A SunCam Online Continuing Education Course

Electronics

Course I Fundamentals

Course II Fundamental Devices—Parts I, II, III

Course III Miscellany Devices—Parts IV, V, VI

Fundamental Devices—Electrical & Electronic / Operational Amplifiers
Pulse Circuits / Diodes / Zeners / Switches / Logic Families / MOS / CMOS

Miscellany Devices—Multivibrators / Phase Locked Loops / RTD / Ladder Logic

by

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Nomenclature[1]

A	constant	$K^{-8} \cdot m^{-8}$
A	gain	-
BW	bandwidth	Hz
C	capacitance	F
c, c	speed of light, 2.9979×10^{8}	m/s
$CMRR$	common-mode rejection ratio	-
D	diffusion constant	m^2 / s
E	energy	J or eV
E	source voltage	V
E, **E**	electric field strength	V/m
f	frequency	Hz
F_m	figure of merit	rad/s
G	gain	-
GBW	gain bandwidth	rad/s
h	Planck's constant, 6.6256×10^{-34}	$J \cdot s$
H	high	V
h_{fe}	CE small-signal (AC) forward current transfer ratio or gain	-
h_{FE}	CE small-signal (DC) forward current transfer ratio or gain	-
i	instantaneous or variable current	A
I, **I**	constant or rms current or effective or DC current	A
I_{CBO}	reverse saturation current	A
I_{EBO}	DC emitter cutoff current	A
I_{ZK}	keep-alive current	A
n	electron concentration [negative charge concentration]	cm^{-3}
n, N	concentration	m^{-3}
N	fan-out (number of loads)	-
p	hole concentration [positive charge concentration]	cm^{-3}
p	pole strength	Wb (N/T)
P	polarization	C/m^2
P	power	W
p^+	charge on a proton, 1.6022×10^{-19}	C
q	electric unit charge; charge of an electron, -1.6022×10^{-19}	C
q, Q	charge, electric unit charge 1.6022×10^{-19}	C

[1] Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related and may be found when reviewing the references listed for further information. Further, all the nomenclature, symbols, or subscripts will be found in of many electrical courses (on SunCam, PDH Academy, and also in many texts). For guidance on nomenclature, symbols, and electrical graphics: IEEE 280-2021, IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering, New York: IEEE; and IEEE 315-1975, Graphic Symbols for Electrical and Electronics Diagrams, New York: IEEE, approved 1975, reaffirmed 1993.

r, R	resistance	Ω
S_R	slew rate	V/s
T	temperature	K
t	time	s
T	temperature	K
U	energy	J
υ or *v*	instantaneous or variable voltage	V
V	constant or rms voltage or effective or DC voltage	V
VR	voltage regulation	%
V_T	voltage equivalent of temperature	V
Z	impedance	Ω

Symbols

α	CB forward current ratio or gain	-
β	CE forward current ratio or gain	-
δ	feedback factor	-
ε	permittivity	F/m $\left(C^2 / N \cdot m^2\right)$
ε_0	permittivity of free-space, 8.854×10^{-12}	F/m $\left(C^2 / N \cdot m^2\right)$
κ	Boltzmann's constant, 1.3807×10^{-23}	J/K
μ	mobility	$m^2 / V \cdot s$
μ	permeability	H/m $\left(N / A^2\right)$
μ_0	free-space permeability, 1.2566×10^{-6}	H/m $\left(N / A^2\right)$
σ	conductivity	S/m
ω	angular frequency	rad/s

Subscripts

0	original, origin, zero value, at-rest value, evaluated value, at 0K, barrier value, resonance	-
A	acceptor	-
A,B	input A, input B	-
ac	ac current or small signal	=
amp	amplifier	-
ave	average	-
B	DC or biased base	-
b, B	base	-
BB	base supply or base biasing	-
be, BE	base to emitter	-
BO	breaker over	-
BQ	base or quiescent point	-
c	conduction band or AC or small-signal collector	-
C	collector	-
C	DC or biased collector	-
c, C	collector or control	-
CB	collector to base	-
CC	collector supply	-
CE	collector to emitter	-
cl	current limiting	-
cm	common mode	-
co, CO	cutoff	-
d	diffusion	-
D	donor, diode, or drain	-
d	delay	-
D	diode	-
DC	direct current	-
DD	drain supply	-
dm	differential mode	-
e	AC or small-signal emitter	-
E	DC or biased emitter	-
e,E	emitter	-
EBO	emitter to base, collector open	-
EE	emitter supply	-
f	forward (AC or instantaneous)	-
F	forward (DC component) or total	-
f	fall, feedback, or forward (AC or instantaneous)	-
F	forward (DC component or total)	-
fl	full load	-

G	gap or gate	-
GS	gate to source	-
H	hold	-
i	intrinsic or input (AC or instantaneous)	-
I, I	current	-
L	load	-
m	maximum	-
m	merit	-
max	maximum	-
min	minimum	-
n	electrons or n-type	-
n	noise	-
nl	no load	-
p	holes or p-type	-
p	power	-
pn	p to n	-
r	reverse (AC or instantaneous)	-
R	reverse (DC component or total)	-
r	rise	-
R	resistance	-
ref	reference	-
RTD	resistance temperature detector	-
s	saturation	-
S	source	-
t	transition	-
T	thermal	-
u	unity	-
υ, ν, ʋ	instantaneous voltage	-
v	valence band	-
V	constant or effective voltage	-
V	voltage	-
Z	zener	-
ZK	zener knee	-
ZM	zener at maximum rated current/voltage	-

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COURSE INTRODUCTION

The theoretical information is primarily from the author's books, Refs. [A] and [B]. The NESC Ref. [C] and NEC Ref. [D] though not covered in this course are useful sources for electrical engineers. Information useful in many aspects of electric engineering may be found in [E] and [F]. Reference [G] has detailed descriptions of analysis techniques. Reference [H] covers many terms in EE with excellent definitions and explanations. The appendices cover information useful in many engineering tasks with App. G providing a side by side comparison of electric and magnetic equations. Use these texts or their counterparts for indepth information. References in bold are highly recommended.

This course will focus on basics, that rarely change, and provide the basis for all other knowledge.

OVERVIEW & PERIODIC TABLE

Electronics involves charge motion through materials other than metals, such as vacuums, gases, or semiconductors. The focus in this course will be on semiconductor materials. Because an understanding of the electron structure is vital to understanding electronics, a periodic table of the elements is shown below.

The Periodic Table of Elements (Long Form)

The number of electrons in filled shells is shown in the column at the extreme left; the remaining electrons for each element are shown immediately below the symbol for each element. Atomic numbers are enclosed in brackets. Atomic weights (rounded, based on carbon-12) are shown above the symbols. Atomic weight values in parentheses are those of the isotopes of longest half-life for certain radioactive elements whose atomic weights cannot be precisely quoted without knowledge of origin of the element.

metals | transition metals | nonmetals

periods	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 0	1.00794 H[1] 1																	4.00260 He[2] 2
2 2	6.941 Li[3] 1	9.01218 Be[4] 2											10.811 B[5] 3	12.0107 C[6] 4	14.0067 N[7] 5	15.9994 O[8] 6	18.9984 F[9] 7	20.1797 Ne[10] 8
3 2,8	22.9898 Na[11] 1	24.3050 Mg[12] 2											26.9815 Al[13] 3	28.0855 Si[14] 4	30.9738 P[15] 5	32.065 S[16] 6	35.453 Cl[17] 7	39.948 Ar[18] 8
4 2,8	39.0983 K[19] 8,1	40.078 Ca[20] 8,2	44.9559 Sc[21] 9,2	47.867 Ti[22] 10,2	50.9415 V[23] 11,2	51.9961 Cr[24] 13,1	54.9380 Mn[25] 13,2	55.845 Fe[26] 14,2	58.9332 Co[27] 15,2	58.6934 Ni[28] 16,2	63.546 Cu[29] 18,1	65.38 Zn[30] 18,2	69.723 Ga[31] 18,3	72.64 Ge[32] 18,4	74.9216 As[33] 18,5	78.96 Se[34] 18,6	79.904 Br[35] 18,7	83.798 Kr[36] 18,8
5 2,8,18	85.4678 Rb[37] 8,1	87.62 Sr[38] 8,2	88.9059 Y[39] 9,2	91.224 Zr[40] 10,2	92.9064 Nb[41] 12,1	95.96 Mo[42] 13,1	(98) Tc[43] 14,1	101.07 Ru[44] 15,1	102.906 Rh[45] 16,1	106.42 Pd[46] 18	107.868 Ag[47] 18,1	112.411 Cd[48] 18,2	114.818 In[49] 18,3	118.710 Sn[50] 18,4	121.760 Sb[51] 18,5	127.60 Te[52] 18,6	126.904 I[53] 18,7	131.293 Xe[54] 18,8
6 2,8,18	132.905 Cs[55] 18,8,1	137.327 Ba[56] 18,8,2	* (57-71)	178.49 Hf[72] 32,10,2	180.948 Ta[73] 32,11,2	183.84 W[74] 32,12,2	186.207 Re[75] 32,13,2	190.23 Os[76] 32,14,2	192.217 Ir[77] 32,15,2	195.084 Pt[78] 32,17,1	196.967 Au[79] 32,18,1	200.59 Hg[80] 32,18,2	204.383 Tl[81] 32,18,3	207.2 Pb[82] 32,18,4	208.980 Bi[83] 32,18,5	(209) Po[84] 32,18,6	(210) At[85] 32,18,7	(222) Rn[86] 32,18,8
7 2,8,18,32	(223) Fr[87] 18,8,1	(226) Ra[88] 18,8,2	† (89-103)	(265) Rf[104] 32,10,2	(268) Db[105] 32,11,2	(271) Sg[106] 32,12,2	(272) Bh[107] 32,13,2	(270) Hs[108] 32,14,2	(276) Mt[109] 32,15,2	(281) Ds[110] 32,17,1	(280) Rg[111] 32,18,1	(285) Cn[112] 32,18,2	(286) Nh[113] 32,18,3	(289) Nh[114] 32,18,4	(290) Mc[115] 32,18,5	(293) Lv[116] 32,18,6	(294) Ts[117] 32,18,7	(294) Og[118] 32,18,8

(referred to carbon-12)

*lathanide series	138.905 La[57] 18,9,2	140.116 Ce[58] 20,8,2	140.908 Pr[59] 21,8,2	144.242 Nd[60] 22,8,2	(145) Pm[61] 23,8,2	150.36 Sm[62] 24,8,2	151.964 Eu[63] 25,8,2	157.25 Gd[64] 25,9,2	158.925 Tb[65] 27,8,2	162.500 Dy[66] 28,8,2	164.930 Ho[67] 29,8,2	167.259 Er[68] 30,8,2	168.934 Tm[69] 31,8,2	173.054 Yb[70] 32,8,2	174.967 Lu[71] 32,9,2
†actinide series	(227) Ac[89] 18,9,2	232.038 Th[90] 18,10,2	231.036 Pa[91] 20,9,2	238.029 U[92] 21,9,2	(237) Np[93] 23,8,2	(244) Pu[94] 24,8,2	(243) Am[95] 25,8,2	(247) Cm[96] 25,9,2	(247) Bk[97] 26,9,2	(251) Cf[98] 28,8,2	(252) Es[99] 29,8,2	(257) Fm[100] 30,8,2	(258) Md[101] 31,8,2	(259) No[102] 32,8,2	(262) Lr[103] 32,9,2

Figure 1: Periodic Table

PART I: INTRODUCTION

An understanding of basic electronics and associated circuits is necessary for the electronics engineer who will apply this knowledge to fabricate more complex circuits from the basic building blocks. A power electrical engineer makes use of such knowledge because of the widespread use of electronic devices in instrumentation, illumination controls, battery charging automatic circuits, power electronics, variable speed drives, protective relays, transmission line monitoring, and control circuits. A computer engineer, while primarily focused on utilizing the computer to accomplish various tasks, must have an appreciation of the underlying hardware in order to take full advantage of the available capabilities.

This course develops the principles of the operational amplifier, various pulse circuits used for waveform shaping and logic circuits, and specific circuits with widespread applications. These principles are essential to an electrical engineer, as they provide a basis for understanding electronics and associated circuits.

PART II: AMPLIFIERS

Fundamentals

An amplifier is any device capable of increasing the magnitude or power level of a physical quantity. The prime concern of most amplifiers is gain, A. The gain can be a voltage (A_V), current (A_i), or power gain (A_p or G_p). The input impedance (Z_{in}) and the output impedance (Z_{out}) are also parameters of concern. The *bandwidth* is the range of frequencies for which the amplifier will respond. The bandwidth (BW) is normally defined as the difference between the upper and lower frequencies at the *3 dB half-power points*, also called the *3 dB down points*, as shown in Fig. 2.

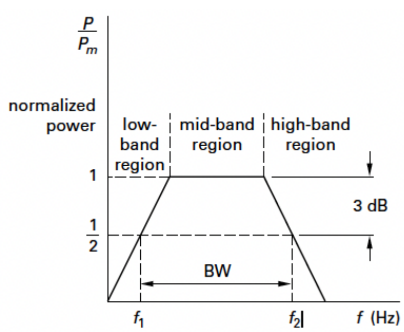

Figure 2: Bandwidth

The amplifier in Fig. 2 is a *band-pass* amplifier. There are also *low-pass*, *high-pass*, and *notch* amplifiers, though they are more commonly called *filters*. The gain-bandwidth product is used as a *figure of merit*, F_m, measured in rad/s, for band-pass and high-pass amplifiers. (Bandwidth is sometimes measured in Hz instead of rad/s, in which case the figure of merit is also given in Hz.) The *gain-bandwidth product* (GBW) for a bandpass amplifier is

Equation 1: Gain-Bandwidth Product

$$\text{GBW} = F_m = A_{ref}(\text{BW})$$

The term A_{ref} is a reference gain, which is either the maximum gain or the gain at the frequency at which the gain is purely real or purely imaginary. For a low pass amplifier, a high-frequency cutoff, ω_0, at the 3 dB down point is defined and the figure of merit is as follows.[2]

Equation 2: Figure of Merit at Cutoff

$$\text{GBW} = F_m = A_{ref}\omega_0$$

The transistors, when biased in the linear region, are amplifiers. The small-signal models given are for the midband region of Fig. 2. In this region, the reactive elements act as short circuits and so are not included in any models.[3] In the low-band region, the impedance of the reactive elements, normally the coupling capacitors and emitter bypass capacitors, is significant.[4] Capacitors must be added to the small-signal models in the low-band region, and the gain decreases.[5] In the high-band region, the impedance caused by the capacitance of the device itself becomes significant, and the gain decreases. In this manner, the device capacitances are added to the models.

Individual transistors amplify. When used in conjunction with other transistors and electronic devices, different types of amplifiers can be constructed. An amplifier with a wide range of applications is the *operational amplifier* (op amp). It takes its name from the multiple configurations with which mathematical operations may be performed. Originally designed as the major component of an analog computer, the op amp is a high-gain DC (direct-coupled or direct current) amplifier with high stability and immunity to oscillation, which uses large amounts of negative feedback. The symbols used for the op amp are shown in Fig. 3.

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[2] The 3 dB down point occurs at the resonant frequency.

[3] In *tuned circuits*, the reactive elements must be accounted for in the mid-frequency region to ensure proper design.

[4] "Significant" means the impedance seen at the terminals of the capacitor is of the same magnitude as the impedance of the capacitor itself.

[5] They would consist of the small-signal low-frequency model with coupling and emitter bypass capacitors added—the same circuit that would be used to determine the AC load line. See Ref [B].

The op amp has two input terminals. The one marked "–" is the inverting terminal; the one marked "+" is the noninverting terminal. The fundamental op amp formula follows.

Equation 3: Op Amp Formula—Constraint Equation

$$v_{\text{out}} = A_V\left(v^+ - v^-\right)$$

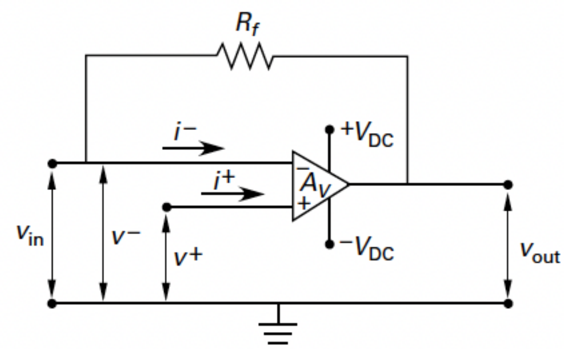


Figure 3: Operation Amplifier Symbols

The voltages marked $+V_{\text{DC}}$ and $-V_{\text{DC}}$ are the power supplies to the op amp, and are often called the *rail voltages*. The range of v_{out} is between these two voltages.[6] In fact, the output voltage should stay 3 V from either value to avoid distortion of the output signal at rated values. The range of the input signal, derived from Eq. 3 and this distortion restriction, is as follows.

Equation 4: Op Amp Input Signal Range

$$\left|v^+ - v^-\right| < \frac{V_{\text{DC}} - 3\text{ V}}{A_V}$$

When the op amp input is within the range of Eq. 4, the operation is linear. Typical values of op amp voltage gain, A_V, range from 10^5 to 10^8. Power supply voltages, $\pm V_{\text{DC}}$, are in the range of the following: ±10 V–15 V. Typical input impedances are greater than 10^5 Ω, while output impedances are very low. The internal circuits of the op amp consist of the items in the block diagram of Fig. 4.

[6] Although the gain is large, the output cannot be greater than the available voltage. The rail voltages represent ±∞ for the output.

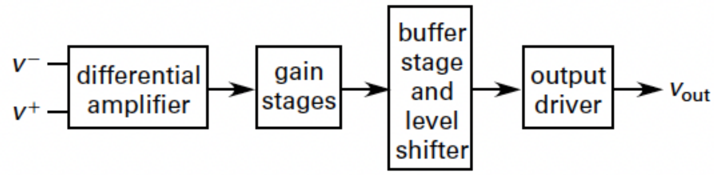


Figure 4: Operational Amplifier Internal Configuration

The *differential amplifier* constitutes the first stage of the op amp. A differential amplifier is shown in Fig. 5. The resulting output current, which is a combination of the emitter currents, is shown in Fig. 6. The linear region for operation restricts the voltage difference at the op amp input, as in Eq. 4, though here the difference is expressed in terms of the thermal voltage, V_T. Ideally, the differential amplifier amplifies only the difference between the signals. In practice, both the *differential-mode signal*, v_{dm}, and the *common-mode signal*, v_{cm}, are amplified.

Equation 5: Differential Mode

$$v_{dm} = v^{+} - v^{-}$$

Equation 6: Common Mode

$$v_{cm} = \tfrac{1}{2}\left(v^{+} - v^{-}\right)$$

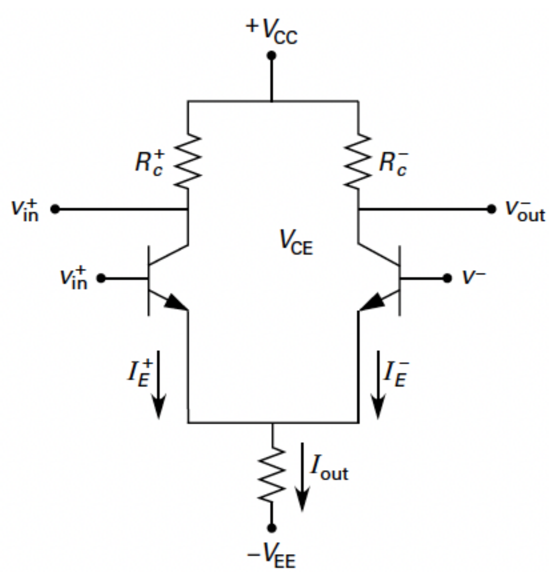


Figure 5: Differential Amplifier

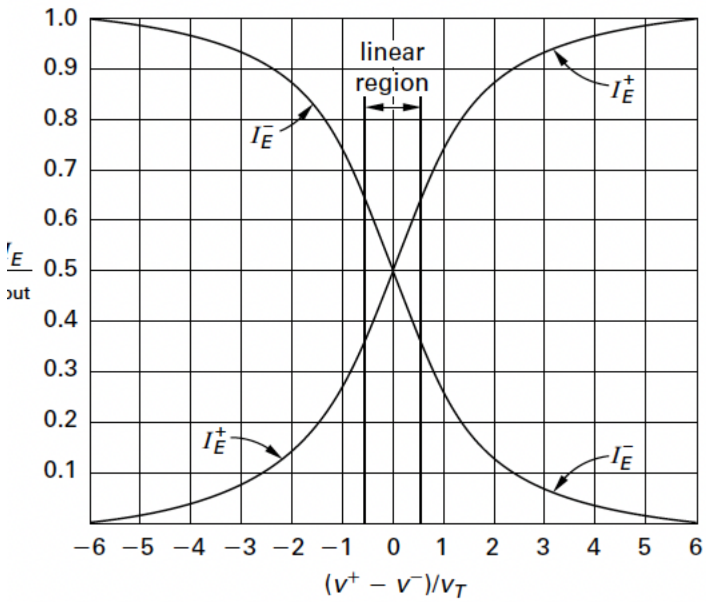


Figure 6: Differential Amplifier, Emitter Current, Ideal Current Source Biasing

To measure how well the differential amplifier amplifies the difference signal and not the common signal, the concept of the *common-mode rejection ratio* (CMRR) is used. The CMRR is defined as follows.

Equation 7: Common Mode Rejection Ratio

$$\text{CMRR} = \left| \frac{A_{dm}}{A_{cm}} \right|$$

The output voltage in terms of the CMRR is as follows.

Equation 8: Output Voltage using CMRR

$$v_{\text{out}} = A_{dm} v_{dm} \left(1 + \left(\frac{1}{\text{CMRR}} \right) \left(\frac{v_{cm}}{v_{dm}} \right) \right)$$

The gain stages in Fig. 4 supply the required amplification and represent cascaded amplifiers; for example, cascaded CE amplifiers. The buffer stage has a high input resistance to prevent it from loading the output driver; for example, an *emitter follower* (CC configuration). Circuitry associated with the buffer shifts the DC bias level so that the output is zero when the input is zero. The output driver provides the necessary large-signal current or voltage gain. For example, for a large current gain, the common collector configuration could be used.[7]

Op amps are designed to closely approximate ideal op amp behavior. Because of this and because of their versatility of use, they are widely used in place of discrete transistor amplifiers. Op amps are used in linear systems such as voltage-to-current converters, DC instrumentation, voltage followers, and filters, and in nonlinear systems such as AC/DC converters, peak detectors, sample-and-hold systems, analog multipliers, and analog-to-digital and digital-to-analog converters, among many others. A low-frequency op amp equivalent circuit is shown in Fig. 7.

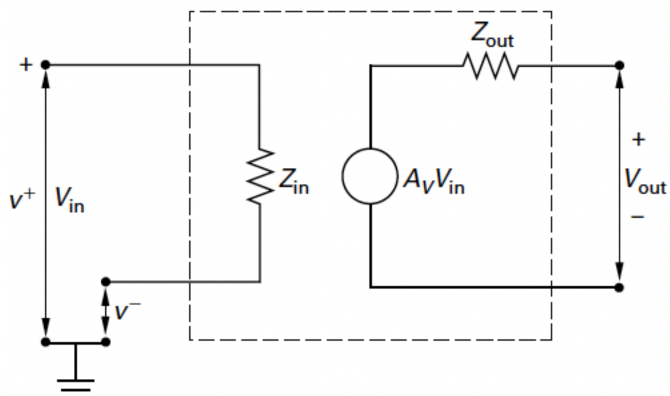


Figure 7: Operational Amplifier Equivalent Circuit

Example 1

An op amp has a voltage gain of 10^8. The DC power supply is ±15 V. What is the maximum input voltage difference for linear operation?

Solution

The input voltage difference is given by Eq. 4.

$$\left|v^+ - v^-\right| < \frac{V_{\text{DC}} - 3\text{ V}}{A_V} = \frac{15\text{ V} - 3\text{ V}}{10^8} = 0.12 \times 10^{-6}\text{ V} \quad (0.12\,\mu\text{V})$$

[7] The common collector (CC) configuration is also called the emitter follower. Voltage gain is approximately one. This means that a change in base voltage appears as an equal change across the load at the emitter. Consequently, the emitter follows the input signal.

The difference between the terminals of the op amp must be kept within 0.12 μV to remain within the linear region. (This is accomplished with negative feedback.) This voltage difference is so small that the terminals are essentially at the same voltage. *One of the assumptions for an ideal op amp is that the voltages are the same*. Consequently, if one of the terminals is connected to ground, the other terminal is also at ground potential. This situation is referred to as having a *virtual ground* at the ungrounded op amp terminal.

Example 2

The input impedance for the op amp in Ex. 1 is 10^5 Ω. What is the input current?

Solution

The voltage difference was calculated as 0.12 μV. The current between the terminals is given by Ohm's law.

$$I = \frac{V}{R} = \frac{0.12 \times 10^{-6}\ \text{V}}{10^5\ \text{V}} = 1.2 \times 10^{-12}\ \text{A} \quad (1.2\ \text{pA})$$

This current is so low as to be negligible. An *ideal op amp implies that the input current is zero.*

Example 3

If the gain given for Ex. 1 is the differential-mode gain and the CMRR is 10,000, what is the output voltage when the op amp is at the edge of linear operation and v^+ = 50 μV?

Solution

The op amp is at the edge of linear operation for the maximum voltage difference allowed by Eq.4, which was calculated in Ex. 1 as 0.12 μV. The output voltage is given by Eq. 8.

$$v_{\text{out}} = A_{dm} v_{dm} \left(1 + \left(\frac{1}{\text{CMRR}} \right) \left(\frac{v_{cm}}{v_{dm}} \right) \right)$$

The differential-mode voltage was calculated as 0.12 μV. The common-mode voltage is unknown. From Eq. 4 and Eq. 5, one gets the following.

$$
\begin{aligned}
v_{dm} &= v^{+} - v^{-} = 0.12\ \mu\text{V} = 50\ \mu\text{V} - v^{-} \\
v^{-} &= 50\ \mu\text{V} - 0.12\ \mu\text{V} = 49.88\ \mu\text{V} \\
&\therefore \\
v_{cm} &= \tfrac{1}{2}\left(v^{+} - v^{-}\right) \\
&= \frac{1}{2}\left(50\ \mu\text{V} + 49.88\ \mu\text{V}\right) \\
&\approx 50\ \mu\text{V}
\end{aligned}
$$

Substituting the calculated and given values gives the following.

$$
\begin{aligned}
v_{\text{out}} &= A_{dm} v_{dm}\left(1 + \left(\frac{1}{\text{CMRR}}\right)\left(\frac{v_{cm}}{v_{dm}}\right)\right) \\
&= \left(10^{8}\right)\left(0.12 \times 10^{-6}\ \text{V}\right)\left(1 + \left(\frac{1}{10{,}000}\right)\left(\frac{50\ \mu\text{V}}{0.12\ \mu\text{V}}\right)\right) \\
&= 12.5\ \text{V}
\end{aligned}
$$

This output voltage is within 3 V of the power supply voltage. Distortion could be caused by the nonlinear operation. In this case, because $A_{dm}v_{dm}$ = 12 V, the negative feedback must be increased.

Ideal Operational Amplifiers

An ideal operational amplifier exhibits the following properties.[8]

- $Z_{\text{in}} = \times$
- $Z_{\text{out}} = 0$
- $A_{V} = \times$
- $\text{BW} = \times$
- $V_{\text{out}} = 0$ when $v^{+} = v^{-}$
- Characteristics do not drift with *Temperature*

[8] The fifth and sixth properties here are sometimes omitted from lists of ideal op amp properties. The fifth property is true for the op amp itself, without feedback; when negative feedback is used, the output voltage is whatever value maintains $v^{+} = v^{-}$.

Because the actual op amp so closely approximates these conditions, it is possible to use ideal op amp analysis for most calculations. Only when high-frequency behavior is desired or circuit limitations have been calculated do the assumptions need to be discarded. A typical ideal op amp configuration with *voltage shunt feedback* is shown in Fig. 8.

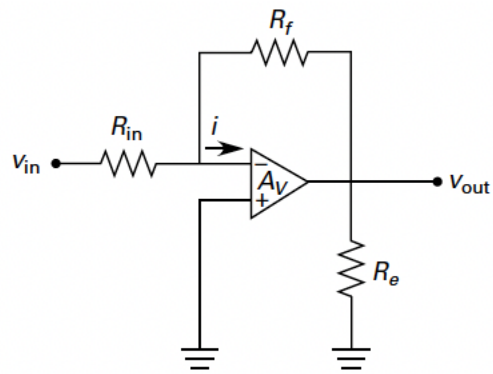


Figure 8: Ideal Operational Amplifier Typical Configuration

The assumptions regarding the properties of the ideal op amp result in the following practical results during analysis.

- The current to each input is zero.
- The voltage between the two input terminal is zero.
- The op amp is operating in the linear range.

The voltage difference of zero between the two terminals is called a *virtual short circuit*, or, because the positive terminal is often grounded, a *virtual ground*. The term "virtual" is used because, although the feedback from the output is used to keep the voltage difference between the terminals at zero, no current actually flows into the short circuit.

The procedure for analyzing an ideal op amp circuit follows.

Step 1: Draw the circuit and label all nodes, voltages, and currents of interest.

Step 2: Write Kirchhoff's current law (KCL) at the op amp input node (normally the inverting terminal).[9]

[9] The current into the node, i_{in}, is shown as negative to be consistent with the assumption that all currents into the node are negative and those out are positive. Ohm's law is written as shown to be consistent with the direction of the input current given.

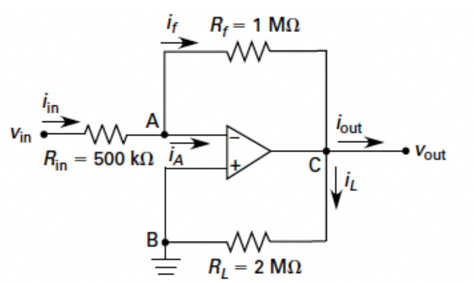
Step 3: Simplify the resulting equation using the ideal op amp assumptions. Specifically, the current into the op amp is zero and the voltage difference between the terminals is zero. The known voltage at one terminal must be the voltage at the other (e.g., if the positive terminal is at ground, the negative terminal voltage is zero).

Step 4: Solve for the desired quantity or expression.

Using the procedure given, the relationships or values for any op amp circuit parameter can be found. The relationship between the output voltage and the input voltage for a variety of op amp circuits is shown in Fig. 9 (shown after the examples).

Example 4

Consider the circuit shown. The op amp is considered ideal with a 1 μV input signal. What is the current through the feedback resistor?



Solution

The circuit is already drawn. Writing KCL (*assuming that currents out of the node are positive*) at the input terminal of the op amp, that is, node A, gives

$$-i_{\text{in}} + i_{\text{A}} + i_f = 0$$

$$i_f = i_{\text{in}} - i_{\text{A}}$$

$$= \frac{v_{\text{in}} - V_{\text{A}}}{R_{\text{in}}} - i_{\text{A}}$$

Using the ideal op amp assumptions, the input current to the op amp is zero, that is, $i_A = 0$. Also, the voltage difference between the terminals is zero. Because the positive terminal is connected to ground, $v^+ = 0 = v^- = V_A$. Substituting gives

$$\begin{aligned} i_f &= i_{in} - i_A = \frac{v_{in} - V_A}{R_{in}} - i_A \\ &= \frac{1\times 10^{-6}\ \text{V} - 0\ \text{V}}{500\times 10^{3}\ \Omega} - 0\ \text{A} \\ &= 2\times 10^{-12}\ \text{A} \end{aligned}$$

The negative sign indicates that the current direction is opposite that shown. Because i_A is zero, the input current is equal to 2×10^{-12} A.

Example 5

For the circuit shown in Example 4, determine the voltage gain.

Solution

The voltage gain is

$$A_V = \frac{v_{out}}{v_{in}}$$

The input voltage is known. The output voltage can be calculated in a number of ways. Using a portion of KCL at node A (or Ohm's law) gives

$$i_f = \frac{v_A - v_{out}}{R_f} = \frac{0\ \text{V} - v_{out}}{1\times 10^{6}\ \Omega} = 2\times 10^{-12}\ \text{A}$$

$$v_{out} = (-1)\left(2\times 10^{-12}\ \text{A}\right)\left(1\times 10^{6}\ \Omega\right) = -2\times 10^{-6}\ \text{V}$$

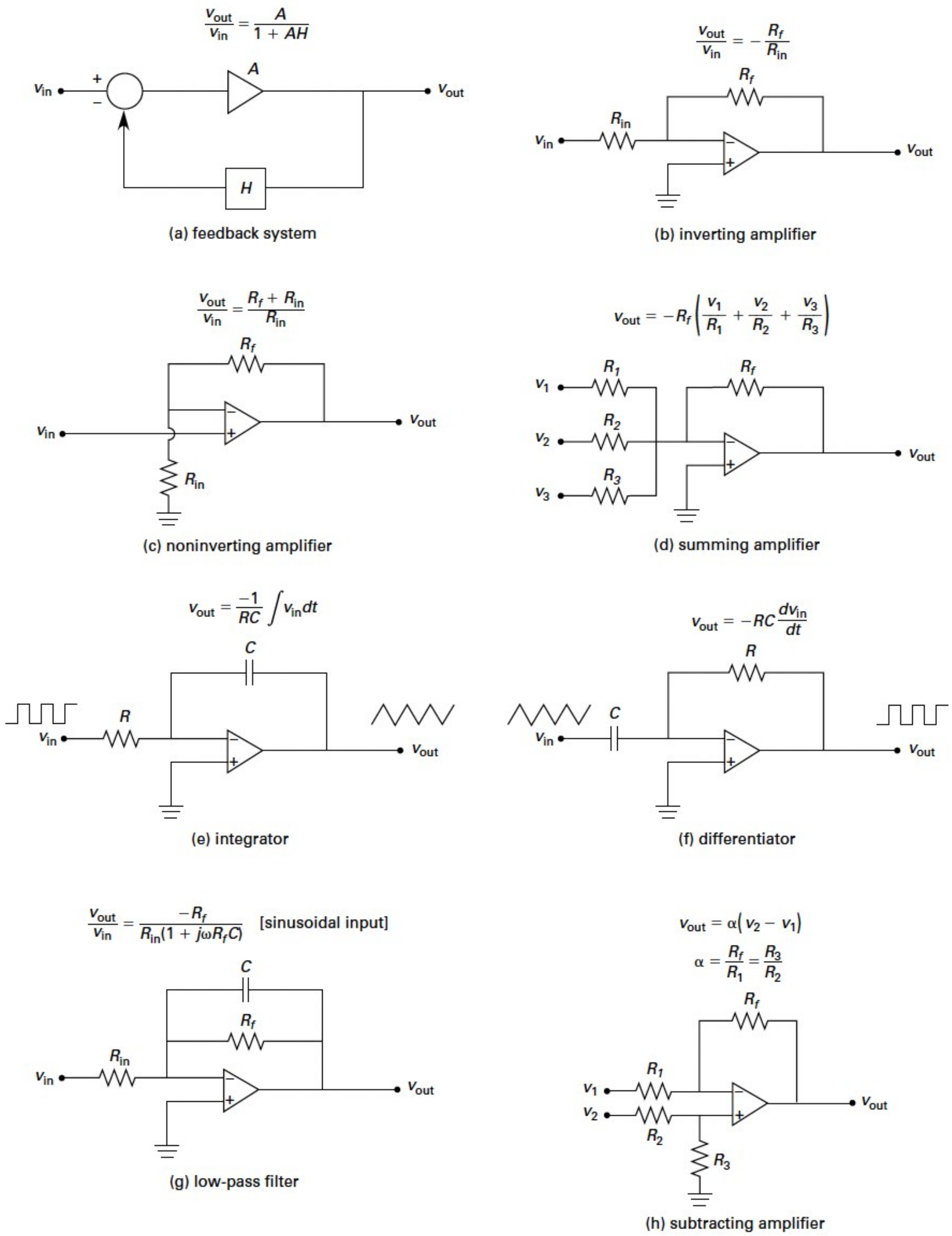


Figure 9: Operational Amplifier Circuit Types

Operational Amplifier Limits

A real op amp has limits that must be accounted for during design or use. Those limits are summarized in this section.

Op amps have a finite bandwidth. Specifically, they have a finite gain-bandwidth product. The product of the gain and the bandwidth is essentially constant (see Eq. 1 and Eq. 2). The first-order approximation of the frequency response of an amplifier is shown in Fig. 10.

The *unity gain* is defined as follows.

Equation 9: Unity Gain

$$|A| = A_0 \frac{\omega_0}{\omega_u} = 1$$

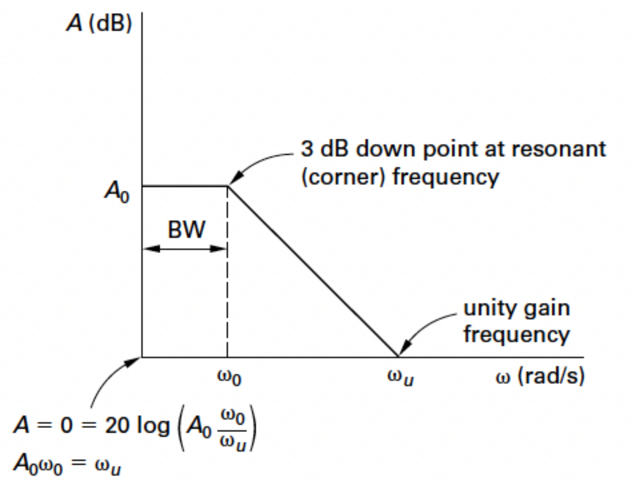


Figure 10: Operational Amplifier Frequency Response

The term ω_u is called the *unity gain frequency* and replaces $A_0\omega_0$ in Eq. 1 and Eq. 2. The unity gain frequency is used to define the bandwidth for a real op amp. As the frequency increases the gain decreases, keeping the gain bandwidth product (GBW), also called the *figure of merit* (F_m), approximately constant.[10]

The output voltage is limited to the range supply voltages. Further, to ensure operation in the linear region and prevent distortion, the output voltage should be approximately 3 V from the maximum

[10] The frequency response of any amplifier circuit can be found by transforming the external electrical components into the s domain. Then obtain the gain equation using the procedures of the previous section. Finally, plot the response on a Bode plot.

supply voltage. This limit is expressed in terms of the gain and the voltage difference that can be used at the terminal inputs (see Eq. 4 and Fig. 6). When operating outside the linear region, the op amp is considered *saturated*.

The op amp amplifies both the difference signal and the common signal. Amplifying the common signal results in problems, especially for precision applications. The common-mode rejection ratio (CMRR) is a measure of the op amp's ability to overcome this obstacle (see Eq. 7 and Eq. 8).

The gain is finite. As such, the output is less than expected (see Ex. 4). The voltage difference is determined from the fundamental op amp equation (see Eq. 3), which is also called the *constraint equation*. Nevertheless, the gain is usually so high that this is a minor problem at low frequencies.

The input currents are small but not zero when the input is zero. A voltage difference does exist when the input is zero. These currents and voltages are called *offset currents* and *offset voltages*. They are compensated for within the op amp by the level shifter (see Fundamental Section and Fig. 4).

If a large signal is applied to an op amp, driving it into saturation, the output signal is limited to the maximum current that the amplifier can supply. This means that an incoming exponential signal would become a ramp signal on the output, and a sinusoidal signal would become a sawtooth signal on the output. The slopes of the output ramps are determined by the *slew rate*, S_R. The slew rate is determined by the internal capacitance of the amplifier and is

Equation 10: Slew Rate

$$S_R = \left.\frac{dv}{dt}\right|_{\text{max}} \approx \frac{I_{\text{max}}}{C}$$

Amplifier Noise

There are numerous sources of noise in amplifiers. The *thermal noise* is produced by the random thermal agitation of electrons. It is also called the *Johnson noise* or the *resistance noise*. The thermal noise is proportional to the root-mean-square of the noise voltage squared. The thermal noise is also called the *noise power*, P_n.

Equation 11: Noise Power

$$P_n = \frac{V^2_{\text{noise, rms}}}{4R} = \kappa T\left(\text{BW}\right)$$

The *shot noise* is due to the random variations in the velocity of the electrons in a current flow. It is proportional to $2qI$. *Low-frequency noise*, also called *flicker noise*, is the noise from slow changes, or drift, in the system parameters, caused by temperature, component aging, and so on. *Interference noise* is electromagnetic induction from sources outside the amplifier. To reduce interference, ensure the *electromagnetic compatibility* of the components.

The total noise in a given circuit is compared to the incoming signal, or power, to ensure the ability of the circuit to discern between the two. The measure of this ability is the signal-to-noise ratio (SNR).

Equation 12: Signal to Noise Ratio

$$\mathrm{SNR} = 10\log\frac{P_s}{P_n} = 20\log\frac{V_s}{V_n}$$

Because the resistance is common to both the signal and the noise, the units of the signal power, P_s, and the noise power, P_n, can be expressed in watts or simply volts squared (in which case the noise power equals $4R\kappa T(\mathrm{BW})$).[11] A signal-to-noise ratio of approximately 10 dB is required for the op amp output to reflect the signal.

PART III: PULSE CIRCUITS—WAVEFORM SHAPING & LOGIC

Pulse Circuit Fundamentals

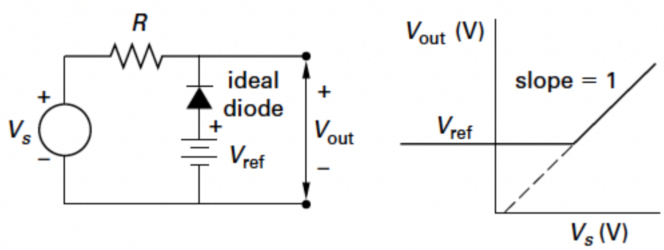
A *pulse circuit* is any active electrical network designed to respond to discrete pulses of current or voltage. Such circuits abound in computer and digital systems. Examples are waveform shaping circuits and logic circuits. An *active circuit*, such as a transistor circuit, is a circuit that is capable of amplifying a current or voltage signal. Waveform shaping is primarily accomplished with active nonlinear circuits. Active nonlinear elements are described Ref [B], though when used as switches, they are considered pulsed circuits—hence their coverage here. Passive linear elements, those elements that are not sources of energy (for example, resistors, capacitors, and inductors) are encountered in the design of pulse, waveform shaping, and logic circuits. *Passive nonlinear element*s, primarily diodes, are widely used for waveform shaping elements and elements of logic circuits. The delta (Dirac) function and the step function are used to mathematically describe the response of pulse circuits. *Pulse generators* are circuits that are designed with high-speed switches, capable of generating very short-duration pulses for physically testing the response of a network

[11] When ratios are involved, such as the signal-to-noise ratio, the only term remaining is the rms noise voltage squared, V^2. This is why the square of the voltage is sometimes referred to as the *noise power*.

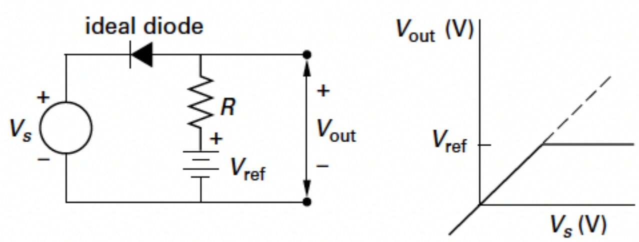
to a delta-like input. The ability to recognize the [12]overall function of a pulse, waveform, or logic circuit is a practical skill required in electrical engineering. Detailed analysis of such circuits involves the application of knowledge of the individual components and requires significant time and specialization.

Clamping Circuits

A diode represents a purely resistive passive nonlinear element. The diode can be used as a simple means of limiting voltage to a desired value. When the voltage is limited in some manner, it is *clamped*. Two types of clamping exist: limiting and clipping. A *limiter* is an electronic circuit designed to prevent a waveform from exceeding a predetermined value. A *clipper* is an electronic circuit designed to maintain a waveform at a predetermined value when the input is above a certain level. The two types of circuits are shown in Fig. 11 for an ideal diode, that is, a diode with no resistance and a zero voltage drop. The ideal circuits can be used for many practical calculations and for determining overall circuit response.



(a) shunt diode limiter circuit



(b) series diode clipper circuit

Figure 11: Ideal Diode Limiter and Clipper Circuits

The diode is not an ideal device. The forward threshold voltage drop, V_F, for silicon diodes is approximately 0.7 V, while that for germanium diodes is approximately 0.2 V. Additionally,

[12] An *RC differentiator* with a switching transistor to control the application of the power is a pulse generator. The RC differentiator is an RC circuit with the output taken across the resistor.

diodes exhibit some forward resistance, R_f, which accounts for the small but finite slope of the forward characteristic. (The reverse resistance or zener resistance is not a concern, because limiter and clipper circuits do not operate in the breakdown region.) The limiter and clipper circuits for an actual diode are shown in Fig. 12. Actual circuits are more complex than ideal circuits but produce more accurate results.

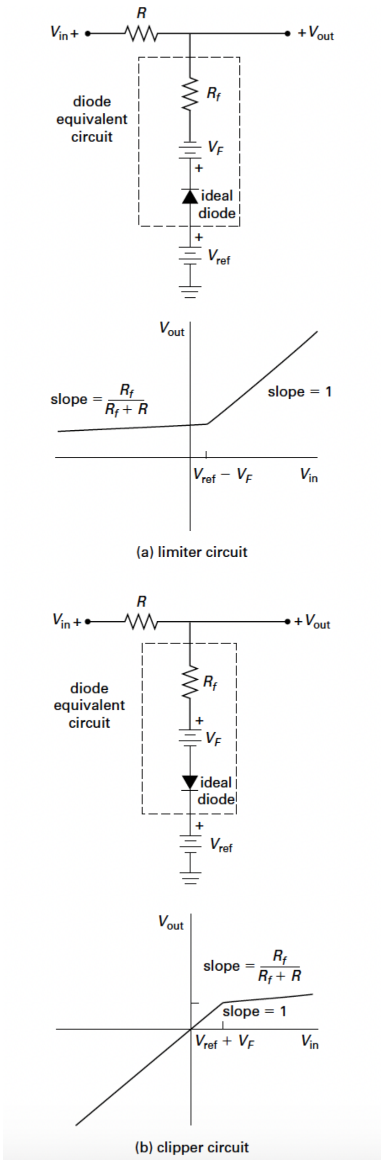


Figure 12: Real Diode Limiter and Clipper Circuits

Analysis of the circuit in Fig. 12(a) yields Eqs. 13 and 14.

Equation 13: Limiter Circuit Forward Biased

$$v_{\text{out}} = \frac{R\left(V_{ref} - V_F\right)}{R + R_f} + \frac{R_f V_{\text{in}}}{R_f + R} \quad \left[V_{\text{in}} < V_{ref} - V_F\right]$$

Equation 14: Limiter Circuit Reversed Biased

$$V_{\text{out}} = V_{\text{in}} \quad \left[V_{\text{in}} > V_{ref} - V_F\right]$$

Equation 37.13 represents the forward-biased case. Equation 37.14 represents the reverse-biased case, ignoring the small reverse saturation current. Analysis of the circuit in Fig. 12(b) yields Eqs. 15 and 16.

Equation 15: Clipper Circuit Forward Biased

$$v_{\text{out}} = \frac{R_f V_{\text{in}}}{R_f + R} + \frac{R\left(V_{ref} + V_F\right)}{R + R_f} \quad \left[V_{\text{in}} > V_{ref} + V_F\right]$$

Equation 16: Clipper Circuit Reversed Biased

$$V_{\text{out}} = V_{\text{in}} \quad \left[V_{\text{in}} < V_{ref} + V_F\right]$$

Equation 15 represents the forward-biased case. Equation 16 represents the reverse-biased case, ignoring the small reverse saturation current.

The ideal limiter or clipper circuit would have R_f equal to zero, and the effect of the threshold voltage would be negligible. This type of circuit can be realized using a *precision diode*. A precision diode circuit, equivalent circuit, and characteristics are shown in Fig. 13. A precision diode is used to rectify very small signals: those less than 0.7 V for silicon or 0.2 V for germanium. In fact, the precision diode rectifies microvolt-level signals.

In Fig. 13(a), a diode is placed between the operational amplifier output and the feedback point of a voltage follower. For negative voltage inputs, $-v_{\text{in}}$, the op amp output voltage, V_{amp}, is negative and the diode is reverse biased.[13] The output voltage is then equal to zero because no current flows through the output resistor, R_{out}. For positive voltage inputs, $+v_{\text{in}}$, the op amp output voltage, V_{amp},

[13] For proper operation, the diode must have a reverse breakdown voltage greater in magnitude than the maximum negative voltage of the op amp, that is, greater in magnitude than the negative supply voltage (not shown in the figure).

is equal to $A_V V_{in}$ until the diode becomes forward biased; that is, until the diode voltage, V_D, exceeds V_F. This forward-biased condition, which occurs when $A_V V_{in} = V_D$ or $V_{in} = V_D/A_V$, is shown in Fig. 13(b). From the forward-biased condition onward, the output voltage across R_{out} is the input voltage magnified by the op amp as shown in Fig. 13(c). The approximate transfer equation for the precision diode of Fig. 13(a) is

Equation 17: Precision Diode Transfer Equation

$$V_{out} = V_{in} - \frac{V_F}{A_V}$$

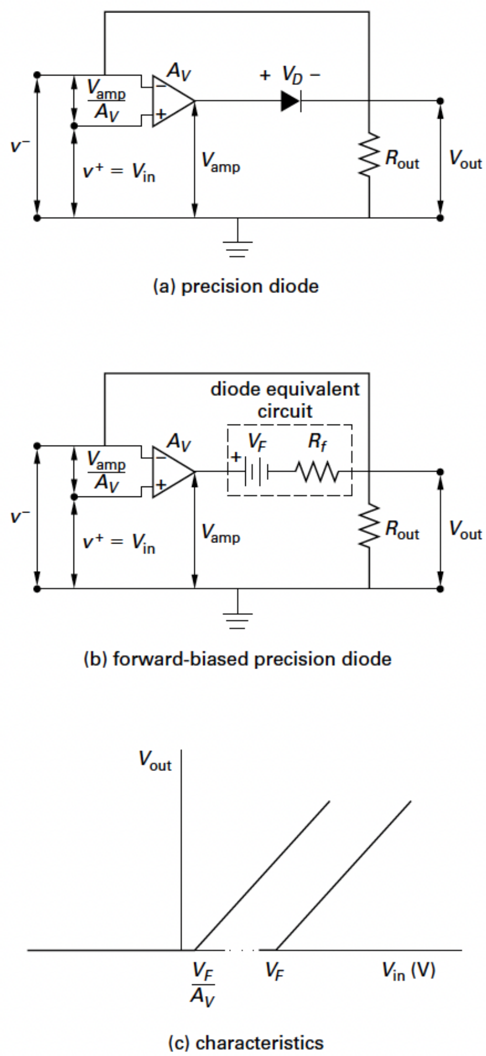


Figure 13: Precision Diode Circuit

Precision diode circuits sensitive to negative voltages are shown in Fig. 14.

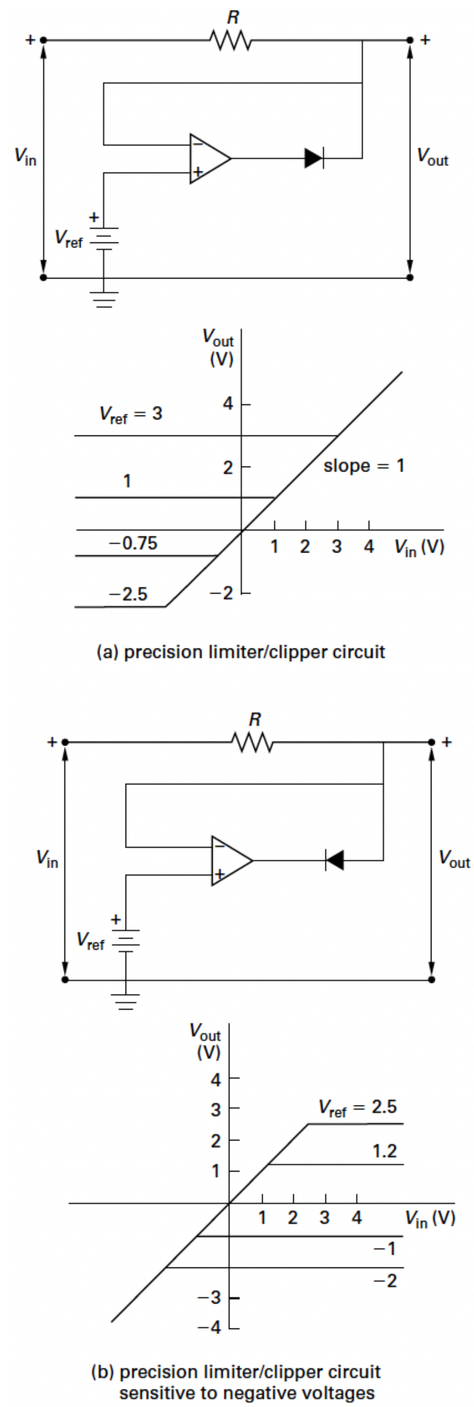
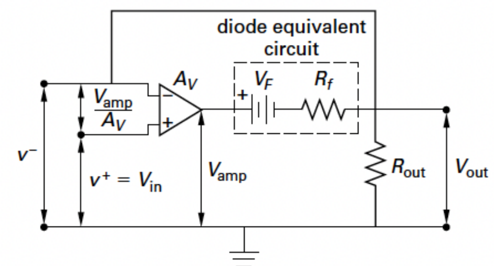


Figure 14: Precision Diode Circuits Sensitive to Negative Voltage Input

Example 6

Consider the precision diode in Fig. 13(b) with a 12 V power supply and a voltage gain of 10^6.



What is the maximum voltage difference that can be amplified without distortion or saturation?

Solution

The maximum voltage difference that can be amplified is determined by the constraint equation, Eq. 3 or Eq. 17 with the diode removed, which changes V_F to V_{amp} (or to v_{out} in Eq. 3).

$$v_{out} = A_V\left(v^+ - v^-\right)$$

or

$$V_{amp} = A_V\left(v^+ - v^-\right)$$

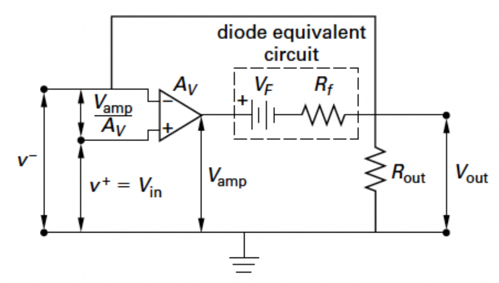
$$v^+ - v^- = \frac{V_{amp}}{A_V}$$

The maximum voltage of the amplifier is 9 V (3 V less than the maximum power supply voltage) to prevent distortion or saturation. Therefore,

$$\begin{aligned} v^+ - v^- &= \frac{V_{amp}}{A_V} \\ &= \frac{9\text{ V}}{10^6} \\ &= 9\square 10^{-6}\text{ V} \quad \left(9\mu\text{V}\right) \end{aligned}$$

Example 7

Use the op amp in Ex. 6 with a *silicon* diode.



What is the output voltage if the input voltage is 9 μV?

Solution

With the silicon diode, the threshold voltage is 0.7 V. Writing KCL at the negative terminal (ignoring the small current into the op amp) gives

$$\frac{v^- - 0 \text{ V}}{R_{\text{out}}} + \frac{v^- + V_F - V_{amp}}{R_F} = 0$$

Note that $v^- = V_{\text{out}}$ and $V_F = 0.7$ V. Substitute these values.

$$\frac{V_{\text{out}}}{R_{\text{out}}} + \frac{V_{\text{out}} + 0.7 \text{ V} - V_{amp}}{R_F} = 0$$

The constraint equation must be used, rather than the assumption used with ideal op amps that $v^+ = v^-$, because the voltage response to an input is the unknown. That is, the *transfer function* must be determined. The constraint equation is

$$\begin{aligned} v^+ - v^- &= V_{\text{in}} - V_{\text{out}} \\ &= \frac{V_{amp}}{A_V} \\ \text{V}_{amp} &= A_V\left(V_{\text{in}} - V_{\text{out}}\right) \end{aligned}$$

Substituting and rearranging gives the following result.

$$\frac{V_{\text{out}}}{R_{\text{out}}} + \frac{V_{\text{out}} + 0.7\ \text{V} - V_{\text{amp}}}{R_f} = 0$$

$$\frac{V_{\text{out}}}{R_{\text{out}}} + \frac{V_{\text{out}} + 0.7\ \text{V} - A_V(V_{\text{in}} - V_{\text{out}})}{R_f} = 0$$

$$\frac{V_{\text{out}}}{R_{\text{out}}} = \frac{V_{\text{out}} + 0.7\ \text{V} - A_V(V_{\text{in}} - V_{\text{out}})}{R_f}$$

$$\frac{V_{\text{out}}}{R_{\text{out}}} + \frac{V_{\text{out}}}{R_f} + \frac{A_V V_{\text{out}}}{R_f} = \frac{A_V V_{\text{in}} - 0.7\ \text{V}}{R_f}$$

$$V_{\text{out}}\left(\frac{1}{R_{\text{out}}} + \frac{1}{R_f} + \frac{A_V}{R_f}\right) = \frac{A_V V_{\text{in}} - 0.7\ \text{V}}{R_f}$$

$$V_{\text{out}} = \frac{A_V V_{\text{in}} - 0.7\ \text{V}}{R_f\left(\frac{1}{R_{\text{out}}} + \frac{1}{R_f} + \frac{A_V}{R_f}\right)} = \frac{V_{\text{in}} - \frac{0.7\ \text{V}}{A_V}}{\frac{R_f}{A_V R_{\text{out}}} + \frac{1}{A_V} + 1}$$

The gain is very large making the denominator approximately 1. The transfer function thus reduces to Eq. 16 with 0.7 V substituted for V_{F}.

$$V_{\text{out}} = V_{\text{in}} - \frac{0.7\ \text{V}}{A_V}$$

$$= 9 \times 10^{-6}\ \text{V} - \frac{0.7\ \text{V}}{10^6}$$

$$= 8.3 \times 10^{-6}\ \text{V} \quad (8.3\ \mu\text{V})$$

The offset voltage of 0.7 V for silicon has been reduced by the precision diode to 0.7×10^{-6} V, or 0.7 μV, which is essentially zero.

Zener Voltage Regulator Circuit—Ideal

Zener diodes operating in the breakdown region are used as voltage regulators. That is, they are used to control the load voltage (output voltage) within a narrow set of values in circuits for which the supply voltage (input voltage) varies and the load current ranges from zero to a maximum value.

An ideal zener diode voltage regulator circuit is shown in Fig. 15(a) with the model for the ideal zener illustrated in Fig. 15(b). The zener will maintain a constant voltage as long as the zener current, I_Z, is positive as shown in Fig. 15(c).

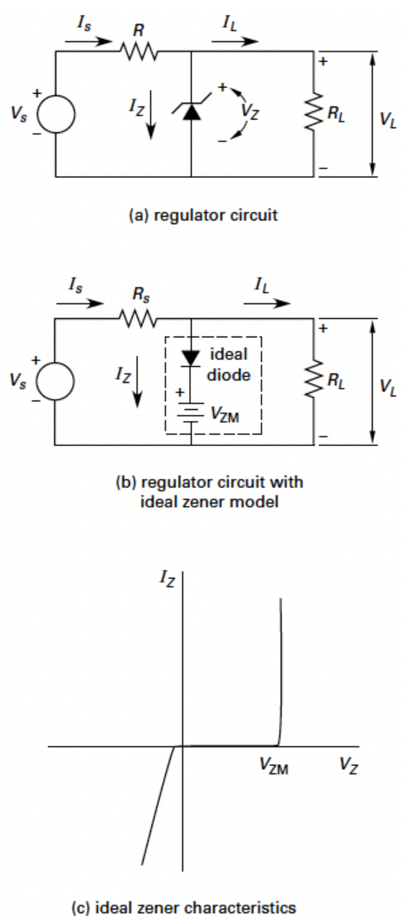


Figure 15: Ideal Zener Diode Voltage Regulator

The design restrictions can be understood from Eq. 18 and Eq. 19, or derived from Ohm's law and the application of KCL to Fig. 15(b).

Equation 18: Zener Supply Current

$$I_s = \frac{V_s - V_Z}{R_s}$$

Equation 19: Zener Current

$$I_Z = I_s - I_L$$

As the load current, I_L, varies, the supply current given by Eq. 18 remains constant, but, according to KCL as stated in Eq. 19, the zener current will vary to compensate. As the load current increases, the zener diode current decreases. However, to keep the ideal diode forward biased, that is, to keep the zener diode in breakdown, the zener current must be zero or positive.[14] That is, the load voltage, which equals the zener voltage, must be equal to V_{ZM}. To minimize the total power consumption, the value of R_s selected should be as small as possible while still compensating for changes in supply voltage and load current. The combination of these requirements gives the following.

Equation 20: Zener Supply Resistance

$$R_s = \frac{V_{\text{in,min}} = V_{\text{ZM}}}{I_{L,\text{max}}}$$

Equation 21: Zener Current Design Restrictions

$$I_{Z,\text{max}} = \frac{V_{\text{in,max}} - V_{ZM}}{R_s} = \left(\frac{V_{\text{in,max}} - V_{ZM}}{V_{\text{in,min}} - V_{ZM}}\right) I_{L,\text{max}}$$

Equation 20 represents the requirement that at the maximum possible load current, no negative current flows in the ideal diode, with an equal sign used to minimize the value of R_s.[15] The phrase "no negative current" means that the minimum supply voltage, $V_{\text{in,min}}$, is greater than or equal to V_{ZM}. Equation 21 represents the maximum current drawn by the zener, setting the power requirements for the diode and the supply resistor, given by Eq. 22 and Eq. 23, respectively.

Equation 22: Zener Power

$$P_D = I_{Z,\text{max}} V_{ZM}$$

Equation 23: Zener Supply Resistor Power

$$P_{R_s} = I_{Z,\text{max}}^2 R_s$$

---

[14] In a practical zener diode, the current must be greater than the keep-alive current, I_{ZK}.

[15] The actual requirement is that the supply resistance be less than or equal to the right-hand side of Eq. 20.

An ideal zener diode voltage regulator circuit is designed as follows.

Step 1: Determine the supply resistance that minimizes power loss but keeps the zener operating in the breakdown region (see Eq. 20). This step accounts for the minimum supply (input) voltage and the maximum load (output) current.

Step 2: Determine the maximum zener current drawn with the supply resistance calculated in step 1 (see Eq. 21). This step accounts for the maximum supply (input) voltage and the minimum load (output) current.

Step 3: Determine the required diode power rating with the current calculated in Step 2 (see Eq. 22).

Step 4: Determine the required resistor power rating with the current calculated in step 2 (see Eq. 23).

Ideal zener diode regulator calculations can be used as first approximations of actual circuits. The inherent assumptions are that the keep-alive current, I_{ZK}, equals zero and that the zener resistance, R_Z, is negligible (see the next section).

Zener Voltage Regulator Circuit—Practical

A practical (equivalent) zener diode voltage regulator circuit, illustrated in Fig. 15, differs from the ideal case in a number of ways. First, the voltages that can be used are standardized. Table 1 lists some of the standard zener diodes and their respective zener impedances, Z_Z, and keep-alive currents, I_{ZK}. Second, equivalent zener diodes exhibit some resistance, R_Z, as shown in the equivalent model of Fig. 16(b). This resistance, R_Z, can cause the zener voltage to vary as much as 20% from the stated value at rated current. The effect is significant for zener diodes rated for 8 V or less, as the zener effect dominates. Third, the zener diode requires a small but finite current called the keep-alive current, I_{ZK}, to remain in the breakdown region. The zener current must be kept above the keep-alive current for the voltage-regulating effect to occur.

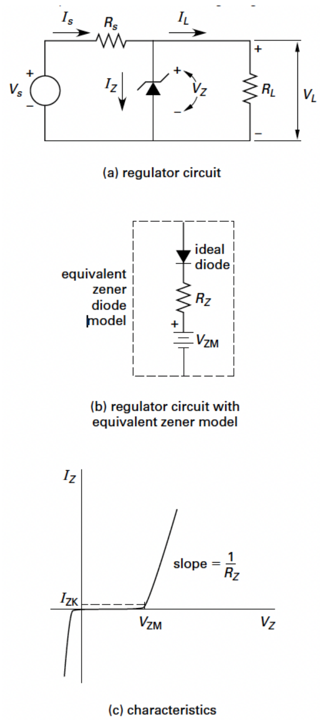


Figure 16: Equivalent Zener Diode Voltage Regulator

The defining equations for the equivalent circuit are derived from Fig. 16(b).[16]

Equation 24: Zener Equivalent Circuit Ohm's Law

$$V_{\text{in}} - V_L = \left(I_Z + I_L\right) R_s$$

Equation 25: Zener Equivalent Load Voltage

$$V_L = V_Z + I_Z R_Z$$

[16] The input voltage is the source voltage.

Table 1: Standard Zener Diodes

zener voltage (nominal)	500 mW		1 W		5 W		50 W	
	$Z_Z{}^a$ (Ω)	$I_{ZK}{}^b$ (mA)	$Z_Z{}^a$ (Ω)	$I_{ZK}{}^b$ (mA)	$Z_Z{}^a$ (Ω)	$I_{ZK}{}^b$ (mA)	$Z_Z{}^a$ (Ω)	$I_{ZK}{}^b$ (mA)
2.4	30	1.0						
2.7	30	1.0						
3.0	29	1.0						
3.3	28	1.0	10	1.0	3.0	1.0		
3.6	24	1.0	10	1.0	2.5	1.0		
3.9	23	1.0	9.0	1.0	2.0	1.0	0.16	5.0
4.3	22	1.0	9.0	1.0	2.0	1.0	0.16	5.0
4.7	19	1.0	8.0	1.0	2.0	1.0	0.12	5.0
5.1	17	1.0	7.0	1.0	1.5	1.0	0.12	5.0
5.6	11	1.0	5.0	1.0	1.0	1.0	0.12	5.0
6.2	7.0	1.0	2.0	1.0	1.0	1.0	0.14	5.0
6.8	5.0	1.0	1.5	1.0	1.0	1.0	0.16	5.0
7.5	5.5	0.5	1.5	1.0	1.5	1.0	0.24	5.0
8.2	6.5	0.5	4.5	0.5	1.5	1.0	0.4	5.0
8.7					2.0	1.0		
9.1	7.5	0.5	5.0	0.5	2.0	1.0	0.5	5.0
10	8.5	0.25	7.0	0.25	2.0	1.0	0.6	5.0
11	9.5	0.25	8.0	0.25	2.5	1.0	0.8	5.0
12	11.5	0.25	9.0	0.25	2.5	1.0	1.0	5.0

Equation 24 and Eq. 25 reduce to the ideal conditions when I_Z equals zero. Because the zener current must be kept above the keep-alive current, Eq. 24 and Eq. 25 must be solved simultaneously, which results in Eq. 26. (Equation 26 is similar to Eq. 20 for the ideal case, in that it is used to determine the source resistance with the input voltage at its minimum value, and the load current at its maximum value.)

Equation 26: Zener Equivalent Limiting Equation

$$V_L = \frac{V_{ZM} R_s + V_{\text{in}} R_Z}{R_s + R_Z} - I_L \left(\frac{R_s R_Z}{R_s + R_Z} \right)$$

The zener current is maximum when the input voltage, V_{in}, is maximum and the load current, I_L, is minimum.

Equation 27: Zener Equivalent Maximum Current

$$I_{Z,\max} = \frac{V_{ZM} R_s + V_{\text{in,max}} R_Z}{R_Z \left(R_s + R_Z \right)} - \frac{V_{ZM}}{R_Z} = \frac{V_{\text{in,max}}}{R_s} - \frac{V_{ZM} R_s + V_{\text{in,max}} R_Z}{R_Z \left(R_s + R_Z \right)}$$

Equation 27 represents the maximum current drawn by the zener and sets the power requirements for the diode and the supply resistor, given by Eq. 28 and Eq. 29, respectively.

Equation 28: Zener Equivalent Max Diode Power

$$P_D = I_{Z,\max} V_{ZM} + I_{Z,\max}^2 R_Z$$

Equation 29: Zener Equivalent Supply Resistor Power

$$P_{R_s} = I_{Z,\max}^2 R_s$$

If the load current, I_L, is allowed to exceed the maximum level, the zener current, I_Z, will decrease below the keep-alive level, I_{ZK}, and the zener diode will cease to regulate. Equation 26 and Eq. 27 will no longer be valid. The load voltage will be given by Eq. 30, which is derived from Eq. 24.

Equation 30: Load Voltage without Zener Regulation

$$V_L = V_{\text{in}} - I_L R_s$$

A practical zener diode voltage regulator circuit is designed as follows.

Step 1: Determine the supply resistance that minimizes power loss but keeps the zener operating in the breakdown region. Use Eq. 25 with the zener current, I_Z, equal to the keep-alive current, I_{ZK}, to determine the load voltage minimum that maintains the keep-alive current. Then use Eq. 26 with the calculated value of the minimum load voltage, the minimum input voltage, and the maximum load current to determine the source resistance. This step accounts for the minimum supply (input) voltage and the maximum load (output) current.

Step 2: Determine the maximum zener current drawn with the supply resistance calculated in step 1 (see Eq. 27). This step accounts for the maximum supply (input) voltage and the minimum load (output) current.

Step 3: Determine the required diode power rating with the current calculated in Step 2 (see Eq. 28).

Step 4: Determine the required resistor power rating with the current calculated in Step 2 (see Eq. 29).

Step 5: If desired, determine temperature effects on the zener diode at maximum expected temperature changes, and recalculate the parameters in the above steps to determine the circuit functions correctly (discussed in Course I).

Step 6: If desired, determine the voltage regulation of the circuit. The regulation is calculated between the maximum input voltage at no load current and the minimum input voltage at maximum current.

Example 8

What is the source resistance for a zener voltage regulator circuit, patterned after Fig. 16(b)? The output voltage requirement is 5.1 V, while the input varies from 10 V to 15 V, and the load current ranges from 0 A to 5 A. The keep-alive current, I_{ZK}, equals 5 mA, and the zener resistance, R_Z, equals 0.12 Ω.

Solution

The supply resistance is determined from Eq. 26 by substituting the value of the minimum load voltage from Eq. 25. (See Step 1 of the procedure in this section.) The minimum load voltage is

$$\begin{aligned} V_{L,\min} &= V_{ZM} + I_{ZK}R_Z \\ &= 5.1\ \text{V} + \left(5\times10^{-3}\ \text{A}\right)\left(0.12\ \Omega\right) \\ &= 5.1006\ \text{V} \end{aligned}$$

Substituting into Eq. 26, using the minimum input voltage and maximum load current, gives

$$\begin{aligned} V_L &= \frac{V_{\text{ZM}}R_s + V_{\text{in,min}}R_Z}{R_s + R_Z} \\ &\quad - I_{L,\max}\left(\frac{R_sR_Z}{R_s + R_Z}\right) \\ 5.1006\ \text{V} &= \frac{(5.1\ \text{V})(R_s) + (10\ \text{V})(0.12\ \Omega)}{R_s + 0.12\ \Omega} \\ &\quad -(5\ \text{A})\left(\frac{(R_s)(0.12\ \Omega)}{R_s + 0.12\ \Omega}\right) \\ R_s &= 0.98\ \Omega \end{aligned}$$

Using a practical zener diode circuit instead of an ideal diode circuit requires a minimal source resistance change.

Example 9

What is the voltage regulation for the circuit in Ex. 11?

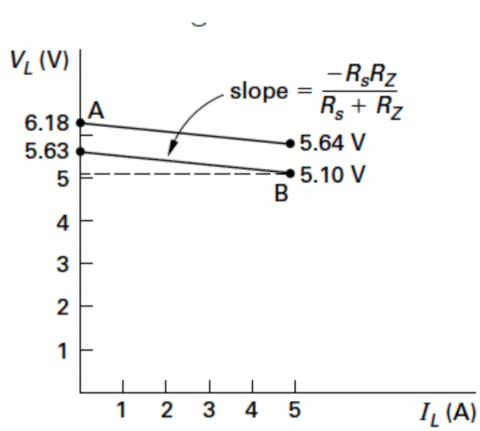
Solution

The voltage regulation (Step 6 of the procedure in this section) is determined by obtaining the equation for the load voltage and current. The value of R_s for the circuit to operate in the breakdown region is 0.98 Ω (from Ex. 11). Substituting this value into Eq. 26 gives the relationship between the load current and voltage.

$$\begin{aligned} V_L &= \frac{V_{ZM}R_s + V_{in}R_Z}{R_s + R_Z} - I_L\left(\frac{R_sR_Z}{R_s + R_Z}\right) \\ &= \frac{(5.1\ \text{V})(0.98\ \Omega) + (V_{in})(0.12\ \Omega)}{0.98\ \Omega\ + 0.12\ \Omega} \\ &\quad - I_L\left(\frac{(0.98\ \Omega)(0.12\ \Omega)}{0.98\ \ \Omega + 0.12\ \Omega}\right) \\ &= 4.54\ \text{V} + 0.109\ \ V_{in} - 0.107 I_L \end{aligned}$$

For an input voltage of 10 V and no-load current, the load voltage is 5.63 V. At the maximum load current of 5 A, the load voltage is 5.10 V. (If the load current goes above the maximum of 5 A, the diode ceases to regulate, and the load voltage is determined by Eq. 30.)

Repeating the calculation for an input voltage of 15 V gives the values of 6.18 V and 5.64 V for no-load current and maximum load current, respectively. Plotting the results gives the following.



The regulation is calculated between the maximum input voltage at no-load current (point A) and the minimum input voltage at maximum current (point B).

$$\text{VR} = \frac{V_{nl} - V_{fl}}{V_{fl}} \square 100\% = 21.2\%$$

Operational Amplifier Regulator Circuit

Zener diodes provide voltage regulation at low-current demands. Better regulation is achieved using a precision diode, but a precision diode also lowers the operational amplifier current output. Using the zener diode as a reference for input to an op amp and passing the output to the base of a transistor improves the regulation and increases the current capability. Such an op amp regulator is shown in Fig. 17.

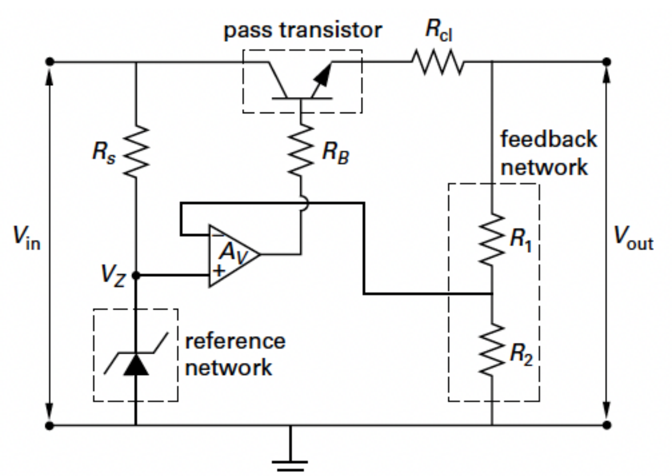


Figure 17: Operational Amplifier Voltage Regulator

The common base transistor is called the *pass transistor* and is used to supply a current gain, α. The zener provides the reference voltage, V_Z, at the noninverting terminal of the op amp. The input to the op amp is $V_Z - \delta V_{out}$ where delta is called the feedback factor and is given by[17]

[17] The feedback factor is sometimes symbolized β. This symbol is avoided here to prevent confusion with the CE current gain, β.

Equation 31: Delta Factor

$$\delta = \frac{R_2}{R_1 + R_2}$$

The output is approximately

Equation 32: Op Amp Regulator Voltage Output

$$V_{\text{out}} \approx \frac{V_Z}{\delta} \approx V_Z \left(\frac{A_V}{1 + \delta A_V} \right)$$

The supply resistance, R_s, is set to ensure that the minimum keep-alive current flows in the zener diode.

Equation 33: Op Amp Regulator Supply Resistance

$$R_s = \frac{V_{\text{in,min}} - V_{ZM}}{I_{ZK}} - R_Z$$

The equivalent circuit for the zener diode, which shows R_Z, is shown in Fig. 16(b). The minimum and maximum zener voltages are

Equation 34: Minimum Zener Voltage

$$V_{Z,\text{min}} = V_{ZM} + I_{ZK} R_Z$$

Equation 35: Maximum Zener Voltage

$$V_{Z,\text{max}} = \frac{V_{\text{in,max}} R_Z + V_{ZM} R_s}{R_s + R_Z}$$

REFERENCES

Items (latest editions) in **bold** are highly recommended for in-depth study.

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B. Camara, John A. *PE Power Reference Manual*. Belmont, CA: PPI (Kaplan), 2021.

C. Marne, David J., and John A. Palmer. *National Electrical Safety Code®* (NESC®) *2023 Handbook*. New York: McGraw Hill, 2023.

D. Earley, Mark, ed. *NFPA 70, National Electrical Code Handbook*. Quincy, Massachusetts: NFPA, 2020.

NOTE

Electrical refers to something related to electricity while "electric" refers to a device or machine that runs on electricity. Nevertheless, the NEC is sometimes referred to as the National Electric Code.

E. IEEE 315-1975. *Graphic Symbols for Electrical and Electronics Diagrams.* New York: IEEE, approved 1975, reaffirmed 1993.

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G. Grainger, John J., and William Stevenson, Jr. *Power System Analysis*. New York, McGraw Hill, 1994.

H. Parker, Sybil P., editor in chief. *McGraw-Hill Dictionary of Scientific and Technical Terms,* 5th ed. New York, McGraw-Hill, 1994.

Appendix A: Equivalent Units Of Derived And Common SI Units

Symbol	Equivalent Units			
A	C/s	W/V	V/Ω	$J/(s\cdot V)$
C	$A\cdot s$	J/V	$(N\cdot m)/V$	$V\cdot F$
F	C/V	C^2/J	s/Ω	$(A\cdot s)/V$
F/m	$C/(V\cdot m)$	$C^2/(J\cdot m)$	$C^2/(N\cdot m^2)$	$s/(\Omega\cdot m)$
H	W/A	$(V\cdot s)/A$	$\Omega\cdot s$	$(T\cdot m^2)/A$
Hz	1/s	s^{-1}	cycles/s	$\text{radians}/(2\pi\cdot s)$
J	$N\cdot m$	$V\cdot C$	$W\cdot s$	$(kg\cdot m^2)/s^2$
m^2/s^2	J/kg	$(N\cdot m)/kg$	$(V\cdot C)/kg$	$(C\cdot m^2)/(A\cdot s^3)$
N	J/m	$(V\cdot C)/m$	$(W\cdot C)/(A\cdot m)$	$(kg\cdot m)/s^2$
N/A^2	$Wb/(N\cdot m^2)$	$(V\cdot s)/(N\cdot m^2)$	T/N	$1/(A\cdot m)$
Pa	N/m^2	J/m^3	$(W\cdot s)/m^3$	$kg/(m\cdot s^2)$
Ω	V/A	W/A^2	V^2/W	$(kg\cdot m^2)/(A^2\cdot s^3)$
S	A/V	1/Ω	A^2/W	$(A^2\cdot s^3)/(kg\cdot m^2)$
T	Wb/m^2	$N/(A\cdot m)$	$(N\cdot s)/(C\cdot m)$	$kg/(A\cdot s^2)$
V	J/C	W/A	C/F	$(kg\cdot m^2)/(A\cdot s^3)$
V/m	N/C	$W/(A\cdot m)$	$J/(A\cdot m\cdot s)$	$(kg\cdot m)/(A\cdot s^3)$
W	J/s	$V\cdot A$	V^2/Ω	$(kg\cdot m^2)/s^3$
Wb	$V\cdot s$	$H\cdot A$	T/m^2	$(kg\cdot m^2)/(A\cdot s^2)$

Appendix B: Physical Constants

Table Note 1

Quantity	Symbol	US Customary	SI Units
Charge			
electron	e		-1.6022×10^{-19} C
proton	p		$+1.6022 \times 10^{-19}$ C
Density			
air [STP][32°F, (0°C)]		0.0805 lbm/ft^3	1.29 kg/m^3
air [70°F, (20°C), 1 atm]		0.0749 lbm/ft^3	1.20 kg/m^3
sea water		64 lbm/ft^3	1025 kg/m^3
water [mean]		62.4 lbm/ft^3	1000 kg/m^3
Distance			
Earth radius[2]	⊕	2.09×10^{7} ft	6.370×10^{6} m
Earth-Moon separation[2]	⊕☾	1.26×10^{9} ft	3.84×10^{8} m
Earth-Sun separtion[2]	⊕⊙	4.89×10^{11} ft	1.49×10^{11} m
Moon radius[2]	☾	5.71×10^{6} ft	1.74×10^{6} m
Sun radius[2]	⊙	2.28×10^{9} ft	6.96×10^{8} m
first Bohr radius	a_0	1.736×10^{-10} ft	5.292×10^{-11} m
Gravitational Acceleration			
Earth [mean]	g	32.174 (32.2) ft/sec^2	9.8067 (9.81) m/s^2
Mass			
atomic mass unit	μ or m_μ $\frac{1}{12}m\left({}^{12}C\right)$	3.66×10^{-27} lbm	1.6606×10^{-27} kg or 10^{-3} kg mol^{-1} / N_A or 931.481 MeV
Earth[2]	⊕	4.11×10^{23} slugs	6.00×10^{24} kg
Earth [customary U.S.][2]	⊕	1.32×10^{25} lbm	-
Moon[2]	☾	1.623×10^{23} lbm	7.36×10^{22} kg
Sun[2]	⊙	4.387×10^{30} lbm	1.99×10^{30} kg
electron rest mass	m_e	2.008×10^{-30} lbm	9.109×10^{-31} kg [0.511 MeV]
neutron rest mass	m_n	3.693×10^{-27} lbm	1.675×10^{-27} kg [939.6 MeV]
proton rest mass	m_p	3.688×10^{-27} lbm	1.672×10^{-27} kg [938.2 MeV]
Pressure			

Quantity	Symbol	US Customary	SI Units
atmospheric		14.696 (14.7) lbf/in^2	1.0133□10^5 Pa
Temperature			
standard		32°F (492°R)	0°C (273 K)
absolute zero		–459.67°F (0°R)	–273.16°C (0 K)
Velocity[3]			
Earth escape		3.67□10^4 ft/sec	1.12□10^4 m/s
light (vacuum)	c, c_0	9.84□10^8 ft/sec	2.9979 (3.00) □10^8 m/s
sound [air, STP]	a	1090 ft/sec	331 m/s
sound [air, 70°F, (20°C), 1 atm]		1130 ft/sec	344 ft/s
Volume			
Volume: molal ideal gas (STP)[4]		359 ft^3 / lbmol	22.41 m^3/kmol

Table 1 Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.
2. Symbols shown for the solar system are those used by NASA. See https://science.nasa.gov/resource/solar-system-symbols/.
3. Velocity technically is a vector. It has direction.
4. The unit "lbmol" is an actual unit, not a misspelling.

Appendix C: Fundamental Constants

Quantity	Symbols	US Customary	SI Units
Avogadro's number	N_A, L		6.022×10^{23} mol^{-1}
Bohr magneton	μ_B		9.2732×10^{-24} J/T
Boltzmann constant	κ	5.65×10^{-24} ft-lbf/ R	1.3805×10^{-23} J/T
electron volt: $\left(\frac{e}{C}\right)$ J	eV		1.602×10^{-19} J
Faraday constant, $N_A e$	F		96485 C/mol
fine structure constant, inverse α^{-1}	α α^{-1}		7.297×10^{-3} (≈1/137) 137.035
gravitational constant	g_c	32.174 lbm-ft/lbf-sec^2	
Newtonian gravitational constant	G	3.44×10^{-8} ft^4 / lbf-sec^4	6.672×10^{-11} N·m^2 / kg^2
nuclear magneton	μ_N		5.050×10^{-27} J/T
permeability of a vacuum	μ_0		1.2566×10^{-6} N/A^2 (H/m)
permittivity of a vacuum, electric constant $1/\mu_0 c^2$	ε_0		8.854×10^{-12} C^2 / N·m^2 (F/m)
Planck's constant	h		6.6256×10^{-34} J·s
Planck's constant: h/2π			1.0546×10^{-34} J·s
Rydberg constant	R_∞		1.097×10^{7} m^{-1}
specific gas constant, air	R	53.3 ft-lbf/lbm- R	287 J/kg·K
Stefan-Boltzmann constant		1.71×10^{-9} BTU/ft^2-hr-°R^4	5.670×10^{-8} W/m^2·K^4
triple point, water		32.02 F, 0.0888 psia	0.01109 C, 0.6123 kPa
universal gas constant	R*	1545 ft-lbf/lbmol- R 1.986 BTU/lbmol- R	8314 J/kmol·K

Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/. The unit in Volume of "lbmol" is an actual unit, not a misspelling.

Appendix D: Mathematical Constants, Signs/Symbols, Maxwell's Equations

Quantity	Symbol	Value
Archimedes' constant (pi)	π	3.1415926536
base of natural logs	e	2.7182818285
Euler's constant	C or τ	0.5772156649

Signs/Symbols	Meaning
$\cdot$	multiplied by
/	divided by
:	ratio
$\gg$	much greater than
$\ll$	much less than
=	equals
$\equiv$	identical with
~	similar to
$\approx$	approximately equals
$\cong$	approximately equals, congruent
$\rightarrow$, $\doteq$	approaches
$\propto$	proportional, varies as
$\therefore$	therefore

Maxwell's Equations

integral form	point form	remarks
$\oint_s \mathbf{D}\cdot ds = \int_V \rho\, dv$	$\nabla\cdot\mathbf{D} = \rho$	Gauss' law
$\oint_s \mathbf{B}\cdot ds = 0$	$\nabla\cdot\mathbf{B} = 0$	nonexistence of magnetic monopoles
$\oint \mathbf{E}\cdot dl = \int_s \left(\frac{-\partial \mathbf{B}}{\partial t}\right)\cdot ds$	$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	Faraday's law
$\oint \mathbf{H}\cdot dl = \int_s \left(\mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}\right)\cdot ds$	$\nabla\times\mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{dt}$	Ampère's law

Free-Space Form

integral form	point form
$\oint_s \mathbf{D}\cdot ds = 0$	$\nabla\cdot\mathbf{D} = 0$
$\oint_s \mathbf{B}\cdot ds = 0$	$\nabla\cdot\mathbf{B} = 0$
$\oint \mathbf{E}\cdot dl = \int_s \left(\frac{-\partial \mathbf{B}}{\partial t}\right)\cdot ds$	$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\oint \mathbf{H}\cdot dl = \int_s \left(\frac{\partial \mathbf{D}}{\partial t}\right)\cdot ds$	$\nabla\times\mathbf{H} = \frac{\partial \mathbf{D}}{dt}$

Electromagnetic Field Vector Equations

$$\mathbf{D} = \epsilon\mathbf{E} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon_0(1+\chi_e)\mathbf{E}$$
$$\mathbf{B} = \mu\mathbf{H} = \mu_0\mathbf{H} + \mu_0\mathbf{M} = \mu_0(1+\chi_m)\mathbf{H}$$
$$\mathbf{J} = \sigma\mathbf{E} = \rho\mathrm{v}$$

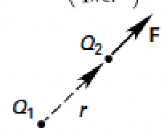
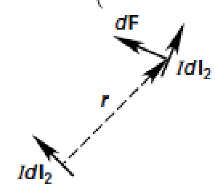
Appendix E: The Greek Alphabet

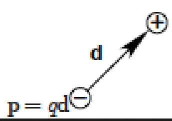
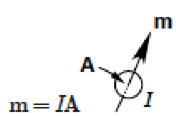
Α	α	alpha	Ν	υ	nu
Β	β	beta	Ξ	ξ	xi
Γ	γ	gamma	Ο	o	omicron
Δ	δ	delta	Π	π	pi
Ε	ε	epsilon	Ρ	ρ	rho
Ζ	ζ	zeta	Σ	σ	sigma
Η	η	eta	Τ	τ	tau
Θ	θ	theta	ϒ	υ	upsilon
Ι	ι	iota	Φ	ϕ	phi
Κ	κ	kappa	Χ	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
Μ	μ	mu	Ω	ω	omega

Appendix F: SI Prefixes

symbol	prefix	value
a	atto	10^{-18}
f	femto	10^{-15}
p	pico	10^{-12}
n	nano	10^{-9}
μ	micro	10^{-6}
m	milli	10^{-3}
c	centi	10^{-2}
d	deci	10^{-1}
da	deka	10
h	hecto	10^{2}
k	kilo	10^{3}
M	mega	10^{6}
G	giga	10^{9}
T	tera	10^{12}
P	peta	10^{15}
E	exa	10^{18}

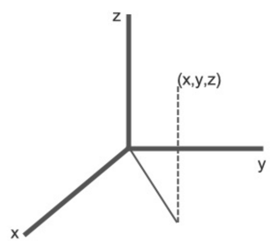
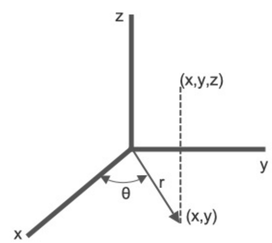
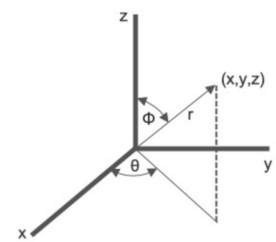
Appendix G: Comparison of Electric & Magnetic Equations

equation description	electric version	magnetic version	remarks
experimental force law	Coulomb's law $\mathbf{F} = \left(\frac{Q_1 Q_2}{4\pi\epsilon r^2}\right)\mathbf{r}$ 	force between two current elements $d\mathbf{F} = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I_2 d\mathbf{l}_2 \times (I_1 d\mathbf{l}_1 \times r)}{r^2}\right)$ 	The term $Id\mathbf{l}$ in the magnetic column is the equivalent of a "magnetic charge" q_m. The I or the $d\mathbf{l}$ can be the vector. The r is a unit vector pointing from 1 to 2.
field definitions from force law	$\mathbf{F} = Q\mathbf{E}$	$d\mathbf{F} = \mathbf{I} \times \mathbf{B}\, dl$ current element $d\mathbf{F} = \mathbf{J} \times \mathbf{B}\, dV$ distributed current element $d\mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$ moving charge	The V used in this row represents volume, not voltage. The v is the velocity.
general force law	$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ $d\mathbf{F} = (\rho\mathbf{E} + \mathbf{J} \times \mathbf{B})\, dV$ where $dQ = \rho\, dV$		The V in this row represents the volume, not voltage. The v is the velocity.
definition of scalar and vector potential	$E = -\nabla V$	$\mathbf{B} = \nabla \times \mathbf{A}$	**A** is the magnetic vector potential.
Poisson's equation for the potential function	$\nabla^2 V = -\frac{\rho}{\epsilon}$	$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J}$	From a knowledge of the charge distribution, the potential can be found and then the **E** and **B** fields determined.
Gauss's law enclosing charge and Ampère's law enclosing current	$\iint \mathbf{D} \cdot d\mathbf{A} =$ $\iiint \rho\, dV = Q$ $\nabla \cdot \mathbf{D} = \rho$	$\oint H \cdot d\mathbf{l} = I$ $\nabla \times \mathbf{H} = \mathbf{J}$	The V in this row represents volume.
constitutive relations	$\mathbf{D} = \epsilon\mathbf{E}$ $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$	$\mathbf{B} = \mu\mathbf{H}$ $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$	The second set of equations is always valid. The first set assumes the medium is linear and isotropic.
definitions of relative permittivity and permeability	$\epsilon_r = \frac{\epsilon}{\epsilon_0}$ $\epsilon_0 =$ 8.854×10^{-12} F/m	$\mu_r = \frac{\mu}{\mu_0}$ $\mu_0 = 4\pi \times 10^{-7}$ H/m	

equation description	electric version	magnetic version	remarks
capacitance and inductance of a field cell	$\epsilon_0 = \frac{C}{l}$	$\mu_0 = \frac{L}{l}$	Field cells are a construct designed to represent free space in terms of a parallel plate capacitor and an inductor. This capacitance and inductance exist regardless of the presence of an electric or magnetic field.
capacitance and inductance	$C = \frac{Q}{V}$	$L = \frac{\Lambda}{I}$	Λ is the flux linkage.
energy density of a field	$U = \frac{1}{2}\epsilon E^2$	$U = \frac{1}{2}\mu H^2$	Both energy and momentum are carried by a field.
energy stored by capacitance and inductance	$W = \frac{1}{2}CV^2$	$W = \frac{1}{2}LI^2$	
electromotive and magnetomotive force with sources present	$\oint \mathcal{E}\cdot d\mathbf{l} = \mathcal{E} = V$	$\oint \mathbf{H}\cdot d\mathbf{l} = NI = F_m = V_m$	The $\mathcal{E}$ is the emf, not the permittivity. Without sources present, both line integrals are equal to zero.
dipole moments			
dipole torque	$\mathbf{T} = \mathbf{p} \times \mathbf{E}$	$\mathbf{T} = \mathbf{m} \times \mathbf{B}$	This torque occurs due to the dipole being immersed in an external **E** or **B** field.
dipole potential energy	$W = -\mathbf{p}\cdot\mathbf{E}$	$W = -\mathbf{m}\cdot\mathbf{B}$	

electric	magnetic
emf $= V = IR$	mmf $= V_m = \phi\mathcal{R}$
current I	flux ϕ
emf $\mathcal{E}$ or V	mmf V_m
resistance $R = \rho l/A = l/\sigma A$	reluctance $\mathcal{R} = l/\mu A$
resistivity ρ	reluctivity $1/\mu$
conductance $G = 1/R$	permeance $P_m = \mu A/l$
conductivity $\sigma = 1/\rho$	permeability μ

Appendix H: Coordinate Systems and Related Operations

Mathematical Operations	Rectangular Coordinates	Cylindrical Coordinates	Spherical Coordinates
Conversion to Rectangular Coordinants	 $x = x$ $y = y$ $z = z$	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	 $x = r\sin\phi\cos\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\phi$
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\boldsymbol{\theta} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\boldsymbol{\phi} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \theta}\boldsymbol{\theta}$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial (A_\phi \sin\phi)}{\partial \phi} + \frac{1}{r\sin\phi}\frac{\partial A_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{r} & \boldsymbol{\theta} & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & A_\theta & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r^2\sin\theta}\mathbf{r} & \frac{1}{r^2\sin\theta}\boldsymbol{\phi} & \frac{1}{r}\boldsymbol{\theta} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ A_r & rA_\phi & rA\theta A_\phi \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial r}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2 f}{\partial \theta^2}\right)$

