reverse bias (or *off condition*) is the application of a negative voltage to the *p*-type material, or equivalently, the flow of current from the *n*-type to the *p*-type material. In a small semiconductor device, reverse bias results in current in the nanoampere range.

Self-biasing is the use of the amplifier's output voltage, rather than a separate power source, as the supply for the input bias voltage. This negative feedback control regulates the output current and voltage against variations in transistor parameters.

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[11] Bias is used both as a verb and a noun in electronics.

Amplifiers

An *amplifier* produces an output signal from the input signal. The input and output signals can be either voltage or current. The output can be either smaller or larger (the usual case) than the input in magnitude. While most amplifiers merely scale the input voltage or current upward, the amplification process can include a sign change, a phase change, or a complete phase shift of 180°.[12] The ratio of the output to the input is known as the *gain* or *amplification factor*, A. A *voltage amplification factor*, A_V, and *current amplification factor*, A_I or β, can be calculated for an amplifier.

Figure 3 illustrates a simplified current amplifier with current amplification factor β. The additional current leaving the amplifier is provided by the bias battery, V_2.

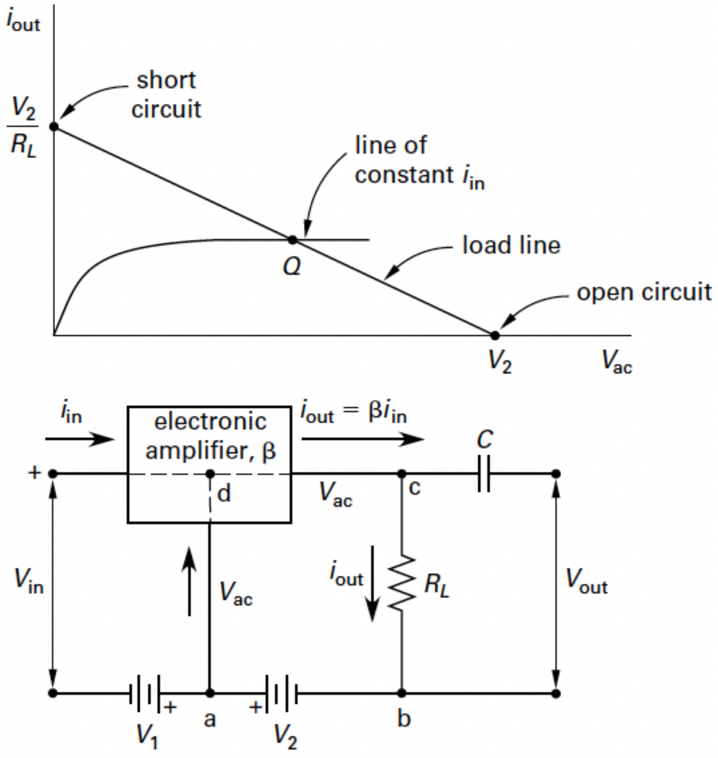

Figure 3: General Amplifier

The current amplification is thus given by the following equation.

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[12] An *inverting amplifier* is one for which $v_{out} = -A_V v_{in}$. For a sinusoidal input, this is equivalent to a phase shift of 180° (i.e., $V_{out} = A_V V_{in} \angle -180°$).

Equation 8: Current Amplification

$$i_{\text{out}} = \beta i_{\text{in}}$$

A capacitor, *C*, is placed in the output terminal to force all DC current to travel through the *load resistor*, R_L. Kirchhoff's voltage law for loop abcd is

Equation 9: Load Resistor DC Current

$$V_2 = i_{\text{out}} R_L + V_{ac} = \beta i_{\text{in}} R_L + V_{ac}$$

If there is no input signal (i.e., $i_{\text{in}} = 0$), then $i_{\text{out}} = 0$ and the entire battery voltage appears across terminals ac ($V_{ac} = V_2$). If the voltage across terminals ac is zero, then the entire battery voltage appears across R_L so that $i_{\text{out}} = V_2/R_L$.

Amplifier Classification

Amplifiers are classified on the basis of how much input is translated into output. A sinusoidal input signal is assumed. The output of an amplifier depends on the bias setting, which in turn establishes the quiescent point.

A *Class A amplifier*, as shown in Fig. 4, has a quiescent point in the center of the active region of the operating characteristics. Class A amplifiers have the greatest linearity and the least distortion. Load current flows throughout the full input signal cycle. Because the load resistance of a properly designed amplifier will equal the Thevenin equivalent source resistance, the maximum power conversion efficiency of an ideal Class A amplifier is 50%.

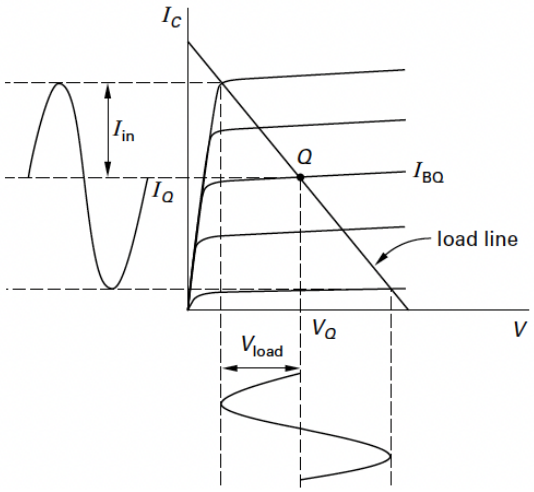


Figure 4: Class A Amplifier

For *Class B amplifiers*, as shown in Fig. 5, the quiescent point is established at the cutoff point. A load current flows only if the signal drives the amplifier into its active region, and the circuit acts like an amplifying half-wave rectifier. Class B amplifiers are usually combined in pairs, each amplifying the signal in its respective half of the input cycle. This is known as *push-pull operation*. The output waveform will be sinusoidal except for the small amount of crossover distortion that occurs as the signal processing transfers from one amplifier to the other. The maximum power conversion efficiency of an ideal Class B push-pull amplifier is approximately 78%.

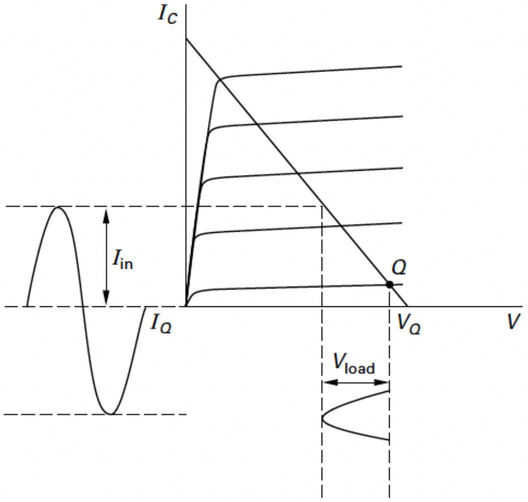


Figure 5: Class B Amplifier

The intermediate *Class AB amplifier* has a quiescent point somewhat above cutoff but where a portion of the input signal still produces no load current. The output current flows for more than half of the input cycle. AB amplifiers are also used in push-pull circuits.

Class C amplifiers, as shown in Fig. 6, have quiescent points well into the cutoff region. Load current flows during less than one-half of the input cycle. For a purely resistive load, the output would be decidedly non-sinusoidal. However, if the input frequency is constant, as in radio frequency (rf) power circuits, the load can be a parallel LRC tank circuit tuned to be resonant at the signal frequency. The LRC circuit stores electrical energy, converting the output signal to a sinusoid. The power conversion efficiency of an ideal Class C amplifier is 100%.

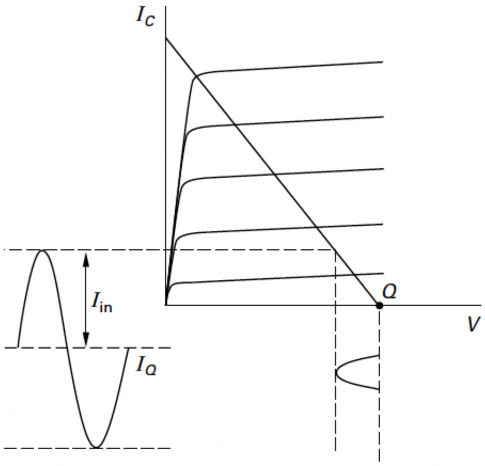


Figure 6: Class C Amplifier

Load Line and Quiescent Point Concept

The i_{in}-v_{out} curves shown above illustrate how amplification occurs. The two known points, (v_{out}, i_{out}) = (V_2, 0) and (v_{out}, i_{out}) = (0, V_2/R_L), are plotted on the voltage-current characteristic curve. The straight load line is drawn between them. The change in output voltage (the horizontal axis) caused by a change in input voltage (parallel to the load line) can be determined. Equation 11 gives the voltage *gain* (*amplification factor*).[13][14]

Equation 10: Voltage Gain

$$A_V = \frac{\partial v_{\text{out}}}{\partial v_{in}} \approx \frac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}}$$

Usually, a nominal current (the *quiescent current*) flows in the abcd circuit even when there is no signal. The point on the load line corresponding to this current is the quiescent point (*Q-point* or *operating point*). It is common to represent the quiescent parameters with uppercase letters (sometimes with a subscript *Q*) and to write instantaneous values in terms of small changes to the quiescent conditions as shown in the following equations.

Equation 11: Transistor Input Instantaneous Voltage

$$v_{\text{in}} = V_Q + \Delta v_{\text{in}}$$

---

[13] Gain can be increased by increasing the load resistance, but a larger biasing battery, V_2, is required. The choice of battery size depends on the amplifier circuit devices, size considerations, and economic constraints.

[14] A high-gain amplifier has a gain in the tens of hundreds of thousands.

Equation 12: Transistor Instantaneous Output Voltage

$$v_{\text{out}} = V_{\text{out}} + \Delta v_{\text{out}}$$

Equation 13: Transistor Instantaneous Output Current

$$i_{\text{out}} = I_{\text{out}} + \Delta i_{\text{out}}$$

Because it is a straight line, the load line can also be drawn if the quiescent point and any other point, usually (V_{BB}, 0), are known.

The ideal voltage amplifier has an *infinite input impedance* (so that all of v_{in} appears across the amplifier and no current or power is drawn from the source) and *zero output impedance* so that all of the output current flows through the load resistor.

Determination of the load line for a generic transistor amplifier is accomplished through the following steps. [An example follows.]

Step 1: For the configuration provided, label the x-axis on the *output characteristic curves* with the appropriate voltage. (For a BJT, this is V_{CE} or V_{CB}. For a FET, this is V_D.)[15]

Step 2: Label the y-axis as the output current. (For a BJT, this is I_C. For a FET, this is I_D.)

Step 3: Redraw the circuit with all three terminals of the transistor open. Label the terminals. (For a BJT, these are base, emitter, and collector. For a FET, these are gate, source, and drain.) Label the current directions all pointing inward, toward the amplifier. (For a BJT, these are I_B, I_E, and I_C. For a FET, these are I_D and I_S.)

Step 4: Perform KVL analysis in the output loop. (For a BJT, this is the collector loop. For a FET, this is the drain loop.) The transistor voltage determined is a point on the x-axis with the output current equal to zero. Plot the point.

Step 5: Redraw the circuit with all three terminals of the transistor shorted. Label as in Step 3.

Step 6: Use Ohm's law, or another appropriate method, in the output loop to determine the current. (For the BJT, this is the collector current. For the FET, this is the drain current.) The transistor current determined is a point on the y-axis with the applicable voltage in Step 1 equal to zero. Plot the point.

---

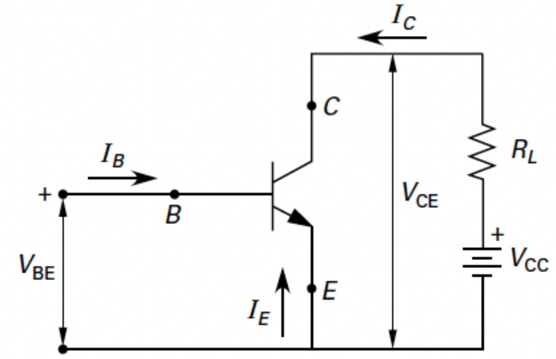
[15] The output characteristic curves for the BJT are also called the *collector characteristics* or the *static characteristics*.

Step 7: Draw a straight line between the two points. This is the DC load line.[16]

Example 3

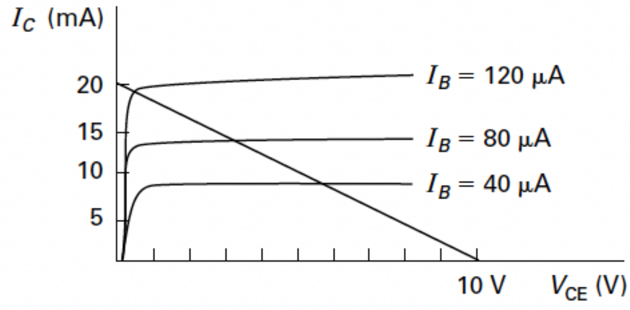
Consider the following common emitter (CE).

If the load resistance is 500 Ω and the collector supply voltage, V_{CC}, is 10 V, determine and draw the load line.



Solution

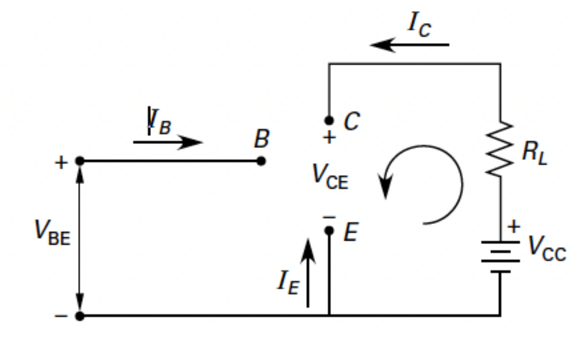
[Step 1 and 2] The *x*- and *y*- axes are drawn as shown. [The values will be calculated in the following steps.]



[Step 3] Redrawing the circuit and labeling gives the following.

---

[16] AC load lines are determined in the same manner, but active components, that is, inductors and capacitors, are accounted for in the analysis.



[Step 4] Write KVL around the indicated loop.

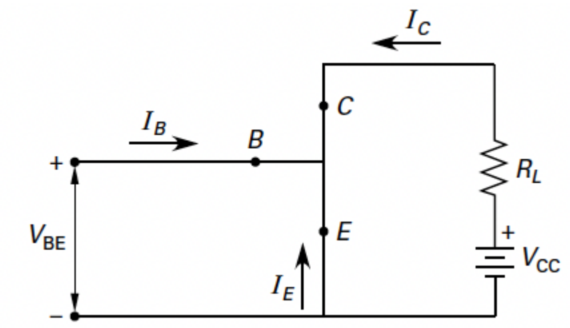
$$V_{CC} - I_C R_L - V_{CE} = 0$$

With the terminal open-circuited, $I_C = 0$. Substitute and rearrange.

$$V_{CE} = V_{CC} = 10 \text{ V}$$

Plot this point (10,0) on the *x*-axis. [See Figure in Step 1]

[Step 5] Redraw and label the circuit with the terminal shorted.

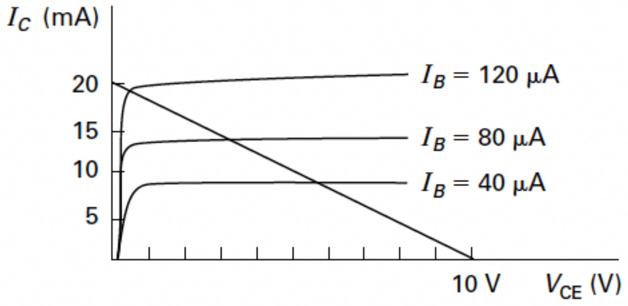


[Step 6] Using Ohm’s law and the given values,

$$I_C = \frac{V_{CC}}{R_L} = \frac{10 \text{ V}}{500 \ \Omega} = 0.20 \text{ A} \quad (20 \text{ mA})$$

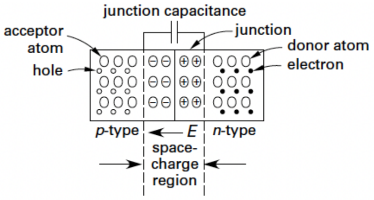
Plot this point on the *y*-axis.

[Step 7] Draw a straight line between the two point. This is the DC load line as shown in the first drawing and repeated here for completion.

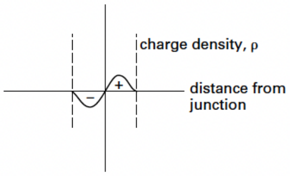

I_C (mA)
20
15
10
5
I_B = 120 μA
I_B = 80 μA
I_B = 40 μA
10 V
V_{CE} (V)

PN JUNCTIONS

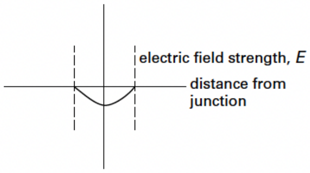
The *pn* junction forms the basis of diode and transistor operation. The *pn* junction is constructed of a *p*-type material (the anode) and an *n*-type material (the cathode) bonded together as shown in the following figure. Some of the acceptor atoms are shown as ions with a minus sign because after an impurity atom accepts an electron, it becomes negatively charged. Some of the donor atoms are shown as ions with a plus sign because after an impurity atom gives up an electron it is positively charged. (Overall, the law of charge neutrality holds, and the *pn* junction is neutral.)



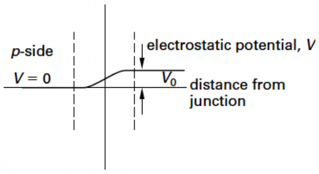
(a) *pn* junction



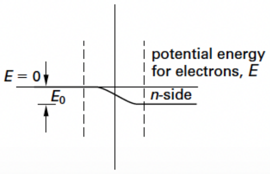
(b) charge density



(c) electric field intensity



(d) electrostatic potential



(e) potential energy

Figure 7: PN Junction Characteristics

The concentration gradient across the junction causes holes to diffuse to the right, and electrons to diffuse to the left. As a result, the concentration of holes on the *p*-side near the junction is depleted and a negative charge exists. The concentration of electrons on the *n*-side near the junction is depleted as well, and a positive charge exists. The result of this diffusion is shown in Fig. 7(a) and 7(b), the shape of which is determined by the level of doping.[17] The diffusion process continues until the electrostatic field set up by the charge separation is such that no further charge motion is possible. The net electric field intensity is shown in Fig. 7(c). The net result is a small region in which no mobile charge carriers exist. This region is called the *space-charge region*, *depletion region*, or *transition region*, Fig. 7(a). Holes have a potential barrier they must overcome to move from left to right just as electrons have an energy barrier they must overcome to move from right to left, as shown in Fig. 7(d) and Fig. 7(e).

The flow of carriers caused by the concentration gradient is called the *diffusion current*, $I_{\text{diffusion}}$, also called the *recombination current* or the *injection current*. The flow of carriers caused by the established electric field is called the *drift current*, I_s, also called the *saturation current*, *thermal current*, or *reverse saturation current*.[18] The movement of carriers caused by recombination (diffusion) and drift (saturation) is continual, though at thermal equilibrium, without any applied voltage, the net current is zero.

Equation 14: PN Junction Carrier Current

$$I_{\text{junction}} = I_{\text{diffusion}} + I_s = 0 \quad \left[\text{algebraic sum}\right]$$

A summary of the movement of carriers is shown below.

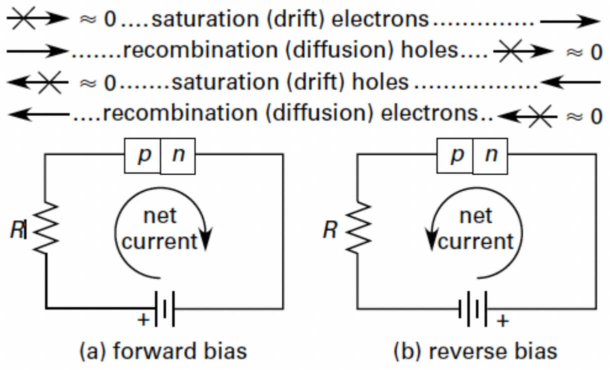


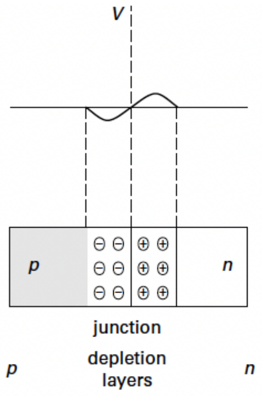
Figure 8: PN Junction Carrier Movement

---

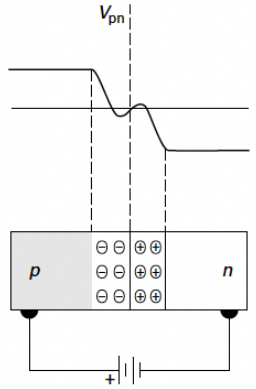
[17] A step-graded junction and a linearly graded junction are two possible types.

[18] Numerous symbols are used, among them I_o and I_{co}.

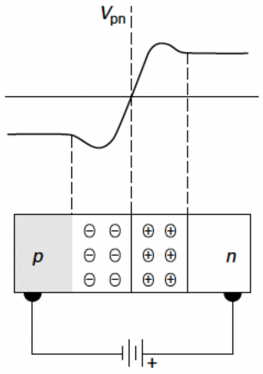
The width of the space-charge region at equilibrium is shown in Fig. 9(a).



(a) no applied voltage



(b) forward bias



(c) reverse bias

Figure 9: PN Junction Space-Charge Region

When an external voltage is applied, that is, a biasing voltage, the width of the space-charge region and the barrier height change. When the *p*-type material is connected to a positive potential, that is, forward biased, holes are repelled across the junction into the *n*-type material, and electrons are repelled across the junction into the *p*-type material. The width of the space-charge region is reduced and the barrier height for *p* to *n* current flow is reduced as shown in Fig. 9(b). A DC forward bias voltage, V_F, of approximately 0.5–0.7 V for silicon and 0.2–0.3 V for germanium is required to overcome the barrier voltage. Once the barrier is overcome, the junction current increases significantly from an increase in the diffusion current. That is, holes cross the junction into the *n*-type material, where they are considered injected minority carriers. Electrons cross the junction into the *p*-type material, where they too are injected minority carriers. Because hole movement in one direction and electron movement in the opposite direction constitute a current in the same direction, the total current is the sum of the hole and electron minority currents.

When the *p*-type material is connected to a negative potential, that is, reverse biased, holes and electrons move away from the junction. The width of the space-charge region is increased and the barrier height for *p* to *n* current flow is increased as shown in Fig. 9(c). The process nominally stops when the holes in the *p*-type material are depleted. However, a few holes in the *n*-type material are thermally generated, as there are electrons in the *p*-type material. These minority carriers thermally diffuse into the depletion region and are swept across by the electric field. The effect is constant for a given temperature and independent of the reverse bias. This is the reverse saturation current, I_s.[19] The reverse saturation current is small ($\approx 10^{-9}$ A). An ideal *pn* junction, excluding the breakdown region, is governed by[20]

Equation 15: PN Junction Current

$$I_{pn} = I_S\left(e^{qV_{pn}/\kappa T} - 1\right)$$

Breakdown is a large, abrupt change in current for a small change in voltage. When a *pn* junction is reverse biased, the saturation current is small up to a certain reverse voltage, where it changes dramatically. Two mechanisms can cause this change. The first is *avalanche breakdown*. Avalanche occurs when thermally generated minority carriers are swept through the space-charge region and collide with ions. If they possess enough energy to break a covalent bond, an electron-hole pair is created. The same effect may occur for these newly generated carriers, resulting in an avalanche effect. Avalanche breakdown occurs in lightly doped materials at greater than 6 V reverse bias. The second breakdown mechanism is *zener breakdown*. Zener breakdown occurs

---

[19] The reverse saturation current also accounts for any current leakage across the surface of the semiconductor

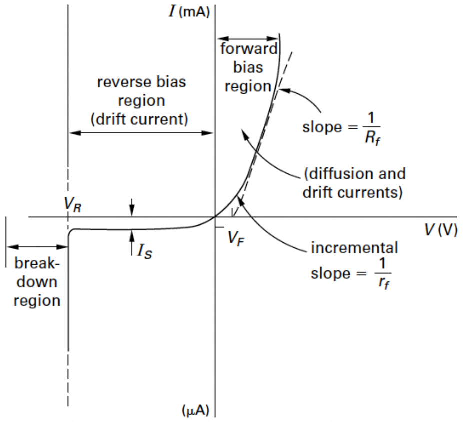
[20] The subscript "pn" is used here for clarification. Standard diode voltage and current directions are defined in the next section on diodes, after which the subscript is no longer used.

through the disruption of covalent bonds caused by the strength of the electric field near the junction. No collisions are involved. The additional carriers created by the breaking covalent bonds increase the reverse current. Zener breakdown occurs in highly doped materials at less than 6 V reverse bias.[21]

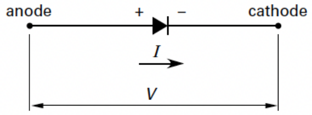
Diode Performance Characteristics

A diode is a two-electrode device. The diode is designed to pass current in one direction only. An *ideal diode*, approximated by a *pn* junction, has a zero voltage drop, that is, no forward resistance, and acts as a short circuit when forward biased (on). When reversed biased (off), the resistance is infinite and the device acts as an open circuit. Diode construction and theory is that of a *pn* junction.

The characteristics and symbol for a typical real semiconductor diode are shown in Fig. 10. The *reverse bias voltage* is any voltage below which the current is small, that is, less than 1% of the maximum rated current. The *peak inverse* (*reverse*) *voltage*, PIV or PRV, is the maximum reverse bias the diode can withstand without damage. The forward current is also limited by heating effects. Maximum forward current and peak inverse voltage for silicon diode rectifiers are approximately 600 A and 1000 V, respectively.



(a) characteristics



(b) symbol

Figure 10: Semiconductor Diode Characteristics and Symbol

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[21] The name zener is commonly used regardless of the breakdown mechanism.

The ideal pn junction current was given in Eq. 15. For practical junctions (*diodes* or *rectifiers*), the equation becomes

Equation 16: Practical Diode/Rectifier Current

$$I = I_S\left(e^{qV/\eta\kappa T} - 1\right) = I_S\left(e^{V/\eta V_T}\right) - 1$$

Equation 16, which is based on the *Fermi-Dirac probability function*, is valid for all but the breakdown region (see Fig. 10). The term η is determined experimentally. For discrete silicon diodes, $\eta = 2$. For germanium diodes, $\eta = 1$. The saturation current, taken from any value of I with a small reverse bias (for example, between 0 and −1 V) gives $I_s \approx 10^{-9}$ A for silicon and 10^{-6} A for germanium. The term V_T represents the *voltage equivalent of temperature* and is related to the diffusion occurring at the junction.[22]

Equation 17: Voltage Equivalent of Temperature

$$V_T = \frac{\kappa T}{q} = \frac{D_p}{\mu_p} = \frac{D_n}{\mu_n}$$

Boltzmann's constant is given by κ. The absolute temperature is T. An electron charge is represented by q. Diffusion constants, measured in m^2/s, are given the symbol D. The mobility, measured in $\text{m}^2/\text{V}\cdot\text{s}$, is given by the symbol μ.

The voltage equivalent of temperature is also known as the *thermal voltage*. The value of V_T is often quoted at *room temperature*, which can vary from 293K (20°C) to 300K (27°C) depending upon the reference used. The temperature has the effect of doubling the saturation current every 10°C. So, the following applies.

Equation 18: Saturation Current vs. Temperature

$$\frac{I_{S2}}{I_{S1}} = \left(2\right)^{T_2 - T_1/10^\circ\text{C}}$$

A *real diode* can be modeled as shown in Fig. 11. The real diode model is composed of an ideal diode, a resistor, R_f, and a voltage source, V_F. The voltage source accounts for the barrier voltage. Typically, for silicon $V_F = 0.7$ V, and for germanium $V_F = 0.2$ V.

---

[22] This is also called the *Einstein relationship*.

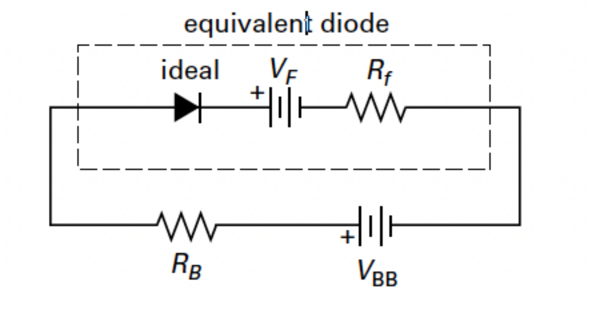

Figure 11: Real Diode Equivalent Circuit

The resistance, R_f, is the slope of a line approximating the linear portion of the characteristic curve as shown in Fig. 10. R_f is the resistance for an ideal diode. It is not the static (average) resistance of the diode, as the average resistance is not a constant. The static (average) resistance is calculated as $R_{\text{static}} = V_D/I_D$. When the diode is forward biased by more than a few tenths of a volt, R_f is equal to the *dynamic forward resistance*, r_f. Disregarding lead contact resistance (less than 2 Ω), the dynamic forward resistance is

Equation 19: Dynamic Forward Resistance

$$R_f = r_f = \frac{\eta V_T}{I_D}$$

A *dynamic reverse resistance*, r_r, also exists and is the inverse of the slope at a point in the reverse bias region.[23] Because the reverse current is very small, the resistance is often considered infinite. Capacitances associated with the junction are also ignored in most models of the diode. The *diffusion capacitance*, C_d, is associated with the charge stored during forward biased operation. The *transition capacitance*, C_t, is associated with the space-charge region width and is the primary capacitance of concern during reverse bias operation. (Diodes designed to take advantage of this voltage-sensitive capacitance are called *varactors*.)

The model of Fig. 11 assumes that (1) the reverse bias current is sufficiently small that the diode acts as an open circuit in that direction, (2) the reverse bias voltage does not exceed the breakdown voltage, and (3) the switching time is instantaneous. The *switching time* is the transient that occurs from the time interval of the application of a voltage to forward (reverse) bias and the achievement of the actual condition. The switching time depends upon the speed of movement of minority carriers near the junction and the junction capacitance.

---

[23] Specifying the reverse current, I_{co}, is equivalent to specifying the reverse resistance.

Example 4

What is the thermal voltage at a room temperature of 300K?

Solution

The thermal voltage is given by Eq. 17.

$$
\begin{aligned}
V_T &= \frac{\kappa T}{q} \\
&= \frac{\left(1.3807\times 10^{-23}\ \frac{\text{J}}{\text{K}}\right)\left(300\ \text{K}\right)}{1.6022\times 10^{-19}\ \text{C}} \\
&= 0.026\ \text{V}
\end{aligned}
$$

Diode Load Line

Figure 12 shows a forward-biased real diode in a simple circuit. V_{BB} is the *bias battery* (hence the subscripts), and R_{B} is a current-limiting resistor. R_f and V_f are equivalent diode parameters, not discrete components. If R_f is known in the vicinity of the operating point, the diode current, I_D, is found from Kirchhoff's voltage law. The diode voltage is found from Ohm's law: $V_D = I_D R_f$.

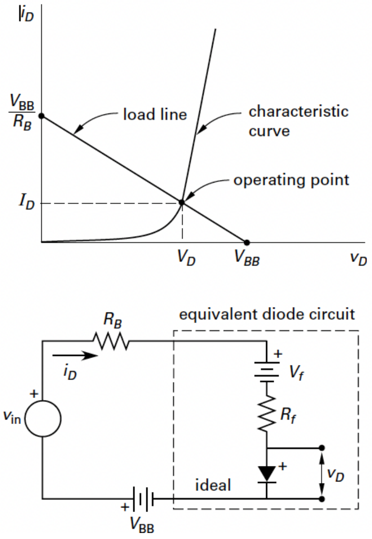

Figure 12: Diode Load Line

Equation 20: Diode Load Line KVL

$$V_{\mathrm{BB}} - V_f = I_D\left(R_B + R_f\right)$$

The diode current and voltage drop can also be found graphically from the *diode characteristic curve* (see Fig. 12) and the load line, a straight line representing the locus of points satisfying Eq. 20.[24] The load line is defined by two points. If the diode current is zero, all of the battery voltage appears across the diode (point (V_{BB}, 0)). If the voltage drop across the diode is zero, all of the voltage appears across the current-limiting resistor (point (0, V_{BB}/R_B)). (Because the diode characteristic curve implicitly includes the effects of V_f and R_f, these terms should be omitted.) The *no-signal operating point*, also known as the *quiescent point*, is the intersection of the diode characteristic curve and the load line.

The *static load line* is derived assuming there is no signal (i.e., $v_{\mathrm{in}} = 0$). With a signal, the *dynamic load line* shifts left or right while keeping the same slope. This is equivalent to solving Eq. 20 with an additional voltage source, as shown below.

Equation 21: Dynamic Load Line

$$i_D = I_D + \Delta i_D = \frac{V_{\mathrm{BB}} - V_f + v_{\mathrm{in}}}{R_B + R_f}$$

Although presented in a slightly different manner, the method for determining the load line is similar to that given in the Load Line section. That is,

Step 1: Open-circuit the electronic component's equivalent circuit and determine the point (x, 0).

Step 2: Short-circuit the electronic component's equivalent circuit and determine the point (0, y).

Step 3: Connect the two points.

The load line superimposed on the characteristics curve is then used to determine the operating point (Q-point) of the overall circuit.

---

[24] The horizontal axis voltage is the voltage across the diode (modeled as a resistor).

Diode Piecewise Linear Model

If the voltage applied to a diode varies over an extensive range, a piecewise linear model may be used.[25] For the real diode characteristic shown in Fig. 10, three regions are evident from V_R to V_F.

(1) *Reverse Breakdown Region*, $V_D < V_R$

Equation 22: Diode Reverse Breakdown Region

$$I_D = \frac{V_D + V_R}{R_r} \quad [V_D \text{ and } V_R \text{ are negative}]$$

(2) *Off Region*, $V_R < V_D < V_F$

Equation 23: Diode Off Region

$$I_D = 0$$

(3) Forward Bias Region, $V_D > V_F$

Equation 24: Diode Forward Bias Region

$$I_D = \frac{V_D - V_F}{R_F}$$

Using ideal diodes, the real diode can be represented by the model in Fig. 13 over the entire range of operation.

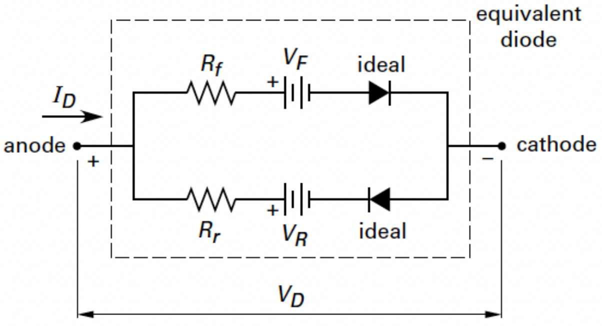

Figure 13: Piecewise Linear Model

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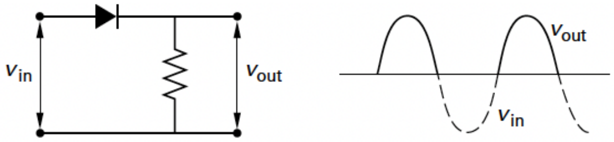
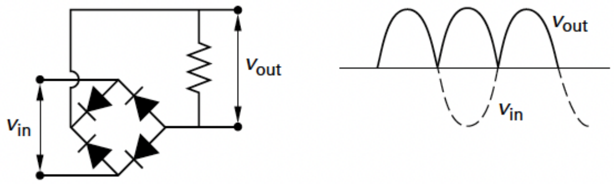
[25] The model in previous section is for the forward bias region only and assumes no reverse current flow.

DIODE APPLICATIONS AND CIRCUITS

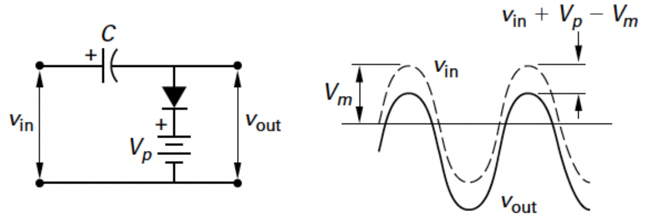
Diodes are readily integrated into *rectifier*, *clipping*, and *clamping circuits*. A clipping circuit cuts the peaks off of waveforms; a clamping circuit shifts the DC (average) component of the signal. Figure 14 illustrates the response to a sinusoid with peak voltage Vm for several simple circuits.



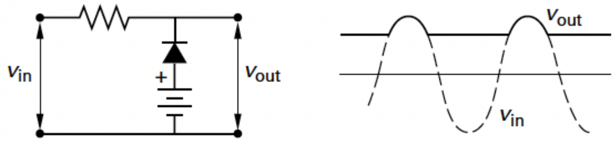
(a) half-wave rectifier



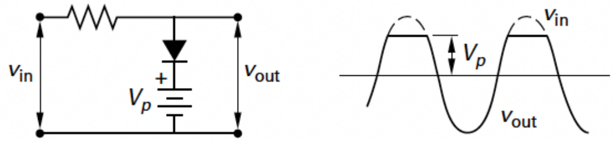
(b) full-wave bridge rectifier



(c) clamping circuit (C charges to V_m)



(d) base clipper

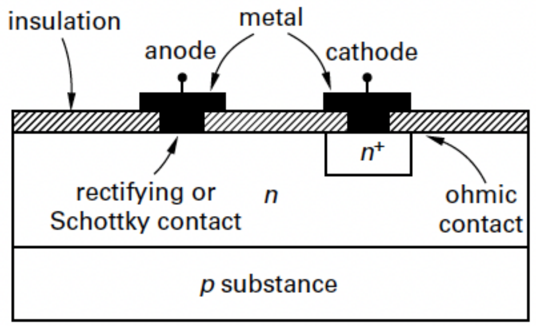


(e) peak clipper

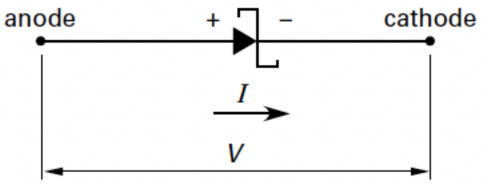
Figure 14: Simple Diode Circuits

Schottky Diodes

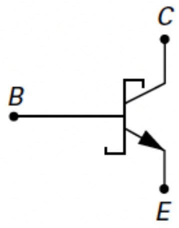
A *Schottky diode*, also called a *barrier diode* or a *hot-carrier diode*, is a diode constructed with a metal semiconductor contact as shown in Fig. 15(a). The Schottky diode symbol is shown in Fig. 15(b). The metal semiconductor rectifying junction is similar to the *pn* junction, but the physical mechanisms are somewhat different.



(a) construction



(b) diode symbol



(c) Schottky contact symbol

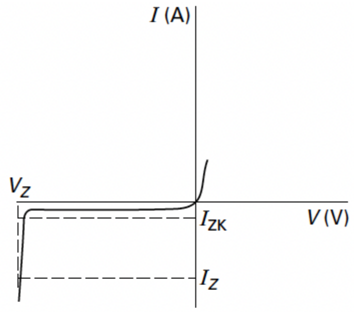
Figure 15: Schottky Diode

In the forward direction, electrons from the lightly doped semiconductor cross into the metal anode, where electrons are plentiful.[26] The electrons in the metal are majority carriers, whereas in a *p*-type material they are minority carriers. Being majority carriers, they are indistinguishable from other carriers and are not stored near the junction. This means that no minority carrier population exists to move when switching occurs from forward to reverse (on to off) bias. Consequently, switching times are extremely short (approximately 10^{-12} s).

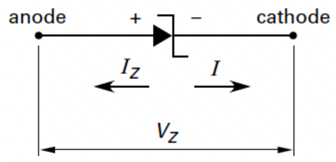
The Schottky symbol is used on any electronic component designed with a *Schottky contact*, that is, a rectifying, rather than an ohmic, contact as shown in Fig. 15(c). In the figure, the Schottky contact lies between the base and the collector.

Zener Diodes

A *zener diode* is a diode specifically designed to operate within the breakdown region. The construction and theory of a zener diode is similar to that of a *pn* junction, with the design allowing for greater heat dissipation capabilities. The characteristics and symbol are shown in Fig. 16.



(a) characteristics



(b) symbol

Figure 16: Zener Diode

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[26] The electrons injected into the metal are above the Fermi energy level , determined by the Fermi-Dirac distribution of electron energies, and are called *hot carriers*.

When a reverse voltage, known as the *zener voltage*, V_Z (which is negative with respect to the anode), is applied, a reverse saturation current, I_Z, flows. The voltage-current relationship is nearly linear. Current flows until the reverse saturation current drops to I_{ZK} near the knee of the characteristic curve. This minimum current is the *keep-alive current*, that is, the minimum current for which the output characteristic is linear.

Because the current is large, an external resistor must be used to limit the current to within the power dissipation capability of the diode. Zener diodes are used as voltage regulating and protection devices. The regulated zener voltage varies with the temperature. The *temperature coefficient* is

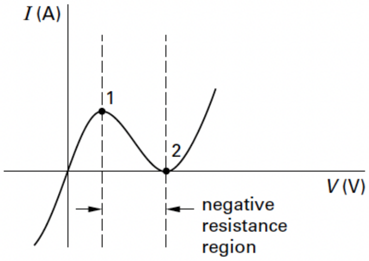
Equation 25: Zener Temperature Coefficient

$$t_C = \frac{\Delta V_Z}{V_Z \Delta T} \times 100\%$$

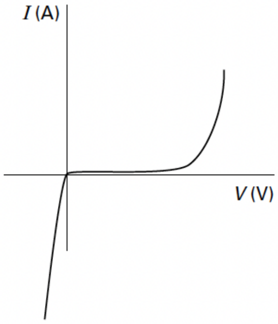
Tunnel Diodes

A *tunnel diode* (*Esaki diode*) is a two-terminal device with an extremely thin potential barrier to electron flow, so that the output characteristic is dominated by the quantum-mechanical tunneling process. To make a thin potential barrier (50 Å to 100 Å), both regions of the diode are heavily doped. The amount of tunneling is limited by the electrons available in the *n*-type material or by the available empty energy states in the *p*-type material to which they can tunnel. The characteristics are shown in Fig. 17(a).

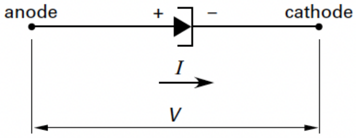
Point 1 in Fig. 17(a) corresponds to a biasing level that allows for maximum tunneling. (At this point, the minimum electron energy in the *n*-type material conduction band equals the Fermi level in the *p*-type material's valence band.) Between points 1 and 2, increases in bias voltage result in a decrease in current as available energy states are filled and the total amount of tunneling drops. This is an area of negative resistance, or negative conductance, and is the primary use for tunnel diodes. If the doping levels are slightly reduced from their typical values of 5×10^{19} cm^{-3}, the forward tunneling current becomes negligible and the output characteristic becomes that of a *backward diode*, Fig. 17(b). The symbol for a tunnel diode is shown in Fig. 17(c).



(a) tunnel diode characteristics



(b) backward diode



(c) symbol

Figure 17 Tunnel Diode

Photodiodes and Light-Emitting Diodes

If a semiconductor junction is constructed so that it is exposed to light, the incoming photons generate electron-hole pairs. When these carriers are swept from the junction by the electric field, they constitute a *photocurrent*, which is seen as an increase in the reverse saturation current. The holes generated move to the *p*-type material and the electrons move to the *n*-type material in response to the electric field that is established whenever *p*- and n-type semiconductors are joined (see Fig. 7(a)). Such devices are used as light sensors and are called *photodiodes*. When the device is designed without a biasing source, it becomes a *solar cell*.

When forward biased, diodes inject carriers across the junction that are above thermal equilibrium. When the carriers recombine, they emit photons from the *pn* junction area.[27] The photon is caused by the recombination of electron-hole pairs. The wavelength depends on the energy band gap and on the material used. Gallium arsenide (GaAs) and other binary compounds are commonly used. When the photon is in the infrared region, the mechanism is called *electroluminescence* and the diodes are called *electroluminescent diodes*. If the photons are in the visible region, the devices are called *light-emitting diodes* (LEDs). The emitted wavelength, λ, is given by Eq. 26. The term h is Planck's constant, with a value of 6.6256×10^{-34} J·s.

Equation 26: LED Wavelength

$$\lambda = \frac{hc}{E_G}$$

Example 5

The manufacturer's data sheet for a gallium arsenide diode shows a band gap energy of 1.43 eV. Will such a band gap result in emitted photons within the wavelength of visible light?

Solution

The photon emitted wavelength is given by

$$\begin{aligned}\lambda &= \frac{hc}{E_G} \\ &= \frac{\left(6.6256\times10^{-34}\ \mathrm{J\cdot s}\right)\left(2.9979\times10^{8}\ \frac{\mathrm{m}}{\mathrm{s}}\right)}{\left(1.43\ \mathrm{eV}\right)\left(1.6022\times10^{-19}\ \frac{\mathrm{J}}{\mathrm{eV}}\right)} \\ &= 8.67\times10^{-7}\ \mathrm{m}\end{aligned}$$

The wavelength of visible light is from approximately 8×10^{-7} m to 4×10^{-7} m. Consequently, GaAs is a good choice for a light-emitting diode.

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[27] They can also release the energy as heat, or *phonons*.

SILICON-CONTROLLED RECTIFIERS

A four-layer *pnpn* device with an anode, cathode, and gate terminal is called a *silicon-controlled rectifier* (SCR) or *thyristor*.[28] A conceptual construction is shown in Fig. 18(a).

When the SCR is reverse biased, that is, when the anode is negative with respect to the cathode, the characteristics are similar to a reverse-biased *pn* junction as shown in Fig. 18(b). When the SCR is forward biased, that is, when the anode is positive with respect to the cathode, four distinct regions of operation are evident.

From point 1 to point 2, junctions J_1 and J_3 are forward biased. Junction J_2 is reverse biased. The external voltage appears primarily across the reverse-biased junctions. The device continues to operate similarly to a reverse-biased *pn* junction. This is called the *off* or *high-impedance region*.

From point 2 to point 3, the current increases slowly to the *breakover voltage*, V_{BO}. At this point, junction J_2 undergoes breakdown and the current increases sharply.

From point 3 to point 4, the current increases as the voltage decreases. This region is called the *negative resistance region*.

From point 4, junction J_2 is forward biased. The voltage across the device is essentially that of a forward-biased *pn* junction (approximately 0.7 V). If the current through the diode is reduced by the external circuit, the diode remains on until the current falls below the *hold current*, I_H, or the *hold voltage*, V_H. Below this point, the diode switches off, to the high-impedance state.

The gate functions to increase the current at the collector junction of the *npn* transistor, J_2, which is an integral part of the SCR. By increasing the current through the reverse-biased junction, J_2, the anode current is increased. This increases the gain of the two transistors, resulting in a lowering of the forward breakover voltage. This is seen in the difference between paths A and B in Fig. 18(b). Consequently, for a given anode to cathode voltage, the gate can be used to turn the SCR on.

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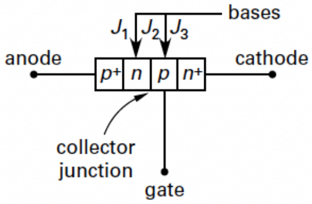
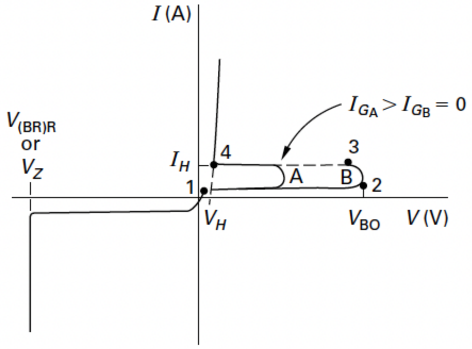
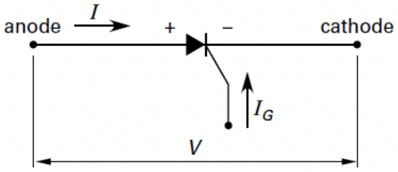
[28] A thyristor is defined as a transistor with thyratron-like characteristics. That is, as collector current is increased to a critical value, the alpha (common base current gain) rises above unity and results in a high-speed triggering action.



(a) conceptual construction



(b) characteristics



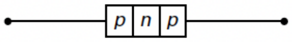
(c) symbol

Figure 18: Silicon-Controlled Rectifier

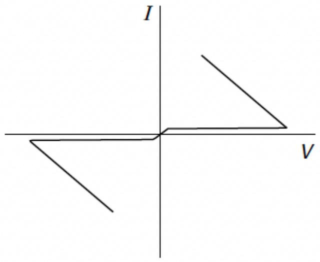
Once on, however, the SCR must be reverse biased to turn off. A thyristor designed to be turned off by the gate is called a *gate turn-off thyristor*.

SCRs are used in power applications to allow small voltages and currents to control much larger electrical quantities. The symbol for an SCR is shown in Fig. 18(c) .

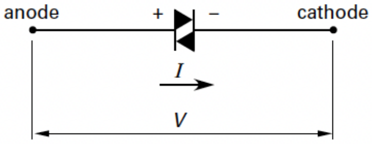
Other devices with multiple *pn* junctions using the same principles are the *diac* and *triac*, shown in Fig. 19 and Fig. 20, respectively.



(a) conceptual construction

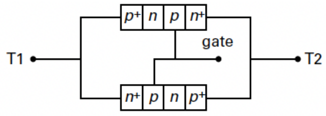


(b) characteristics

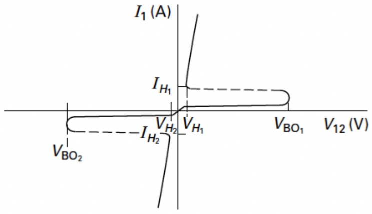


(c) symbol

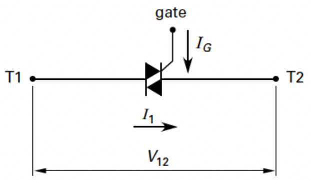
Figure 19: Diac



(a) conceptual construction



(b) characteristics



(c) symbol

Figure 20: Triac

Most electronic components are unable to handle large amounts of current. When properly designed for power dissipation, semiconductors handling large amounts of power are called *power semiconductors*. Such devices constitute a branch of electronics called *power electronics*. Silicon-controlled rectifiers and Schottky diodes are traditional power semiconductors. Newer designs include the *high-power bipolar junction transistor* (HPBT), power metal-oxide semiconductor field-effect transistor (MOSFET), *gate turn-off thyristor* (GTO), and *insulated gate bipolar transistor* (IGBT), sometimes called a *conductivity-modulated field-effect transistor* (COMFET).

Power semiconductors are classified as either trigger or control devices. *Trigger devices*, such as GTOs, start conduction by some trigger input and then behave as diodes. *Control devices* are normally BJTs and FETs used in full-range amplifiers.

REFERENCES

Items (latest editions) in **bold** are highly recommended for in-depth study.

A. Camara, John A. *Electronics, Controls, and Communications Manual, 2nd ed.* Belmont, CA: PPI, 2020.
B. Camara, John A. *PE Power Reference Manual*. Belmont, CA: PPI (Kaplan), 2021.
C. Marne, David J., and John A. Palmer. *National Electrical Safety Code®* (NESC®) *2023 Handbook*. New York: McGraw Hill, 2023.
D. Earley, Mark, ed. *NFPA 70, National Electrical Code Handbook*. Quincy, Massachusetts: NFPA, 2020.

NOTE

Electrical refers to something related to electricity while "electric" refers to a device or machine that runs on electricity. Nevertheless, the NEC is sometimes referred to as the National Electric Code.

E. IEEE 315-1975. *Graphic Symbols for Electrical and Electronics Diagrams.* New York: IEEE, approved 1975, reaffirmed 1993.
F. IEEE 280-2021. *IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering.* New York: IEEE.
G. Grainger, John J., and William Stevenson, Jr. *Power System Analysis*. New York, McGraw Hill, 1994.
H. Parker, Sybil P., editor in chief. *McGraw-Hill Dictionary of Scientific and Technical Terms,* 5th ed. New York, McGraw-Hill, 1994.

Appendix A: Equivalent Units Of Derived And Common SI Units

Symbol	Equivalent Units			
A	C/s	W/V	V/Ω	$\mathrm{J/(s\cdot V)}$
C	$\mathrm{A\cdot s}$	J/V	$\mathrm{(N\cdot m)/V}$	$\mathrm{V\cdot F}$
F	C/V	$\mathrm{C^2/J}$	s/Ω	$\mathrm{(A\cdot s)/V}$
F/m	$\mathrm{C/(V\cdot m)}$	$\mathrm{C^2/(J\cdot m)}$	$\mathrm{C^2/(N\cdot m^2)}$	$\mathrm{s/(\Omega\cdot m)}$
H	W/A	$\mathrm{(V\cdot s)/A}$	$\mathrm{\Omega\cdot s}$	$\mathrm{(T\cdot m^2)/A}$
Hz	1/s	$\mathrm{s^{-1}}$	cycles/s	$\mathrm{radians/(2\pi\cdot s)}$
J	$\mathrm{N\cdot m}$	$\mathrm{V\cdot C}$	$\mathrm{W\cdot s}$	$\mathrm{(kg\cdot m^2)/s^2}$
$\mathrm{m^2/s^2}$	J/kg	$\mathrm{(N\cdot m)/kg}$	$\mathrm{(V\cdot C)/kg}$	$\mathrm{(C\cdot m^2)/(A\cdot s^3)}$
N	J/m	$\mathrm{(V\cdot C)/m}$	$\mathrm{(W\cdot C)/(A\cdot m)}$	$\mathrm{(kg\cdot m)/s^2}$
$\mathrm{N/A^2}$	$\mathrm{Wb/(N\cdot m^2)}$	$\mathrm{(V\cdot s)/(N\cdot m^2)}$	T/N	$\mathrm{1/(A\cdot m)}$
Pa	$\mathrm{N/m^2}$	$\mathrm{J/m^3}$	$\mathrm{(W\cdot s)/m^3}$	$\mathrm{kg/(m\cdot s^2)}$
Ω	V/A	$\mathrm{W/A^2}$	$\mathrm{V^2/W}$	$\mathrm{(kg\cdot m^2)/(A^2\cdot s^3)}$
S	A/V	1/Ω	$\mathrm{A^2/W}$	$\mathrm{(A^2\cdot s^3)/(kg\cdot m^2)}$
T	$\mathrm{Wb/m^2}$	$\mathrm{N/(A\cdot m)}$	$\mathrm{(N\cdot s)/(C\cdot m)}$	$\mathrm{kg/(A\cdot s^2)}$
V	J/C	W/A	C/F	$\mathrm{(kg\cdot m^2)/(A\cdot s^3)}$
V/m	N/C	$\mathrm{W/(A\cdot m)}$	$\mathrm{J/(A\cdot m\cdot s)}$	$\mathrm{(kg\cdot m)/(A\cdot s^3)}$
W	J/s	$\mathrm{V\cdot A}$	V^2/Ω	$\mathrm{(kg\cdot m^2)/s^3}$
Wb	$\mathrm{V\cdot s}$	$\mathrm{H\cdot A}$	$\mathrm{T/m^2}$	$\mathrm{(kg\cdot m^2)/(A\cdot s^2)}$

Appendix B: Physical Constants

Table Note 1

Quantity	Symbol	US Customary	SI Units
Charge			
electron	e		$-1.6022\square 10^{-19}$ C
proton	p		$+1.6022\square 10^{-19}$ C
Density			
air [STP][32°F, (0°C)]		0.0805 lbm/ft^3	1.29 kg/m^3
air [70°F, (20°C), 1 atm]		0.0749 lbm/ft^3	1.20 kg/m^3
sea water		64 lbm/ft^3	1025 kg/m^3
water [mean]		62.4 lbm/ft^3	1000 kg/m^3
Distance			
Earth radius^2	⊕	$2.09\square 10^{7}$ ft	$6.370\square 10^{6}$ m
Earth-Moon separation^2	⊕☾	$1.26\square 10^{9}$ ft	$3.84\square 10^{8}$ m
Earth-Sun separtion^2	⊕⊙	$4.89\square 10^{11}$ ft	$1.49\square 10^{11}$ m
Moon radius^2	☾	$5.71\square 10^{6}$ ft	$1.74\square 10^{6}$ m
Sun radius^2	⊙	$2.28\square 10^{9}$ ft	$6.96\square 10^{8}$ m
first Bohr radius	a_0	$1.736\square 10^{-10}$ ft	$5.292\square 10^{-11}$ m
Gravitational Acceleration			
Earth [mean]	g	32.174 (32.2) ft/sec^2	9.8067 (9.81) m/s^2
Mass			
atomic mass unit	μ or m_μ $\frac{1}{12}m\left({}^{12}C\right)$	$3.66\square 10^{-27}$ lbm	$1.6606\square 10^{-27}$ kg or 10^{-3} kg mol^{-1} / N_A or 931.481 MeV
Earth^2	⊕	$4.11\square 10^{23}$ slugs	$6.00\square 10^{24}$ kg
Earth [customary U.S.]^2	⊕	$1.32\square 10^{25}$ lbm	-
Moon^2	☾	$1.623\square 10^{23}$ lbm	$7.36\square 10^{22}$ kg
Sun^2	⊙	$4.387\square 10^{30}$ lbm	$1.99\square 10^{30}$ kg
electron rest mass	m_e	$2.008\square 10^{-30}$ lbm	$9.109\square 10^{-31}$ kg [0.511 MeV]
neutron rest mass	m_n	$3.693\square 10^{-27}$ lbm	$1.675\square 10^{-27}$ kg [939.6 MeV]
proton rest mass	m_p	$3.688\square 10^{-27}$ lbm	$1.672\square 10^{-27}$ kg [938.2 MeV]

Quantity	Symbol	US Customary	SI Units
Pressure			
atmospheric		14.696 (14.7) lbf/in^2	$1.0133\square10^{5}$ Pa
Temperature			
standard		32˚F (492˚R)	0˚C (273 K)
absolute zero		–459.67˚F (0˚R)	–273.16˚C (0 K)
Velocity[3]			
Earth escape		$3.67\square10^{4}$ ft/sec	$1.12\square10^{4}$ m/s
light (vacuum)	c, c_0	$9.84\square10^{8}$ ft/sec	2.9979 (3.00) $\square10^{8}$ m/s
sound [air, STP]	a	1090 ft/sec	331 m/s
sound [air, 70°F, (20°C), 1 atm]		1130 ft/sec	344 ft/s
Volume			
Volume: molal ideal gas (STP)[4]		359 ft^3 / lbmol	22.41 m^3/kmol

Table 1 Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.
2. Symbols shown for the solar system are those used by NASA. See https://science.nasa.gov/resource/solar-system-symbols/.
3. Velocity technically is a vector. It has direction.
4. The unit “lbmol” is an actual unit, not a misspelling.

Appendix C: Fundamental Constants

Quantity	Symbols	US Customary	SI Units
Avogadro's number	N_A, L		6.022×10^{23} mol^{-1}
Bohr magneton	μ_B		9.2732×10^{-24} J/T
Boltzmann constant	κ	5.65×10^{-24} ft-lbf/ R	1.3805×10^{-23} J/T
electron volt: $\left(\frac{e}{C}\right)$ J	eV		1.602×10^{-19} J
Faraday constant, $N_A e$	F		96485 C/mol
fine structure constant, inverse α^{-1}	α α^{-1}		7.297×10^{-3} (≈1/137) 137.035
gravitational constant	g_c	32.174 lbm-ft/lbf-sec^2	
Newtonian gravitational constant	G	3.44×10^{-8} ft^4 / lbf-sec^4	6.672×10^{-11} N$\cdot$m^2 / kg^2
nuclear magneton	μ_N		5.050×10^{-27} J/T
permeability of a vacuum	μ_0		1.2566×10^{-6} N/A^2 (H/m)
permittivity of a vacuum, electric constant $1/\mu_0 c^2$	ε_0		8.854×10^{-12} C^2 / N$\cdot$m^2 (F/m)
Planck's constant	h		6.6256×10^{-34} J$\cdot$s
Planck's constant: h/2π			1.0546×10^{-34} J$\cdot$s
Rydberg constant	R_∞		1.097×10^{7} m^{-1}
specific gas constant, air	R	53.3 ft-lbf/lbm- R	287 J/kg$\cdot$K
Stefan-Boltzmann constant		1.71×10^{-9} BTU/ft^2-hr-°R^4	5.670×10^{-8} W/m^2$\cdot$K^4
triple point, water		32.02 F, 0.0888 psia	0.01109 C, 0.6123 kPa
universal gas constant	R*	1545 ft-lbf/lbmol- R 1.986 BTU/lbmol- R	8314 J/kmol$\cdot$K

Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/. The unit in Volume of "lbmol" is an actual unit, not a misspelling.

Appendix D: Mathematical Constants, Signs/Symbols, Maxwell's Equations

Quantity	Symbol	Value
Archimedes' constant (pi)	π	3.1415926536
base of natural logs	e	2.7182818285
Euler's constant	C or τ	0.5772156649

Signs/Symbols	Meaning
$\cdot$	multiplied by
/	divided by
:	ratio
$\gg$	much greater than
$\ll$	much less than
$=$	equals
$\equiv$	identical with
$\sim$	similar to
$\approx$	approximately equals
$\cong$	approximately equals, congruent
$\rightarrow$, $\doteq$	approaches
$\propto$	proportional, varies as
$\therefore$	therefore

Maxwell's Equations

integral form	point form	remarks
$\oint_s \mathbf{D}\cdot d\mathbf{s} = \int_V \rho\, dv$	$\nabla\cdot\mathbf{D} = \rho$	Gauss' law
$\oint_s \mathbf{B}\cdot d\mathbf{s} = 0$	$\nabla\cdot\mathbf{B} = 0$	nonexistence of magnetic monopoles
$\oint \mathbf{E}\cdot d\mathbf{l} = \int_s \left(\frac{-\partial\mathbf{B}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$	Faraday's law
$\oint \mathbf{H}\cdot d\mathbf{l} = \int_s \left(\mathbf{J}_c + \frac{\partial\mathbf{D}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{H} = \mathbf{J}_c + \frac{\partial\mathbf{D}}{dt}$	Ampère's law

Free-Space Form

integral form	point form
$\oint_s \mathbf{D}\cdot d\mathbf{s} = 0$	$\nabla\cdot\mathbf{D} = 0$
$\oint_s \mathbf{B}\cdot d\mathbf{s} = 0$	$\nabla\cdot\mathbf{B} = 0$
$\oint \mathbf{E}\cdot d\mathbf{l} = \int_s \left(\frac{-\partial\mathbf{B}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
$\oint \mathbf{H}\cdot d\mathbf{l} = \int_s \left(\frac{\partial\mathbf{D}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{H} = \frac{\partial\mathbf{D}}{dt}$

Electromagnetic Field Vector Equations

$$\mathbf{D} = \epsilon\mathbf{E} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon_0(1 + \chi_e)\mathbf{E}$$

$$\mathbf{B} = \mu\mathbf{H} = \mu_0\mathbf{H} + \mu_0\mathbf{M} = \mu_0(1 + \chi_m)\mathbf{H}$$

$$\mathbf{J} = \sigma\mathbf{E} = \rho\mathrm{v}$$

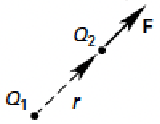
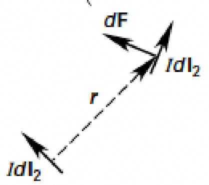
Appendix E: The Greek Alphabet

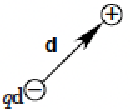
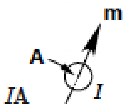
A	α	alpha	N	υ	nu
B	β	beta	Ξ	ξ	xi
Γ	γ	gamma	O	o	omicron
Δ	δ	delta	Π	π	pi
E	ε	epsilon	P	ρ	rho
Z	ζ	zeta	Σ	σ	sigma
H	η	eta	T	τ	tau
Θ	θ	theta	ϒ	υ	upsilon
I	ι	iota	Φ	ϕ	phi
K	κ	kappa	X	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
M	μ	mu	Ω	ω	omega

Appendix F: SI Prefixes

symbol	prefix	value
a	atto	10^{-18}
f	femto	10^{-15}
p	pico	10^{-12}
n	nano	10^{-9}
μ	micro	10^{-6}
m	milli	10^{-3}
c	centi	10^{-2}
d	deci	10^{-1}
da	deka	10
h	hecto	10^{2}
k	kilo	10^{3}
M	mega	10^{6}
G	giga	10^{9}
T	tera	10^{12}
P	peta	10^{15}
E	exa	10^{18}

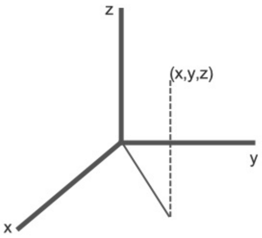
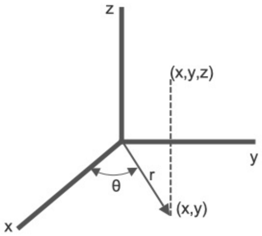
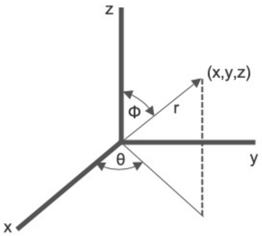
Appendix G: Comparison of Electric & Magnetic Equations

equation description	electric version	magnetic version	remarks
experimental force law	Coulomb's law $\mathbf{F} = \left(\frac{Q_1 Q_2}{4\pi\epsilon r^2}\right)\mathbf{r}$ 	force between two current elements $d\mathbf{F} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I_2 d\mathbf{l}_2 \times (I_1 d\mathbf{l}_1 \times r)}{r^2}\right)$ 	The term $I d\mathbf{l}$ in the magnetic column is the equivalent of a "magnetic charge" q_m. The I or the $d\mathbf{l}$ can be the vector. The r is a unit vector pointing from 1 to 2.
field definitions from force law	$\mathbf{F} = Q\mathbf{E}$	$d\mathbf{F} = \mathbf{I}\times\mathbf{B}\,d l$ current element $d\mathbf{F} = \mathbf{J}\times\mathbf{B}\,dV$ distributed current element $d\mathbf{F} = q\,\mathbf{v}\times\mathbf{B}$ moving charge	The V used in this row represents volume, not voltage. The v is the velocity.
general force law	$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$ $d\mathbf{F} = (\rho\mathbf{E} + \mathbf{J}\times\mathbf{B})\,dV$ where $dQ = \rho\,dV$		The V in this row represents the volume, not voltage. The v is the velocity.
definition of scalar and vector potential	$E = -\nabla V$	$\mathbf{B} = \nabla \times \mathbf{A}$	A is the magnetic vector potential.
Poisson's equation for the potential function	$\nabla^2 V = -\frac{\rho}{\epsilon}$	$\nabla^2\mathbf{A} = -\mu_0\mathbf{J}$	From a knowledge of the charge distribution, the potential can be found and then the E and B fields determined.
Gauss's law enclosing charge and Ampère's law enclosing current	$\iint \mathbf{D}\cdot d\,\mathbf{A} =$ $\iiint \rho dV = Q$ $\nabla\cdot\mathbf{D} = \rho$	$\oint H\cdot d\mathbf{l} = I$ $\nabla \times \mathbf{H} = \mathbf{J}$	The V in this row represents volume.
constitutive relations	$\mathbf{D} = \epsilon\mathbf{E}$ $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$	$\mathbf{B} = \mu\mathbf{H}$ $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$	The second set of equations is always valid. The first set assumes the medium is linear and isotropic.
definitions of relative permittivity and permeability	$\epsilon_r = \frac{\epsilon}{\epsilon_0}$ $\epsilon_0 =$ 8.854×10^{-12} F/m	$\mu_r = \frac{\mu}{\mu_0}$ $\mu_0 = 4\pi \times 10^{-7}$ H/m	

equation description	electric version	magnetic version	remarks
capacitance and inductance of a field cell	$\epsilon_0 = \frac{C}{l}$	$\mu_0 = \frac{L}{l}$	Field cells are a construct designed to represent free space in terms of a parallel plate capacitor and an inductor. This capacitance and inductance exist regardless of the presence of an electric or magnetic field.
capacitance and inductance	$C = \frac{Q}{V}$	$L = \frac{\Lambda}{I}$	Λ is the flux linkage.
energy density of a field	$U = \frac{1}{2}\epsilon E^2$	$U = \frac{1}{2}\mu H^2$	Both energy and momentum are carried by a field.
energy stored by capacitance and inductance	$W = \frac{1}{2}CV^2$	$W = \frac{1}{2}LI^2$	
electromotive and magnetomotive force with sources present	$\oint \mathcal{E} \cdot d\mathbf{l} = \mathcal{E} = V$	$\oint \mathbf{H} \cdot d\mathbf{l} = NI = F_m = V_m$	The $\mathcal{E}$ is the emf, not the permittivity. Without sources present, both line integrals are equal to zero.
dipole moments	$\mathbf{p} = q\mathbf{d}$ 	$\mathbf{m} = I\mathbf{A}$ 	
dipole torque	$\mathbf{T} = \mathbf{p} \times \mathbf{E}$	$\mathbf{T} = \mathbf{m} \times \mathbf{B}$	This torque occurs due to the dipole being immersed in an external **E** or **B** field.
dipole potential energy	$W = -\mathbf{p} \cdot \mathbf{E}$	$W = -\mathbf{m} \cdot \mathbf{B}$	

electric	magnetic
emf $= V = IR$	mmf $= V_m = \phi \mathcal{R}$
current I	flux ϕ
emf $\mathcal{E}$ or V	mmf V_m
resistance $R = \rho l/A = l/\sigma A$	reluctance $\mathcal{R} = l/\mu A$
resistivity ρ	reluctivity $1/\mu$
conductance $G = 1/R$	permeance $P_m = \mu A/l$
conductivity $\sigma = 1/\rho$	permeability μ

Appendix H: Coordinate Systems and Related Operations

Mathematical Operations	Rectangular Coordinates	Cylindrical Coordinates	Spherical Coordinates
Conversion to Rectangular Coordinants	 $x = x$ $y = y$ $z = z$	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	 $x = r\sin\phi\cos\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\phi$
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\boldsymbol{\theta} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\boldsymbol{\phi} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \theta}\boldsymbol{\theta}$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial\left(rA_r\right)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial\left(r^2 A_r\right)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial\left(A_\phi \sin\phi\right)}{\partial \phi} + \frac{1}{r\sin\phi}\frac{\partial A_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{r} & \boldsymbol{\theta} & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & A_\theta & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r^2\sin\theta}\mathbf{r} & \frac{1}{r^2\sin\theta}\boldsymbol{\phi} & \frac{1}{r}\boldsymbol{\theta} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ A_r & rA_\phi & rA\theta A_\phi \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial r}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2 f}{\partial \theta^2}\right)$