A SunCam Online Continuing Education Course

Materials Science & Engineering

Electrical & Magnetic Materials

by

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Nomenclature[1]

A	area	m^2
B	magnetic flux density	T
c	specific heat capacity [capacity compared to water]	J/g·°C
C	heat capacity	J/g·°C
c	perimeter of the cross section	m
C_{H2O}	heat capacity of water	$1\ \frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}} = 4.184\ \frac{\text{J}}{\text{g}\cdot{}^\circ\text{C}}$ or $1\ \frac{\text{BTU}}{\text{lbm}\cdot{}^\circ\text{F}}$
cmil	circular mils	cmil
d	diameter	mm
e	strain, fractional deformation	$\Delta L/L$ unitless
E	elastic modulus [s/e] Young's Modulus	$\text{Pa} = \text{N/m}^2$ or $\text{psi} = \text{lbf/in}^2$
E	voltage (source), electromotive force	V
f	frequency	Hz
H	magnetic field strength	A/m
Hardness	resistance to plastic indentation	empirical units Brinell [BHN] Rockwell [R] Vickers [DPH]
I	fusing current [melting point]	A
k	thermal conductivity	W/m·K
k	fusing coefficient	-
l	length [of magnet]	m

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[1] Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related and may be found when reviewing the references listed for further information. Further, all the nomenclature, symbols, or subscripts will be found in of many electrical courses (on SunCam, PDH Academy, and also in many texts). For guidance on nomenclature, symbols, and electrical graphics: IEEE 280-2021, IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering, New York: IEEE; and IEEE 315-1975, Graphic Symbols for Electrical and Electronics Diagrams, New York: IEEE, approved 1975, reaffirmed 1993.

lb, lbs	pounds [mass]	lbm
m	mass	g or kg
MSE	Materials Science & Engineering	-
Q	heat energy [total]	J
R	resistance	Ω
S, Strength	stress at failure	Pa [psi]
s, σ	stress	Pa [psi]
S_t	tensile stress, maximum strength	Pa [psi]
S_y	yield stress	Pa [psi]
T	temperature	°F, °C, K
Toughness	energy for failure by fracture	joules [ft-lbf]
U	magnetic potential across an air gap	A
V	volume	m^3
W	magnetic energy in air gap	J
YP	yield point	-

Greek Symbols

η	efficiency	-
ϕ	phase	-
ρ	resistivity	Ω·m
σ	stress	Pa
σ	conductivity	S/m
μ	permeability	H/m
Φ	magnetic flux	Wb

Subscripts

0	initial	-
0	standard conditions (if superscript): 1 molar concentration / 25°C / 1 bar≈1 atm	
b	breaking	-
f	final, fracture	-
m	magnetizing	-
p	protection	-
pu	per unit	-
r	reseal	-
S	surface	-
t	tensile	-
V	volume	-
w	withstand	-
y	yield	-

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COURSE REFERENCES

The theoretical information is primarily from one of the author's books, Ref. [A]. The NEC Ref. [B] is always a useful source for electrical engineers. Information useful in many aspects of electric engineering may be found in [C] and [D]. Reference [E] has detailed descriptions of analysis techniques. Reference [F] covers many terms in EE with excellent definitions and explanations. Reference [G] is one of the most comprehensive and best explained texts on motor and generator theory. The appendices (A-F) cover information useful in many engineering tasks with (G) and (H) focused on this course. Reference (I) is compendium of material properties. Reference (J) is an excellent book for specifics about given materials. App. (G) provides a side by side comparison of electric and magnetic equations. Use these texts or their counterparts for indepth information. References in bold are highly recommended. The latest editions are recommended.

This course will focus on basics, that rarely change, that provide the basis for all other knowledge.

MATERIALS SCIENCE & ENGINEERING

INTRODUCTION

The use of materials is ingrained in our culture, to the point where we even named ages after them. There is the Stone Age, from about 10,000 BCE to 4000 BCE. Then the Bronze Age, transitioning from the stone age but generally from about 3000 BCE to about 1100 BCE.[2] Then the Iron Age, overlapping the bronze age from about 1200 BCE to about 400 CE.

Materials are any substances that make them useful in structures, machines, devices, products, and for our specific focus, electrical materials.[3] See the basic intersections in Fig. 1.

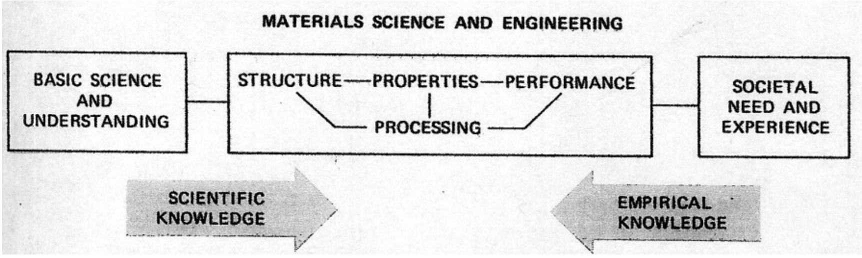

Figure 1: Central Elements of Materials Science & Engineering
[Source: Ref. H]

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[2] Bronze: copper and tin, generally.
[3] The materials of "life" are generally dealt with in the life and agricultural sciences.

The electrical properties of materials impact all engineering designs. An understanding of the properties and potential reactions of chosen materials within the component parts and the surrounding environment is necessary both to ensure proper operation over the service lifetime of the components and to minimize long-term effects on the environment.

The fundamental properties of conducting, insulating and dielectric, semiconductor, magnetic, electron emitting, and radiation-emitting materials are important to the electrical engineer. Materials handbooks are available that provide specific values for these properties for elements and compounds under various conditions.

Materials Science and Engineering is fundamentally interdisciplinary in nature. The difference between the science and the engineering is vague. But, in general, the following is true.

- **Materials Scientists** often focus on the *fundamental* understanding: how atomic structure (synthesis/processing) dictates properties (strength, conductivity).
- **Materials Engineers** focus on *applying* that knowledge: designing, processing, and integrating materials into functional products, considering performance and cost.

Knowing this, the focus of this course will be major/important—and basic— impacts to electrical materials.

SELECTED CHARACTERISTICS

The engineer requires an understanding of the base of the MSE Tetrahedron in order to characterize the materials used in components to obtain the desired performance.

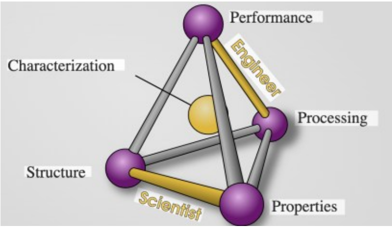
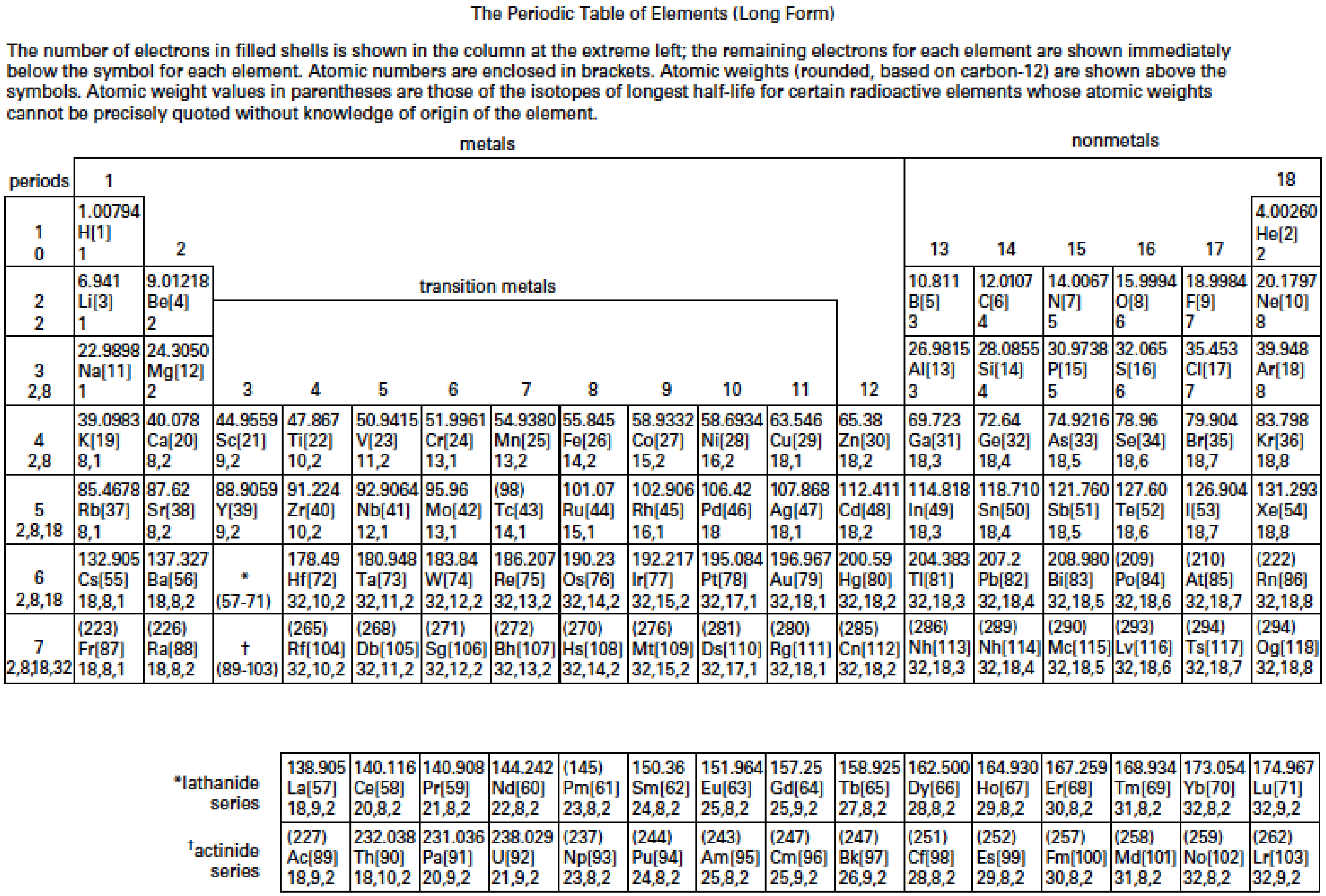


Figure 2: The MSE Tetrahedron

[Source: *https://msestudent.com/what-is-materials-science-tetrahedron-paradigm/*]

To understand materials one must know the basics of the periodic table, the long form of which is shown below.



The Periodic Table of Elements (Long Form)

The number of electrons in filled shells is shown in the column at the extreme left; the remaining electrons for each element are shown immediately below the symbol for each element. Atomic numbers are enclosed in brackets. Atomic weights (rounded, based on carbon-12) are shown above the symbols. Atomic weight values in parentheses are those of the isotopes of longest half-life for certain radioactive elements whose atomic weights cannot be precisely quoted without knowledge of origin of the element.

metals

nonmetals

transition metals

periods	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 0	1.00794 H[1] 1																	4.00260 He[2] 2
2 2	6.941 Li[3] 1	9.01218 Be[4] 2											10.811 B[5] 3	12.0107 C[6] 4	14.0067 N[7] 5	15.9994 O[8] 6	18.9984 F[9] 7	20.1797 Ne[10] 8
3 2,8	22.9898 Na[11] 1	24.3050 Mg[12] 2											26.9815 Al[13] 3	28.0855 Si[14] 4	30.9738 P[15] 5	32.065 S[16] 6	35.453 Cl[17] 7	39.948 Ar[18] 8
4 2,8	39.0983 K[19] 8,1	40.078 Ca[20] 8,2	44.9559 Sc[21] 9,2	47.867 Ti[22] 10,2	50.9415 V[23] 11,2	51.9961 Cr[24] 13,1	54.9380 Mn[25] 13,2	55.845 Fe[26] 14,2	58.9332 Co[27] 15,2	58.6934 Ni[28] 16,2	63.546 Cu[29] 18,1	65.38 Zn[30] 18,2	69.723 Ga[31] 18,3	72.64 Ge[32] 18,4	74.9216 As[33] 18,5	78.96 Se[34] 18,6	79.904 Br[35] 18,7	83.798 Kr[36] 18,8
5 2,8,18	85.4678 Rb[37] 8,1	87.62 Sr[38] 8,2	88.9059 Y[39] 9,2	91.224 Zr[40] 10,2	92.9064 Nb[41] 12,1	95.96 Mo[42] 13,1	(98) Tc[43] 14,1	101.07 Ru[44] 15,1	102.906 Rh[45] 16,1	106.42 Pd[46] 18	107.868 Ag[47] 18,1	112.411 Cd[48] 18,2	114.818 In[49] 18,3	118.710 Sn[50] 18,4	121.760 Sb[51] 18,5	127.60 Te[52] 18,6	126.904 I[53] 18,7	131.293 Xe[54] 18,8
6 2,8,18	132.905 Cs[55] 18,8,1	137.327 Ba[56] 18,8,2	* (57-71)	178.49 Hf[72] 32,10,2	180.948 Ta[73] 32,11,2	183.84 W[74] 32,12,2	186.207 Re[75] 32,13,2	190.23 Os[76] 32,14,2	192.217 Ir[77] 32,15,2	195.084 Pt[78] 32,17,1	196.967 Au[79] 32,18,1	200.59 Hg[80] 32,18,2	204.383 Tl[81] 32,18,3	207.2 Pb[82] 32,18,4	208.980 Bi[83] 32,18,5	(209) Po[84] 32,18,6	(210) At[85] 32,18,7	(222) Rn[86] 32,18,8
7 2,8,18,32	(223) Fr[87] 18,8,1	(226) Ra[88] 18,8,2	† (89-103)	(265) Rf[104] 32,10,2	(268) Db[105] 32,11,2	(271) Sg[106] 32,12,2	(272) Bh[107] 32,13,2	(270) Hs[108] 32,14,2	(276) Mt[109] 32,15,2	(281) Ds[110] 32,17,1	(280) Rg[111] 32,18,1	(285) Cn[112] 32,18,2	(286) Nh[113] 32,18,3	(289) Nh[114] 32,18,4	(290) Mc[115] 32,18,5	(293) Lv[116] 32,18,6	(294) Ts[117] 32,18,7	(294) Og[118] 32,18,8

*lathanide series	138.905 La[57] 18,9,2	140.116 Ce[58] 20,8,2	140.908 Pr[59] 21,8,2	144.242 Nd[60] 22,8,2	(145) Pm[61] 23,8,2	150.36 Sm[62] 24,8,2	151.964 Eu[63] 25,8,2	157.25 Gd[64] 25,9,2	158.925 Tb[65] 27,8,2	162.500 Dy[66] 28,8,2	164.930 Ho[67] 29,8,2	167.259 Er[68] 30,8,2	168.934 Tm[69] 31,8,2	173.054 Yb[70] 32,8,2	174.967 Lu[71] 32,9,2
†actinide series	(227) Ac[89] 18,9,2	232.038 Th[90] 18,10,2	231.036 Pa[91] 20,9,2	238.029 U[92] 21,9,2	(237) Np[93] 23,8,2	(244) Pu[94] 24,8,2	(243) Am[95] 25,8,2	(247) Cm[96] 25,9,2	(247) Bk[97] 26,9,2	(251) Cf[98] 28,8,2	(252) Es[99] 29,8,2	(257) Fm[100] 30,8,2	(258) Md[101] 31,8,2	(259) No[102] 32,8,2	(262) Lr[103] 32,9,2

Figure 3: Long Form Periodic Table

[Source: Ref. A]

Some of the basic properties of importance to the engineer are generalized in the annotated periodic table shown below.

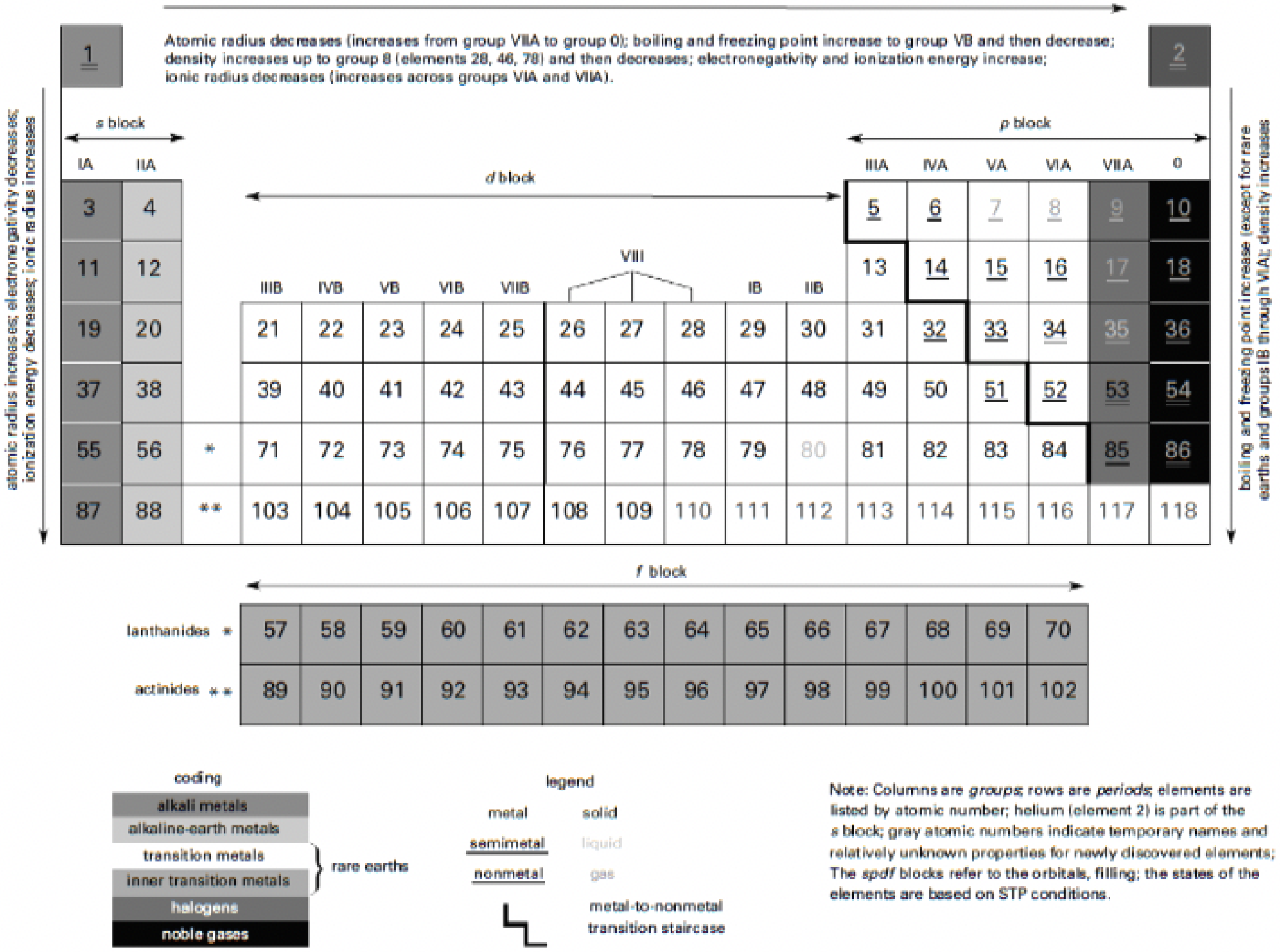


Figure 4: Electrical Properties Periodic Table

[Source: Ref. A]

Types of Materials

Materials are broadly classified as four main types: metals, polymers, ceramics, and composites.

Metals are materials characterized by high thermal and electrical conductivity. They are opaque and can generally be polished to a high luster. Further, they are normally heavy and deformable.

Metals are located to the left of the metal-to-nonmetal transition staircase shown in Fig. 4. Elements adjacent to the staircase exhibit characteristics of both metals and nonmetals and are sometimes called metalloids or semiconductors. Further, in some texts, they are called metals and in others they are called semiconductors. Items to the right of the staircase are called nonmetals.

Polymers, or *plastics*, are materials characterized by low thermal and electrical conductivity. They are generally transparent or translucent and poor reflectors of light. Finally, they are normally flexible and subject to deformation. Polymers are composed of those elements listed in Table 1.

Table 1: Elements in Polymers

Element Atomic Number	Element Name
1	hydrogen
6	carbon
7	nitrogen
8	oxygen
9	fluorine
14	silicon

Ceramics are compounds that contain metallic and nonmetallic elements. Concrete, rocks, glass, and nuclear fuel (UO_2) are all ceramics, to name but a few. Ceramics have extremely low thermal and electrical conductivity and for this reason are also called refractory materials. Indeed, they are highly chemically resistant (inert), can be opaque or translucent, and are very hard and brittle. Ceramics are composed of metallic elements combined with carbon, nitrogen, oxygen, silicon, or phosphorus, or some combination thereof.

Composite materials are the results of two or more materials, each having its own, usually different characteristics, becoming combined, giving useful properties for specific applications.

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Structure⇔Properties⇔Processing

The principle of value to the Materials Engineer or Scientist is that *the properties of a material originate from the internal structures of the materials.* [See Fig. 3 for the electron structure and Fig. 4 for a few of the properties impacting the Electrical Engineer.] This includes elements but also how atoms associate with their neighbors as in molecules, crystals, and microstructures [see Refs. K and L].

While *properties* come from the *structure*, *processing* can more than change an objects shape. often it changes the *properties* of the materials. As a single example, copper is used for its electrical conductivity in making wire. But if drawn through a die, it hardens and is less valuable as a conductor but more desirable if increase strength is desired. Thermal processes can change the internal structure of the copper, hence required temperature rise limits on wires, and more.

Processing can also be considered to be changes wrought by service life, environmental conditions, loading/heating cycles, radiation exposure, and more.

Mechanical Behavior

Many electrical components are housed in structures of a mechanical nature. Therefore, an understanding of some of the more common terms/properties will be helpful.

Deformation occurs when forces interact with materials. *Strain*, e, while technically unitless, is deformation per unit length: $\Delta L/L$. Elongation (final), e_f, and reduction of area, R of A, are also unitless.

Equation 1: Final Elongation

$$e_f = \frac{L_f - L_0}{L_0}$$

Equation 2: Reduction of Area

$$R \text{ of } A = \frac{A_0 - A_f}{A_0}$$

Stress, s, is the force per unit area on a given material. *Strength, S*, is a measure of the level of stress required to make a material fail. *Ductility*, is the amount of permanent strain prior to fracture. And, finally, the *toughness* is the amount of energy absorbed during failure. The YP or *yield point* is the point of the stress-strain curve where deformation begins. Example of these terms for various materials is shown in the figure below.

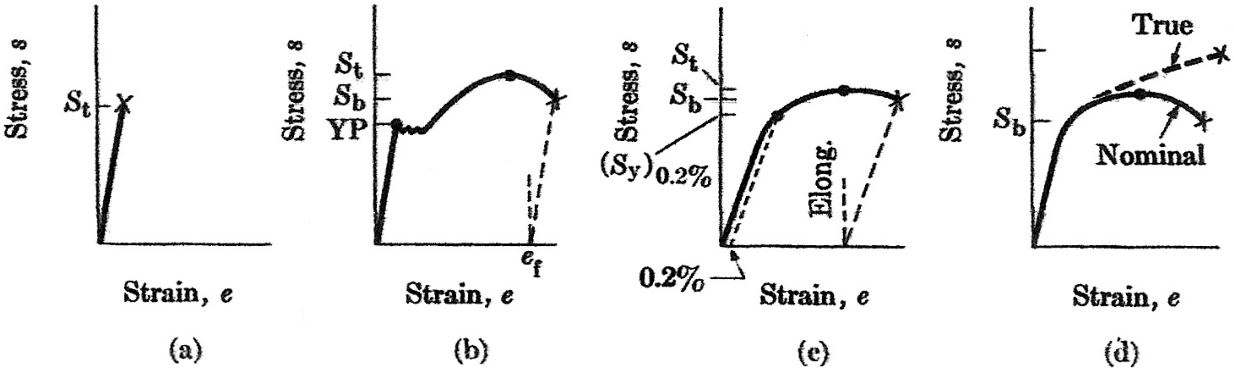

Figure 5: Stress-Strain Diagrams
(a) Non-Ductile, no Plastic Deformation [Cast Iron]
(b) Ductile with Yield Point [Low-Carbon Steel]
(c) Ductile with no marked Yield Point [Aluminum]
(d) True vs Nominal Stress-Strain Curve
[Source: Ref. H]

Per all the diagrams the initial stress is proportional to the strain; that is, it is linear and reversible. Once the stress is gone, the strain is removed—this is lineal strain, more often called *elastic strain.* The ratio between the stress and the reversible strain is called the *modulus of elasticity* (*Young's Modulus*), *E*.

Equation 3: Young's Modulus

$$E = \frac{s}{e}$$

Example 1

A given copper wire nominally has a breaking strength of 225 MPa (≈32,600 psi). Its ductility is such that it undergoes 77% reduction in area before the break. [See Fig. 5(d).]

What is the true stress, σ_f, for fracture?

Solution

Based on the original area, A_0,

$$\frac{F}{A_0} = 225 \text{ MPa} = \left(225 \times 10^6 \ \frac{\text{N}}{\text{m}^2}\right) A_0$$

Now, calculate for the true stress, which needs the true area, one gets the following.[4]

$$\sigma_f = \frac{F}{A_{\text{true}}} = \frac{F}{(1-0.77)A_0} = \frac{\left(225 \times 10^6 \ \frac{\text{N}}{\text{m}^2}\right) A_0}{(0.23)A_0} = 9.78 \text{ MPa}$$

Or, in psi, 142,000 psi.

Comment

The true strain, ε, may be found from the cross sectional dimensions. Define the true strain as the following.

$$\varepsilon = \int_{l_0}^{l} \frac{dl}{l} = \ln\left(\frac{l}{l_0}\right)$$

Now assume a constant volume, $Al = A_0 l_0$ resulting in the following.

$$\varepsilon = \ln\left(\frac{l}{l_0}\right) = \ln\left(\frac{A_0}{A}\right)$$

This represents the true strain and holds for all strains, while being independent of gage length.

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[4] The true values incorporate the concept of the localized elongation and the gage length. Both of which are avoided by using the reduction in area—and result in the "true" curve of Fig. 5(e).

Thermal Behavior

Temperature and heat content differ. Temperature is the level of thermal activity. Heat content is thermal energy. The two are related via the heat capacity, in terms of total energy, Q, in a system, the following is the basic formula.

Equation 4: Total Heat Energy

$$Q = mc\Delta T$$

Transformation occurs when heat is added to or removed from materials. The first, and somewhat obvious one is a change in length. In the following the term α_L is called the *linear expansion coefficient.* The same generally occurs to the volume.

Equation 5: Linear Expansion

$$\frac{\Delta L}{L} = \alpha_L \Delta T$$

Other important heats of transformation include the *heat of fusion* [melting] and the *heat of vaporization* [gasification]. Solids can remain solids and still undergo transformations of their structures resulting in a change of characteristics.

An example is shown in Fig. 6, where copper melts at 1084°C and iron undergoes solid transformation at 912°C.

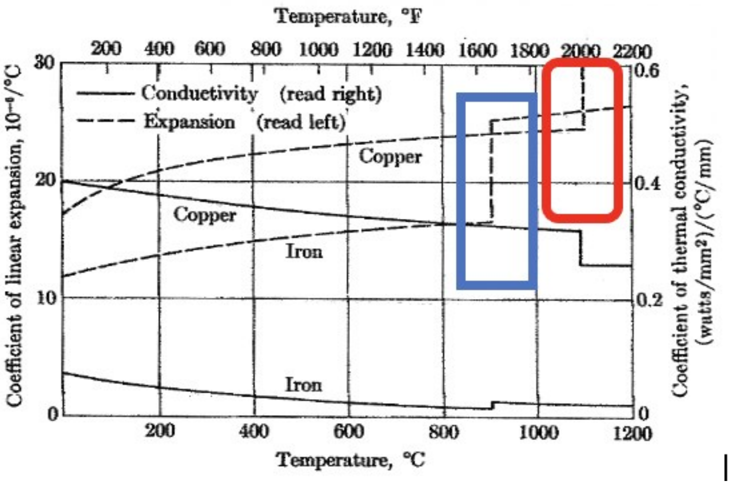


Figure 6: Thermal Properties of Copper and Iron

[Source: Fig. 1-3.1 of Ref. H]

Heat transfer through solids takes place mostly by conduction. The heat flux **Q** depends on the thermal gradient $\Delta T/\Delta x$ and is related by the constant of proportionality called the thermal conductivity, *k*.

Equation 6: Heat Flux

$$\mathbf{Q} = \vec{Q} = k\left(\frac{T_2 - T_1}{x_2 - x_1}\right)$$

In terms of units, the heat flux is as follows. [See Fig. 6 as well.]

$$\frac{\text{energy}}{\text{area} \cdot \text{time}} = k\left[\frac{\text{temperature difference}}{\text{thickness}}\right]$$

Example 2

Given an aluminum wire stressed to 34.5 MPa (5000 psi) in tension.

What temperature increase is required to change its length by the same amount?

Solution

Data used is from various tabulated sources. Start with the formula that has the desired quantity. Then combine the concepts and definitions.

$$\frac{\Delta L}{L} = \alpha_L \Delta T$$

$$\Delta T = \frac{\Delta L / L}{\alpha_L}$$

But noting that Young's Modulus is

$$E = \frac{s}{e} = \frac{s}{\Delta L / L}$$

Combining

$$
\begin{aligned}
\Delta T &= \frac{\Delta L/L}{\alpha_L} = \frac{s/E}{\alpha_L} \\
&= \frac{34.5\times 10^{6}\ \text{Pa}/70{,}000\times 10^{6}\ \text{Pa}}{22.5\times 10^{-6}\ {}^{\circ}\text{C}^{-1}} \\
&= 22^{\circ}\text{C} \quad \left(22\ \text{K}\right)
\end{aligned}
$$

Electrical Behavior

The electrical properties of materials can be beneficial or detrimental. On the beneficial side consider *battery cells* and *electroplating*. On the detrimental side consider *galvanic corrosion.*

Battery cells offer benefits like **portable, on-demand power, high energy density** for compact devices, long life, fast charging, grid stabilization by storing renewable energy, and reduced emissions.

Electroplating benefits include enhanced **corrosion resistance, improved durability, and better appearance** by applying a thin layer of metal, plus it boosts **electrical conductivity, reduces friction, increases hardness, and can be cost-effective** by using cheaper base metals with precious or functional finishes, extending product life and performance across many industries.

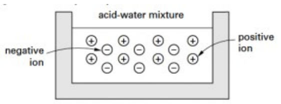
Galvanic corrosion is detrimental for obvious reasons—mechanical failure. It occurs when two different materials of different solution potential are in contact. This results in the loss of the anode material.

Battery Cells

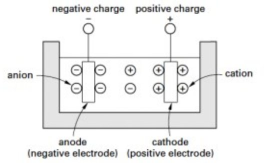
A battery [an electrochemical cell] is a direct-current voltage source that converts chemical, thermal, nuclear, or solar energy into electrical energy. Chemical conversion is used in the majority of batteries and will be the focus of this section. The advantages of a battery as a source of electrical energy are its portability, storage capability, and nonpolluting operation. The disadvantages are its general energy inefficiency and low power capability, though both these disadvantages are lessening as battery technology advances.[5]

A battery may be made up of one or more units called cells. A *primary cell* undergoes chemical change in such a manner that one or both electrodes are unusable following discharge. A *secondary cell* undergoes chemical change in a reversible manner and can be recharged following operation. *Shelf life* is the time a battery can be stored without use before its capacity drops to a value that makes it uneconomical to store any further. The *cycle life* is the number of times a battery can be discharged or charged before the output voltage drops to a prescribed value. A cycle is a predetermined number of amp-hours (A·h) of discharge or charge.

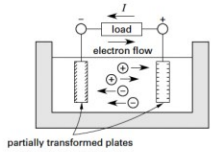
An electrolyte is a compound that, when dissolved in certain solvents (usually water), will conduct electricity. When placed in a solution, an electrolyte dissociates into ions as shown in Fig. 7(a). Placing two dissimilar electrodes in the solution results in a migration of the ions according to each ion's affinity. The result is a potential difference between the two electrodes, as shown in Fig. 7(b).



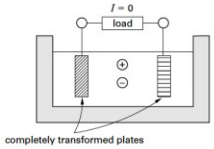
(a) electrolyte dissociation



(b) potential due to electrode affinity



(c) current flow



(d) cell discharged (electrode chemical change complete)

Figure 7: Battery Theory

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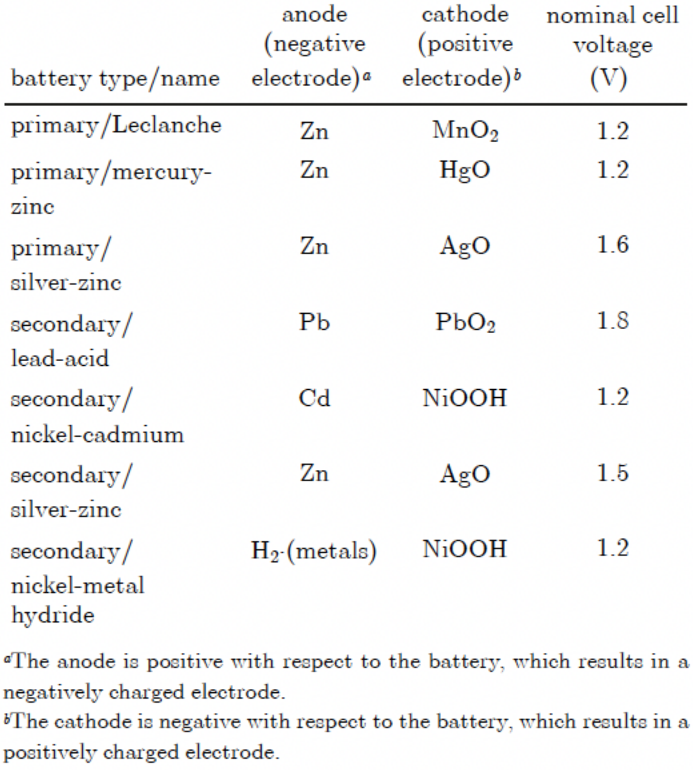
[5] Energy efficiency is the ratio of the discharging energy (in J) to the charging energy (in J). Values between 50% and 70% are common.

The magnitude of the potential difference depends on the materials used for the electrodes and is determined by the specific electronic structures, of those materials. Some typical combinations are shown in Table 2.

Table 2: Cell Voltages



battery type/name	anode (negative electrode)[a]	cathode (positive electrode)[b]	nominal cell voltage (V)
primary/Leclanche	Zn	MnO_2	1.2
primary/mercury-zinc	Zn	HgO	1.2
primary/silver-zinc	Zn	AgO	1.6
secondary/lead-acid	Pb	PbO_2	1.8
secondary/nickel-cadmium	Cd	NiOOH	1.2
secondary/silver-zinc	Zn	AgO	1.5
secondary/nickel-metal hydride	H_2·(metals)	NiOOH	1.2

[a] The anode is positive with respect to the battery, which results in a negatively charged electrode.

[b] The cathode is negative with respect to the battery, which results in a positively charged electrode.

The potential difference between two electrodes, or the *standard cell electromotive force* (*emf*), E^0, can be calculated from the standard electrode potentials, e^0, using Eq. 7.

Equation 7: Standard Cell EMF

$$E^0 = e_1^0 - e_2^0$$

The standard voltage of the cell, also called the theoretical voltage of a cell, may also be represented-directly by the materials used as the anode and the cathode. The two equations are equivalent, but using Eq. 8 automatically provides the correct sign for the potential.[6]

Equation 8: Standard Potential of a Cell

$$E_{\text{cell}}^0 = E_{\text{cathode}}^0 - E_{\text{anode}}^0 = E_{\text{reduction}}^0 - E_{\text{oxidation}}^0$$

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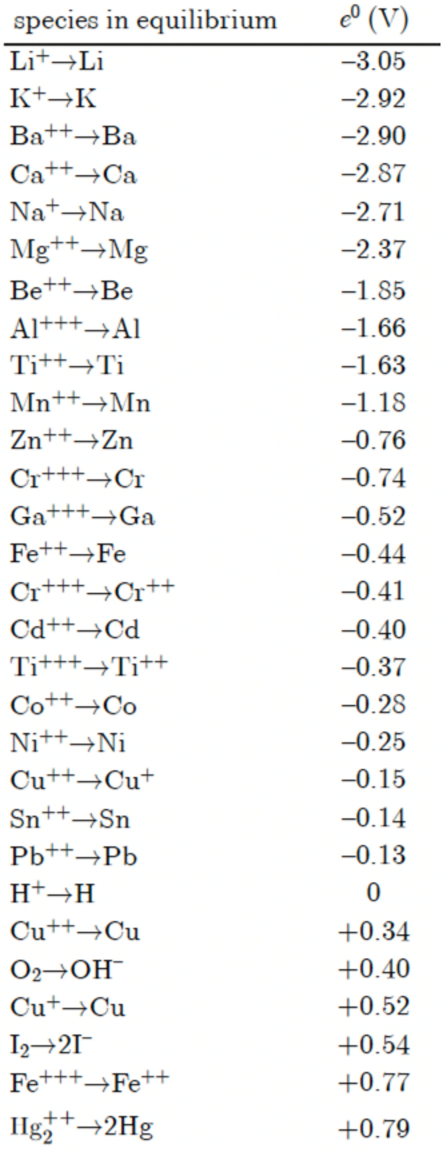
[6] A positive value of the cell potential indicates a spontaneous reaction.

Table 3 lists standard electrode potentials for common electrode reactions. By convention, this table gives the potentials for reactions based on setting *the hydrogen reaction at zero volts as an arbitrary reference*. However, the reactions are reversible, and multiplying the tabulated electrode potential by −1 gives the potential for the opposite reaction in which one or more electrons are lost. The larger the potential, the more easily the reaction can occur; a reversible reaction will tend to occur spontaneously in the direction with a positive potential, unless external voltages force the reaction to occur in the opposite direction.

Table 3: Standard Reversible Potentials at 25°C



species in equilibrium	$\mathcal{E}^0$ (V)
$Li^+ \rightarrow Li$	–3.05
$K^+ \rightarrow K$	–2.92
$Ba^{++} \rightarrow Ba$	–2.90
$Ca^{++} \rightarrow Ca$	–2.87
$Na^+ \rightarrow Na$	–2.71
$Mg^{++} \rightarrow Mg$	–2.37
$Be^{++} \rightarrow Be$	–1.85
$Al^{+++} \rightarrow Al$	–1.66
$Ti^{++} \rightarrow Ti$	–1.63
$Mn^{++} \rightarrow Mn$	–1.18
$Zn^{++} \rightarrow Zn$	–0.76
$Cr^{+++} \rightarrow Cr$	–0.74
$Ga^{+++} \rightarrow Ga$	–0.52
$Fe^{++} \rightarrow Fe$	–0.44
$Cr^{+++} \rightarrow Cr^{++}$	–0.41
$Cd^{++} \rightarrow Cd$	–0.40
$Ti^{+++} \rightarrow Ti^{++}$	–0.37
$Co^{++} \rightarrow Co$	–0.28
$Ni^{++} \rightarrow Ni$	–0.25
$Cu^{++} \rightarrow Cu^+$	–0.15
$Sn^{++} \rightarrow Sn$	–0.14
$Pb^{++} \rightarrow Pb$	–0.13
$H^+ \rightarrow H$	0
$Cu^{++} \rightarrow Cu$	+0.34
$O_2 \rightarrow OH^-$	+0.40
$Cu^+ \rightarrow Cu$	+0.52
$I_2 \rightarrow 2I^-$	+0.54
$Fe^{+++} \rightarrow Fe^{++}$	+0.77
$Hg_2^{++} \rightarrow 2Hg$	+0.79

From Table 3, then, the available potential from a pair of electrode materials can be found, as well as the direction of the associated reaction. For example, if the electrodes are of lithium and copper, the two electrode reactions would be

$$\mathrm{Li}^+ + \mathrm{e}^- \rightarrow \mathrm{Li}$$

$$\mathrm{Cu}^+ + \mathrm{e}^- \rightarrow \mathrm{Cu}$$

From Table 3, when a lithium ion (Li^+) gains an electron to become lithium (Li), the reaction has a potential of –3.05 V. Because this potential is negative, however, this reaction tends to occur spontaneously in the opposite direction, with lithium losing an electron to become a lithium ion. When a copper ion (Cu^+) gains an electron to become copper (Cu), the reaction has a potential of +0.52 V; as this potential is positive, this reaction tends to occur spontaneously in this direction rather than the reverse.

If a copper electrode and a lithium electrode are placed together in solution, with no outside forces acting, the net reaction will be

$$\mathrm{Cu}^+ + \mathrm{Li} \rightarrow \mathrm{Cu} + \mathrm{Li}^+$$

Electrons will flow from the lithium to the copper, and the net reaction will have a valence change of one electron and a potential of 3.05 V + 0.52 V = 3.57 V.[7]

The potential is determined entirely by the chemical reactions that result from the materials used. *The potential cannot be increased or decreased by changing the amount of material in the electrodes; it can only be changed by using a different combination of materials.*[8]

Regardless of the material used and the potential developed in a battery, the chemical reaction that generates current starts when an external load is attached to the terminals. When an external load is attached to the terminals, current flows from the positive electrode (cathode) to the negative electrode (anode) in the external circuit as shown in Fig. 25.1(c). In the internal circuit, current flows from the anode to the cathode. Positive ions (cations) migrate toward the cathode (positive electrode), where they capture incoming electrons and become neutral atoms, transforming the material on the cathode. Negative ions (anions) migrate toward the anode where they lose their electrons by combining with the active material of the anode. When the reactive material on either plate is consumed, the chemical reaction stops and the current ceases as shown in Fig. 25.1(d).

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[7] Using Eq. 8 gives the correct sign.

[8] The amount of material determines the capacity. See Ref. A.

Positive and negative potential on a battery can be referred to in different ways. *The anode is positive with respect to the internal circuit (that is, the battery itself). The cathode is negative with respect to the internal circuit (that is, the battery itself).* In electrical circuit schematics showing a battery, the power is shown flowing out of the positively charged electrode (the cathode) during discharge, because it is positive with respect to the external circuit. *The positive, or conventional, current flows from the anode to the cathode within the battery and from the cathode to the anode in the external circuit (that is, from the negative electrode to the positive electrode within the battery and from the positive to the negative in the external circuit).* **On an actual battery terminal, however, the anode (negative electrode) is labeled as positive even though it has a negative charge. Alternatively, the anode (negative electrode) can be defined as the terminal at which conventional current enters a battery.** This plus and minus convention applies to electrochemical cells.[9]

Electrochemical vs Galvanic Cells

Consider the difference between electrochemical [batteries] cells and galvanic cells [dissimilar metals in solution] and the reason for the sign convention.[10]

The sign convention differs because galvanic cells (voltaic) use spontaneous reactions to produce electricity (anode negative, cathode positive), while electrolytic cells use external power to force non-spontaneous reactions (anode positive, cathode negative), essentially reversing the flow; however, the fundamental definition—oxidation at the anode, reduction at the cathode—always holds true.

In summary,

Galvanic (Voltaic) Cells

- **Spontaneous:** Convert chemical energy to electrical energy.
- **Anode:** Negative (-) because electrons are released here (oxidation), causing an electron buildup.
- **Cathode:** Positive (+) because it attracts electrons (reduction), accepting them from the anode.
- **Electron Flow:** From negative anode to positive cathode.

Electrolytic Cells

- **Non-Spontaneous:** Use electrical energy to drive a reaction (like recharging a battery).
- **Anode:** Positive (+) because an external source pulls electrons from it, making it electron-deficient.

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[9] The sign convention for electrochemical cells is the opposite of the sign convention for galvanic cells. The sign convention depends upon the flow reference: inside the cell or external to it.

[10] A battery is a galvanic (voltaic) cell when providing power (discharging) and electrolytic cell when being recharged. And, it is called an electrochemical cell in general.

- **Cathode:** Negative (-) because the external source forces electrons onto it.
- **Electron Flow:** Still from anode to cathode (oxidation at anode, reduction at cathode), but forced by the power source.

Galvanic Cells

If two electrodes are identical, place in solution and connected to a battery, the conditions necessary for electroplating occur, see the figure below. Electroplating is essentially corrosion in reverse.[11] The driving force is the battery, but the ultimate source of the electrons is the anode.

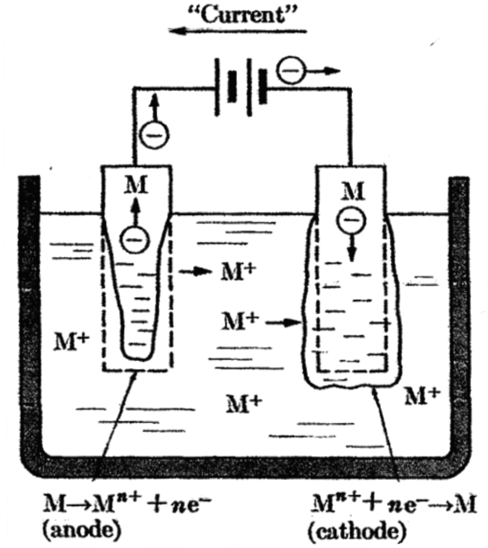

Figure 8: Electroplating

One mole of monovalent ions (0.6022×10^{24})[12] requires the same number of electrons for plating. This amount of charge is called a Faraday, $\mathscr{F}$.

Equation 9: Faraday

$$Q_C = \left(0.6022 \times 10^{24}\ \frac{\text{electrons}}{\text{mole}}\right)\left(0.1602 \times 10^{-18}\ \frac{\text{C}}{\text{electron}}\right) = 96{,}500\ \frac{\text{C}}{\text{mole}} = 1\ \text{Faraday}$$

Although the reaction above is indeed corrosion and galvanic in nature, it is atypical because it's assisted by a battery, and importantly uses identical electrodes.

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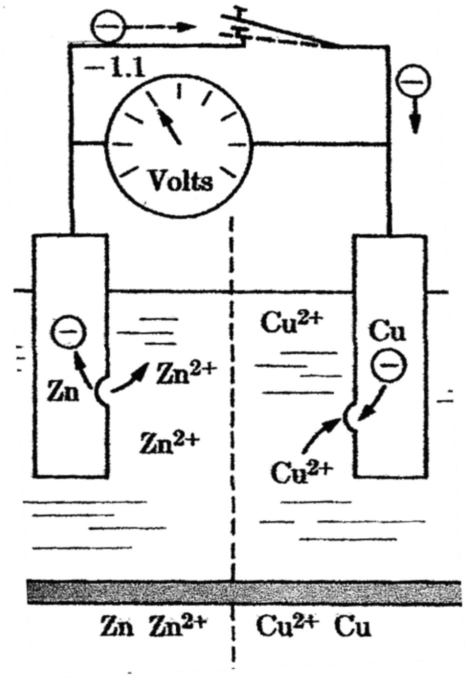
[11] Corrosion still occurs on the anode, but it's the cathode that is being plated, which is the desired outcome.

[12] The 10^{24} is used because this relates the number to "grams" of a material.

Corrosion as normally encountered involves dissimilar metals and no power source. This is called a *galvanic couple*. Use Table 3 with the following example to explain the concept. The take away is always check the galvanic series for the materials used and attempt to use those closest in potential or use another means of protection—such as sacrificial anodes.

Example 3

Consider two medals in a 1-molar solution as shown.[13]



The reactions taking place follow.

$$\mathrm{Zn}^0 \rightleftarrows \mathrm{Zn}^{2+} + 2e^-$$

$$\mathrm{Cu}^0 \leftrightarrows \mathrm{Cu}^{2+} + 2e^-$$

---

[13] One molar solutions are not usually encountered and are used for reference. Dilute concentrations can make adjustments from the standard potential using the Nernst equation. And, nominally, the "solution" may just be the air, moisture, some salt, et cetera.

The arrows are related to the potentials from Table 3, which indicates the zinc reaction goes more strongly in the direction shown, −0.76 V, than copper does in the same direction, +0.34 V. When connected by the switch shown The total voltage potential is −1.1 V.

The zinc is thus the material corroded.

ELECTRICAL MATERIALS

Insulating Materials

Types of Insulators

Insulating materials are generally of two types: organic and inorganic. Organic insulators include rubber, paper, oil, cotton, thermoplastic, silk, air, and synthetics,[14] among others. Inorganic insulators include glass, wool, and ceramics, among others.

An *insulator* is a device or material having a high electrical resistance (i.e., a low conductivity). A *dielectric* is an electrically insulating material or a material in which an electric field can be maintained with minimum dissipation of power. Insulators generally have a conductivity on the order of 10^{-12} S/m.

Dielectrics or insulators are materials that inhibit the passages of charges. Insulators have a conductivity range of 10^{-18} S·m up to approximately 10^{-4} S·m. The determining factors for the conductivity of insulators are the application and the voltage stress (that is, the magnitude of the voltage being insulated).

Plastics are widely used because of their low cost, ability to be molded into many shapes, and good mechanical properties. Ceramics, because of their very low electrical conductivity, also find many uses in electrical insulating applications. Oils, gases, and papers are inexpensive insulators. Mica, with its stability, arc resistance, and high dielectric strength, is another common insulator.

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[14] Combinations of insulators creates advantages. For example, glass fibers with synthetic varnishes produce an insulator that resists high temperatures and extreme vibration and shock.

Solid Insulators

Polymers are simple molecules linked to form thousands of original molecules [polymerization]. All synthetic insulators are polymers: rubber, resins, varnishes, bakelite, et cetera. Such polymers can be subdivided into numerous classes. Regardless, they are all used to insulate wires in motors, transformers, electromagnets, relays, and more.

Natural insulators are still found in use: cotton, paper, mica, and unfortunately asbestos. See the table below for a summary.

Table 4: Solid Insulators

Ceramic/Porcelain	Durable, used in spark plugs, heating elements, and high-voltage power line supports.
Drywood & Cotton	Older or less demanding applications.
Fiberglass	Combines glass fibers with resins for rigid insulation boards and coatings.
Glass	Used in high-voltage applications, from power line insulators to quartz.
Mica	A mineral good for high-temperature capacitors and transistors.
Paper/Cardboard	Used in transformers and capacitors where heat isn't extreme.
Plastics	PVC, nylon, PTFE (Teflon) are widely used for wire insulation due to flexibility and resistance.
Rubber	Natural or synthetic, excellent for flexible cable coatings.

Insulators have tightly bound electrons. They have dielectric strength meaning they can withstand strong electric fields without breakdown—that is with minimum dissipation of power.

Liquid Insulators

Mineral oil is used in high-voltage transformers both as an insulator and a heat transfer agent. It also minimizes oxidation since it surrounds windings and prevents air access. Oxidation is an important problem in high-voltage system because of the corona discharges produced. Such discharges generate ozone, which is a strong oxidizing agent.

As always, there are engineering tradeoffs. Oil is flammable at about 150ºC (≈300ºF).

Other types of liquid insulators include vegetable oils (natural esters), silicon oils, synthetic esters, and others.

Gaseous Insulators

The best known insulator is air. It has excellent thermal properties, acts as a cooling agent, and cost nothing. But, at very high temperatures $\geq 5000°C$ its conductivity approaches that of sea water—due to ionization.

A material found in high-voltage vacuum breaker is SF_6 (sulfur hexafluoride). It has the ability to absorb free electrons, which gives it 10 times the dielectric strength of air at 400 kPa.

Hydrogen is an insulating gas used to cool large rotating machines. Its lower density and viscosity compared to air produces less friction and windage losses. Plus, it absorbs about 14 times the heat air does. And, as long as oxygen content is kept below 10%, hydrogen will not explode or burn.

Insulator Properties

Deterioration of Organic Insulators

Organic insulators deteriorate do to heat, extreme cold, humidity, acidity, oxidation, vibration, and time. The insulators crystallize as temperature rises becoming hard and brittle. Cold conditions result in insulators becoming frozen and then cracking.

Although they can last a significant amount of time, if properly installed and kept dry, most recommend inspection/replacement approximately every 10 years. Less if temperature exceeds 100ºC. Synthetics can function well up to 200ºC and down to −60ºC.

Life Expectancy of Electric Equipment

Apart from actual failures, the life expectancy of electrical apparatus is limited by the temperature conditions of the insulation. A useful *thumb rule is that useful life of electrical equipment decreases by one-half for every 10ºC temperature increase*. [Ref G, Sec. 4-6]

So, if equipment is expected to last 10 years operating at 100ºC it will only last 5 years operating at 110ºC.

Thermal Classification of Insulators

Thermal classifications exist in IEEE 1 [Inactive Reserve], NEMA [letters], and internationally in IEC 60085 [more detailed ranges]. Underwriters Lab (UL) requirements vary by application. IEEE 1/NEMA

- Class A (105): Up to 104.9°C (e.g., cotton, silk)
- Class B (130): 130°C up to 154.9°C (e.g., epoxy, epoxy-mica)
- Class F (155): 155°C up to 179.9°C (e.g., mica, glass fiber, esterimide)
- Class H (180): 180°C up to 199.9°C (e.g., pure polyimide, mica, glass fiber)
- Class N (200): 200°C up to 219.9°C
- Class R (220): 220°C up to 239.9°C
- Class S (240): Above 240°C

Insulator Resistivity

As expected, no insulator is able to stop all current flow. The resistivity of an insulator has two components. On is the surface resistivity, R_S, which varies with the cleanliness of the surface and humidity—and the distance around the perimeter (roughly *c for circumference,* though it may not be a circle).[15] The other is the volume resistivity, R_V, which generally is very high, in the teraohm-meter range [TΩ·m].

Equation 10: Surface Resistance

$$R_S = \rho_s \frac{l}{c}$$

Equation 11: Volume Resistance

$$R_V = \rho \frac{l}{A}$$

---

[15] One could think of this as the perimeter of the object.

Example 4

An insulating spacer has a length of 76 mm and a cross-section of 12 mm by 12 mm. It is mounted between two flat pieces of metal. The published surface resistivity is 0.5 TΩ, while the volume resistivity is 0.01 TΩ·m. The voltage on the plates is 120,000 V.

What is the surface leakage current?

Solution

The surface resistance is as follows.

$$R_S = \rho_s \frac{l}{c} = \left(0.5\times 10^{12}\ \frac{\Omega}{\text{M}}\right)\left(\frac{0.076\ \text{m}}{4\times 0.012\ \text{m}}\right) = 792\times 10^{9}\ \Omega \quad (0.79\ \text{T}\Omega)$$

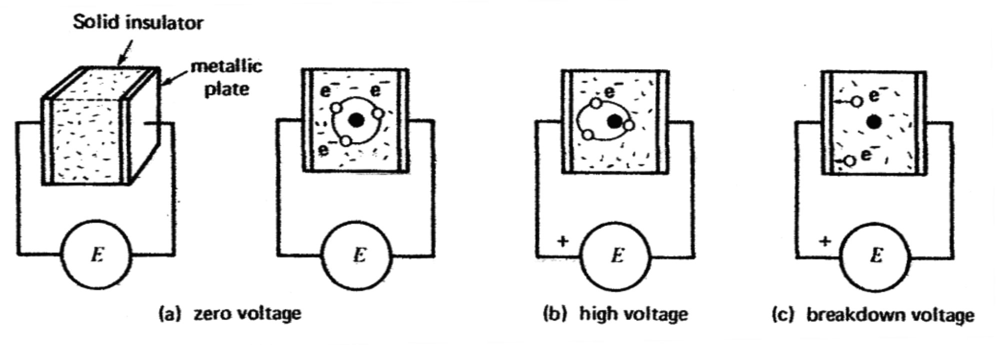
The surface leakage current is as follows.

$$E = I_S R_S$$

$$I_S = \frac{E}{R_s} = \frac{120\times 10^{3}\ \text{V}}{792\times 10^{9}\ \Omega} = 151.5\times 10^{-9}\ \text{A} \quad \left(0.152\ \mu\text{A}\right)$$

Dielectric Breakdown

A dielectric's purpose is to prevent current flow. As the voltage across the dielectric increases, it can continue to do so as long as electrons remain in their orbits (see the figure below, (a) and (b)). As the voltage increase the orbits change in response (see (c) below). Once the voltage is high enough to breakdown the dielectric, the electrons are freed and current flows.



(a) zero voltage

(b) high voltage

(c) breakdown voltage

Figure 9: Dielectric Breakdown
[Source: Ref. G, Fig. 4-4]

The ratio of the breakdown voltage to the insulator thickness is called the [in kV/m or MV/m].

If the dielectric was a gas, much the same process occurs only with ionization causing the resultant current.[16]

Lightning Insulators

Lightning insulators are devices, often made of glass or porcelain, used in lightning protection systems to *physically separate the lightning rod conductor from the structure it protects*, preventing current from jumping to the building while guiding it safely down to the ground via a conductive path.

Fundamentals

Lightning is an atmospheric electrical discharge resulting from the creation and separation of electric charges in cumulonimbus clouds. The exact mechanism causing the charge separation is unknown, with several theories explaining some, but not all, of the effects. Among these theories are the precipitation theory, the convection theory, and the theory of graupel-ice crystal interaction with charge-reversal temperature. The combination of all three is nominally a tripolar-electrified thundercloud as shown in the figure below.

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[16] This actually is the process in fluorescent lamps, and sodium vapor street lamps.

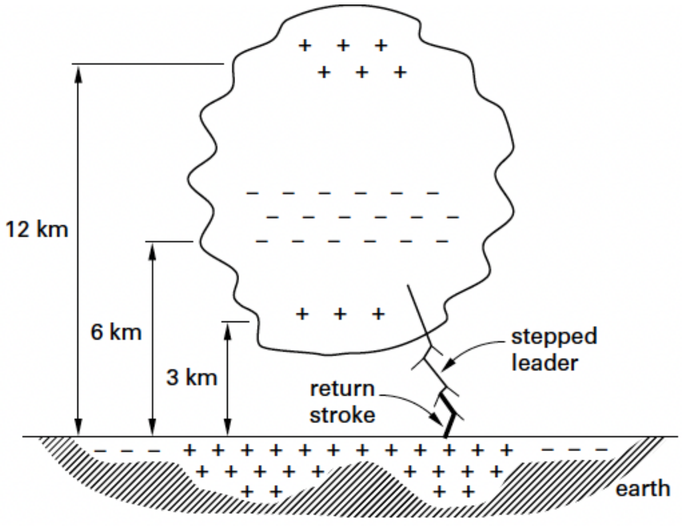

Figure 10: Lightning Development

Lightning occurs when the charge accumulation is such that the electric field between the cloud and the earth is high enough to break down the dielectric between them (i.e., the air). During the first phase of the process, local ionization occurs and discharges called *pilot streamers* create a path for the stepped leaders that follow.[17] The luminous stepped leaders move at 15% to 20% the speed of light, approximately 50 m at a time. Each step is separated by a few microseconds. The second phase occurs when the stepped leader reaches the earth or a building or power line on the earth's surface. When this occurs, an extremely luminous high intensity discharge occurs between the charge centers of the earth and cloud. The discharge moves at 10% to 50% of the speed of light and results in a current ranging from several thousand amperes to 200,000 A, with the peak between 1 μs and 10 μs. This discharge is called the *return stroke* or the *lightning stroke*.[18] In the final phase, strokes may occur between the charge centers within the cloud or to the ground through the established conducting channel because of the shifting potentials within the cloud. Approximately one-third of all lightning is a single stroke. When multiple strokes occur, the mean number is three. A stroke is any one of a series of repeated discharges that comprise a single discharge, which is called a *lightning flash* or *strike*.

The voltage of a lightning stroke varies from 10 MV to 1000 MV. The voltage of concern to the electrical engineer, however, is that which appears on the equipment. The overvoltage on the equipment depends on the impedance and the current. Lightning strokes are considered current sources at the point of the strike. The current varies from 1 kA to 200 kA.

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[17] Stepped leaders may originate from the earth and move skyward.

[18] The strokes mentioned are called ground flashes. Other strokes occur between clouds but are not of concern in power electrical engineering.

Concepts and Definitions

A power system must use a coordinated design to protect from internally generated surges, usually caused by switching, and externally generated surges, which are usually caused by lightning. Coordinated design includes shielding, grounding, surge arresters, switching resistors, breaker timing, and surge capacitors. The first three are primarily used to protect against lightning strikes, and the last three are used to protect against switching transients. Protection devices include spark gaps and surge arresters. The main purpose of the protection is to prevent insulation breakdown, damage to equipment, and outages.

Power system protection terminology has been standardized.[19] Some of the more important terms follow.

- *Withstand voltage* is the voltage that electrical equipment can handle without failure or disruptive discharge.
- *Transient insulation level* (TIL) is an insulation level specified in terms of the crest value of the withstand voltage, for a specified waveform.
- *Lightning impulse insulation level* is an insulation level specified for the crest value of a lightning impulse withstand voltage.
- *Basic lightning impulse insulation level* (BIL) is an insulation level specified in terms of the crest value of the standard lightning impulse.
- *Standard lightning impulse* is a full impulse with a front time of 1.2 μs and a half-value time of 50 μs, normally termed a 1.2/50 impulse.[20] (See the figure below.)

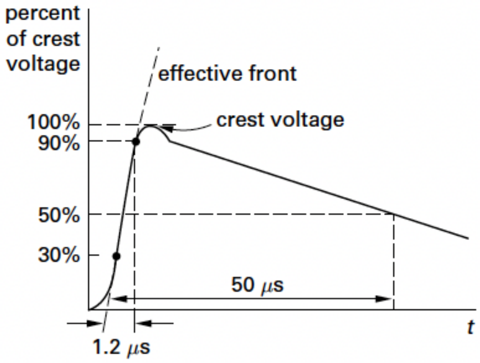

Figure 11: Standard Lightning Impulse

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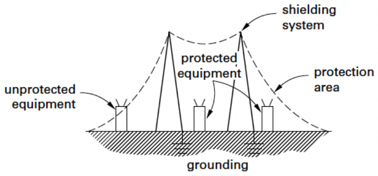
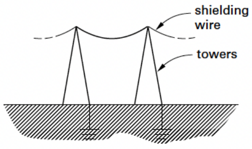
[19] Refer to the American National Standards Institute (ANSI) Std. C92.1.

[20] This is taken from American National Standard Measurement of Voltage in Dielectric Tests, C68.1.

Systems of approximately 230 kV or less cannot be insulated against direct lightning strikes because of the high voltage of the lightning, on the order of several million volts. To protect such systems a combination of shielding and grounding is used as shown in the next figure.



(a) equipment shielding



(b) line shielding

Figure 12: Shielding System

Indirect lightning strikes result in voltage surges by means of conductive coupling through the soil or by means of inductive and capacitive coupling to the lines. Induced voltage surges are typically less than 400 kV and can damage systems of 35 kV or less. Systems of 69 kV and more have sufficient insulation to withstand such surges. To protect such systems, surge arresters are used.

Methods of Analysis

The first step in analysis is the determination of the models to use for the various components of a distribution system. The models can be lumped parameters, distributed parameters, nonlinear equations, time varying equations, or logical equations. Once the models have been determined, three primary methods of analysis are used. *Graphical analysis* is based on the representation of voltages and currents as traveling waves and the use of a *Bewley diagram* to determine the waves that exist at any given point. *Analytical analysis* consists of a systematic approach to obtaining the solution of the differential equations that are used to represent a system, usually with Laplace transforms. *Numerical analysis* transforms the differential equations into discrete time equations, which are algebraic. The algebraic equations can be interpreted as *resistive companion circuits* and analyzed using standard techniques, such as KVL or KCL. Additionally, *Monte Carlo techniques* are often applied to the many uncertainties in the analysis of lightning overvoltages.

Two basic grounding models are used: low frequency and high frequency. Low-frequency models consider grounds to be purely resistive and require DC analysis.[21] High-frequency models require a complete electromagnetic analysis.[22] A general rule for determining which model to use is to compare the largest dimension of the grounding system, say length *l*, to the skin depth, *δ*, and then apply the rule of thumb given by the following.

Equation 12: Analysis Rule of Thumb

$$\frac{l}{\delta} = \frac{l}{\dfrac{1}{\sqrt{\pi f \mu \sigma}}} \begin{cases} < 0.1 \text{ DC Analysis} \\ >0.1 \text{ Electromagnetic Analyis} \end{cases}$$

Grounding for protection against lightning overvoltages is in addition to individual equipment grounding and power system grounding. The relationship among the different grounding systems is shown in next figure.

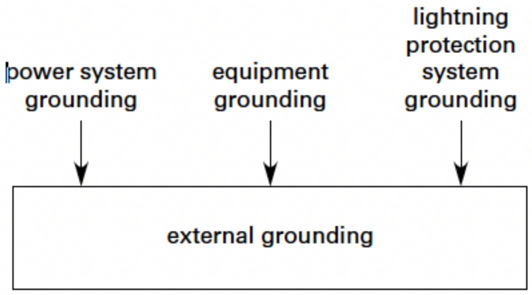


Figure 13: Grounding Systems

Protective Devices

When overvoltages occur, the transient is minimized and the system is protected by connecting the overvoltage condition to the ground until the energy is dissipated. This connection must be temporary and ideally will not allow current flow at normal operating voltages. This is accomplished by the use of strategically placed arresters, also called *surge arresters*. The quality of surge arresters is roughly measured by a *protection quality index* (PQI) given by

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[21] DC analysis is based on the method of moments or the relaxation method.

[22] Electromagnetic analysis is based on the *method of moments* or *finite-element analysis*.

Equation 13: Protection Quality Index

$$\text{PQI} = \frac{V_r}{V_p}$$

The denominator term is the *voltage protection level*, the maximum voltage allowed by the arrester. The numerator term is the *reseal level*, the voltage level at which minimal current flow through the arrester occurs. *The ideal protection quality index is one*.

Basic protection is provided by *air gaps*, also known as *spark gaps*. These devices are composed of two electrodes, the shape of which varies depending upon the application, separated by a distance. When the voltage across the air gap exceeds a given value, an arc is initiated. *Gapped surge arresters* use a series of spark gaps in conjunction with a nonlinear resistor as shown in the figure below. The nonlinear resistors have low resistance at high voltages, thus allowing the surge current. As the voltage of the lightning approaches that of the system, the resistance of the nonlinear resistors increases (typically as the fourth power of the voltage), extinguishing the arc without tripping protective devices.

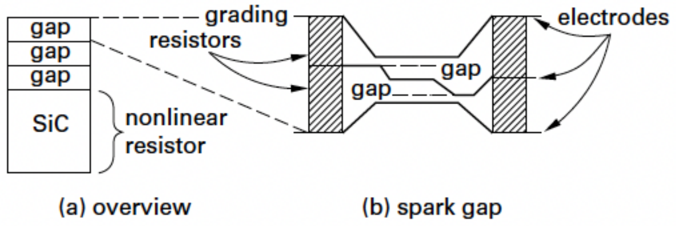

Figure 14: Gapped Surge Arrestor

Improved protection is provided by *metal-oxide varistor arresters* (MOVs). An arrester is shown in the next figure. The device is manufactured in such a manner as to approximate an ideal protective device, that is, a device that clips the voltage level at the maximum allowed and reseals at the same level. An MOV functions like back-to-back diodes. The MOV conducts current at the voltages normally experienced by the electrical system. Heating effects that could deteriorate the device must be accounted for in the design. The performance of MOVs at high discharge currents is improved by adding a shunt gap much like the construction in Fig. 14(a). When constructed in such a manner, they are called *shunt-gap MOVs*.[23]

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[23] Testing methods for MOVs, as well as for other electrical power equipment and systems, can be found in the InterNational Electrical Testing Association (NETA) Standard for Maintenance Testing Specifications for Electrical Power Equipment and Systems (ANSI/NETA MTS).

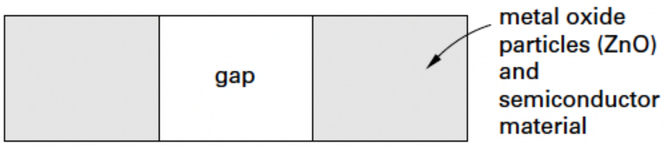

Figure 15: Metal Oxide Varistor (MOV) Arrestor

The effectiveness of surge arresters for a particular system is measured by the *protective margin* (PM) provided. The protective margin is determined by

$$\text{PM} = \frac{V_w - V_p}{V_p}$$

The term V_w is the voltage a given device can withstand before damage or insulation breakdown occurs. The term V_p is the voltage protection level provided by the arrester.[24]

Conducting Materials, Wire & Cable

Conductors

Good Conductors

Many conducting materials exist. The most common, with wide usage The best are metals, led by **Silver**, followed closely by **Copper**, **Gold**, and **Aluminum**, due to their free-moving electrons, with Copper being most common in wiring for its balance of conductivity and cost, while silver is best for heat/electricity but expensive. Other excellent conductors include Zinc, Nickel, Tungsten, Iron, and even saltwater and graphite (carbon).

Copper and aluminum are common for the transfer of large amounts of electricity. Aluminum is used in high power lines due to cost. Copper is used in smaller electrical machines because aluminum, for a given resistance, is nearly 1.7 times the volume of copper. See the table below for some common properties.

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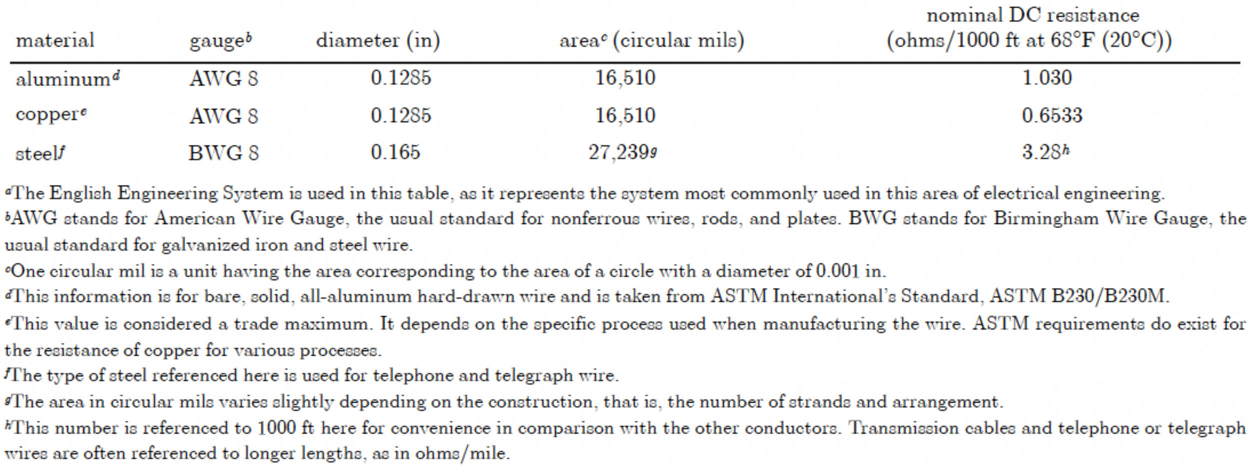
[24] The protection level provided is obtained from manufacturer's nameplate data on the arrester.

Table 5: Common Conductors



material	gauge[b]	diameter (in)	area[c] (circular mils)	nominal DC resistance (ohms/1000 ft at 68°F (20°C))
aluminum[d]	AWG 8	0.1285	16,510	1.030
copper[e]	AWG 8	0.1285	16,510	0.6533
steel[f]	BWG 8	0.165	27,239[g]	3.28[h]

[a] The English Engineering System is used in this table, as it represents the system most commonly used in this area of electrical engineering.

[b] AWG stands for American Wire Gauge, the usual standard for nonferrous wires, rods, and plates. BWG stands for Birmingham Wire Gauge, the usual standard for galvanized iron and steel wire.

[c] One circular mil is a unit having the area corresponding to the area of a circle with a diameter of 0.001 in.

[d] This information is for bare, solid, all-aluminum hard-drawn wire and is taken from ASTM International's Standard, ASTM B230/B230M.

[e] This value is considered a trade maximum. It depends on the specific process used when manufacturing the wire. ASTM requirements do exist for the resistance of copper for various processes.

[f] The type of steel referenced here is used for telephone and telegraph wire.

[g] The area in circular mils varies slightly depending on the construction, that is, the number of strands and arrangement.

[h] This number is referenced to 1000 ft here for convenience in comparison with the other conductors. Transmission cables and telephone or telegraph wires are often referenced to longer lengths, as in ohms/mile.

Conductor Shapes

Conductor shapes vary by purpose and may be round, square, flat, stranded or solid, some may simply be a bare rectangular bar—a bus bar.[25]

Stranding is used to improve flexibility. Some used steel cores to increase mechanical strength. Example illustrations follow.

Figure 16: Steel Reinforce Cable

[Source: *https://www.electrical4u.com/electrical-power-cable/*]

---

[25] Bus bar sizing is in IEC 61439 and a host of additional requirements in the NEC.

Figure 17: Cable Types

[Source: *https://www.electricaltechnology.org/2020/04/types-wires-cables.html#google_vignette*]

Round Conductors

Round conductors are standardized according to the Standard American Wire Gage (AWG). A thumbrule of some use follows. 1) A conductor with twice the cross section of another has a gauge number 3 times smaller. Example: AWG #15 has twice the cross section of AWG #18. 2) A conductor with 10 times the cross section of another has a gauge number 10 numbers smaller. Example: #4 gauge wire has the same cross section as ten #14 gauge wires.[26,27,28]

Circular Mils

Though SI system units are the standard, the units of area, A, are often put in the English Engineering System, especially in many tabulated values in the *National Electrical Code*. The area is given and tabulated for various conductors in *circular mils*, abbreviated cmil. One cmil is the area of a 0.001 in diameter circle. The concept of area in circular mils is represented by the following.

Equation 14: Definition of Circular Mils

$$A_{\text{cmil}} = \left(\frac{d_{\text{inches}}}{0.001}\right)^2$$

---

[26] The AWG system is a geometric progression. Wire gauge #4/0 and #36 are set at 460 mils and 5 mils, respectively. There are 39 wire sizes between the two; therefore, the ratio between two successive sizes is as follows: $\sqrt[39]{460/5} = 1.1229\rightleftharpoons$.

[27] The technical information to this point can be found in detail in the following locations. Insulator properties are found in the NEC Table 310.4(1). Ampacity of the conductors is in NEC Tables 310.17-21. Conductor and Insulator properties are in the NEC Chap. 9.

[28] IEEE 400 Series: Guides field testing and evaluation for power cable insulation (e.g., IEEE 400-2023 for cables 5kV+). See also NETA Acceptance Testing Specification. IEEE 383: Standard for qualifying electrical cables for nuclear facilities. Many other standards may apply.

Equation 15: Conversion—cmil to inches²

$$A_{\text{in}^2} = 7.854 \times 10^{-7} \times A_{\text{cmil}}$$

Equation 16: Conversion—cmil to cm²

$$A_{\text{cm}^2} = 5.067 \times 10^{-6} \times A_{\text{cmil}}$$

Stranded Cable

The cross section of stranded cable equals the sum of the cross sections of all the strands. So #10 stranded has a cross section of #10 solid wire.

Additionally, conductors bigger than #0000 (or 4/0) are usually identified by their cross section in thousands of circular mils (MCM). [You may see kCM or kcmil as well.]

Square Wires

Square wires have gauge numbers that correspond to those of round wires. If thee side of the square wire has the same dimensions as the diameter of the round wire, the two have the same gauge number.

Figure 18: Round vs Square Gauge

Fuses

Metals tend to melt at about 1200°C (≈2200°F). Their melting points determine their uses. For fuses, zinc is used at a melting point of 420°C (≈800°F). To determine when melting will occur one needs to know the diameter of the wire and the *fusing coefficient*. See the following table and equation.

Table 6: Metal Fusing Coefficients

Metal	k
copper	69
silver	62
aluminum	30

chromel	29
zinc	14
lead	6

Equation 17: Fusing Current

$$I = kd^{3/2}$$

For the indicated fusing coefficients, the value of d is in mm.

Fuses are inserted into contacts. Even with very low contact resistance, such resistance may consume large amounts of power resulting in carbonizing insulation and oxidation of metallic parts.

Example 5

The terminals of circuit-breaker are bolted to a bus bar with 1 mΩ of resistance. The breaker amp-frame (AF) is 4000 A. If carrying the maximum load, what is the power loss across the contacts?

Solution

The heat released is given by

$$\begin{aligned} P &= I^2R \\ &= (4000\ \text{A})^2(0.001\ \Omega) \\ &= 16{,}000\ \text{W} \end{aligned}$$

MAGNETIC MATERIALS

Theoretical Overview

Stationary electrons, electric field. Moving electrons, magnetic fields. Accelerating electrons, electromagnetic waves. These are spatial fields produced by charges, which can be static, slowly changing, or propagate as waves. The energy is stored in the field, which after all represents the configuration of electrons underlying it. A point in the field represents the amount of force another electron will feel in that spot. Why, because the electron in the original field is attempting to rearrange the original electrons and change (sometime collapse) the field.[29]

Energy is stored in electric and magnetic fields through the creation of potential energy in a defined space, essentially functioning as a "stress" or "pressure" within that region. Electric fields store energy in capacitors via separated charge, while magnetic fields store energy in inductors through current flow. This energy, representing the work required to arrange these fields, can be released when the field is allowed to collapse.

Magnetic Materials

Iron, cobalt, and nickel are the only elements that exhibit strong, room-temperature magnetism (ferromagnetism) because they possess a unique combination of partially filled $3d$ electron shells, unpaired electron spins, and a crystalline structure that allows these atomic magnets to align, creating a permanent, strong, and lasting magnetic field. Two views of the spin arrangements follow. One in terms of the orbital shells, the other in terms of individual spins.[30]

Magnetic materials establish fields without losing energy because the magnetism originates from the fixed internal alignment of electron spins, rather than a continuous power source. Permanent magnets utilize pre-aligned "magnetic domains" to produce a constant field, maintaining their strength indefinitely unless damaged or heated.[31]

---

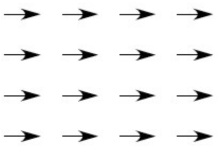
[29] The accelerating electrons are a bit different in that the electromagnetic field generated removes the energy from the accelerating electrons thus slowing them—changing their original field. And, the new field produced is now separated from the electrons that produced it. Electromagnetic Waves require a power source.

[30] From the orbital figure, it would appear that Chromium and Manganese should have strong magnetic properties, but in a crystalline structure they undergo changes called *exchange interaction* that prevent the strong magnetization.

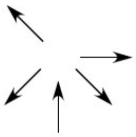
[31] The Curie point (or *Curie temperature,* T_C), is the critical temperature at which ferromagnetic materials lose their permanent magnetic properties and transition into paramagnetic materials. At this threshold, intense heat disrupts the internal atomic alignment, making it impossible for the material to maintain spontaneous magnetization.

Magnetic moment	Element	Number of electrons	Electronic structure 3*d* shell					4 *s* electrons
1	Sc	21	↑					2
2	Ti	22	↑	↑				2
3	V	23	↑	↑	↑			2
5	Cr	24	↑	↑	↑	↑	↑	1
5	Mn	25	↑	↑	↑	↑	↑	2
4	Fe	26	↑↓	↑	↑	↑	↑	2
3	Co	27	↑↓	↑↓	↑	↑	↑	2
2	Ni	28	↑↓	↑↓	↑↓	↑	↑	2
0	Cu	29	↑↓	↑↓	↑↓	↑↓	↑↓	1

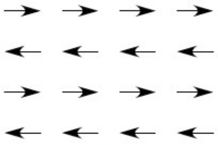
Figure 19: Spin Arrangements by Orbital Shells



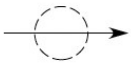
(a)
ferromagnetic



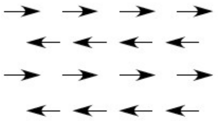
(b)
paramagnetic



(c)
antiferromagnetic



(d)
diamagnetic*



(e)
ferrimagnetic

*The circle represents an internal magnetic field

Figure 20: Spin Arrangements

Magnetic Phenomena

Magnetic phenomena are caused by the directed motion of charges. The effects occur perpendicular to this motion. In magnetic materials, the phenomena are associated with the orientation of the orbiting electrons and, to some extent, the spin of the electrons. Magnetic phenomena occur in a relatively small number of materials whose outer shell orbits fill prior to the inner shells. This occurs because the outer shell orbit configuration is actually a lower energy configuration than the inner shell.

Several types of magnetism exists.

Ferromagnetism is produced by the exchange of forces between atomic moments. This type of magnetism produces strongly magnetic materials with high permeability. Large clusters of atoms group together to form magnetic domains, each of which has the same atomic moment alignment within the domain. Ferromagnetism, named for iron, is the common magnetism one associates with horseshoes or toy magnets. See Fig. 21 for an example of a crystalline structure with domains and Fig. 22 for an example of magnetization curves.

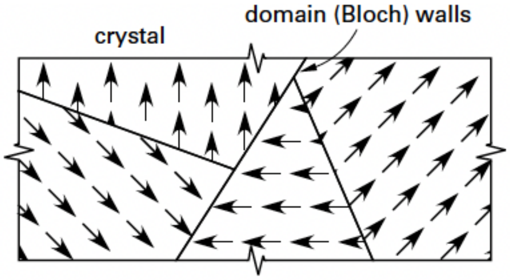


Figure 21: Magnetic Domains

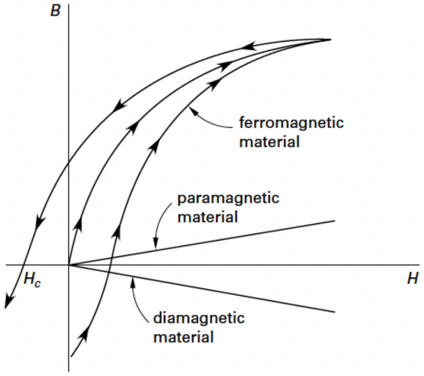


Figure 22: Magnetization Curves

Paramagnetism is produced by the orbital or spin moments of the electrons or both. The atomic alignment is minimal, that is, there is little or no domain formation, and as a result, paramagnetic materials are not strongly magnetic.

Antiferromagnetism is produced by the exchange of forces between atomic moments. Antiferromagnetic materials have an antiparallel arrangement of equal spins. Their magnetism is of a similar strength to that of paramagnetic materials.

Diamagnetism is produced by electron spins in antiparallel pairs in closed electron shells. Diamagnetic materials are weakly repulsive to external magnetic fields. These materials possess an internal magnetic field that opposes any externally applied magnetic field.

Ferrimagnetism is produced by the moment that results from the combination of two antiferromagnetic lattices. Ferrimagnetic materials contain two kinds of magnetic ions, arranged antiparallel but with unequal spins.

The materials and their magnetic properties are summarized in the special periodic table below.

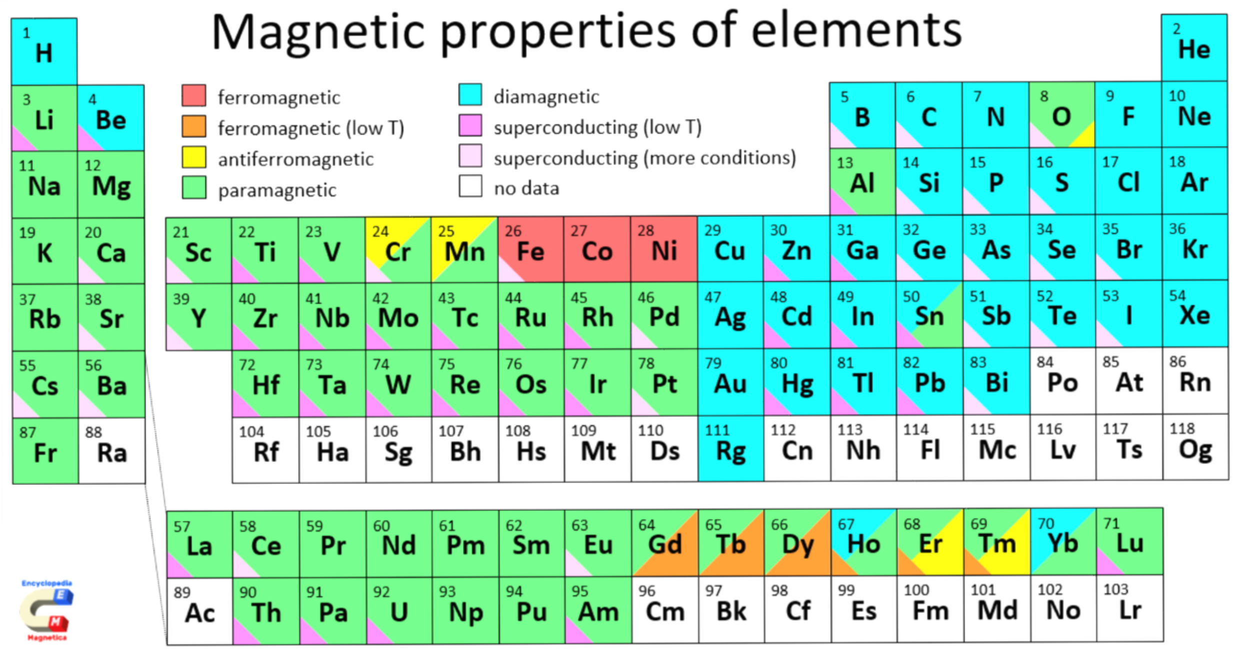


Figure 23: Magnetic Periodic Table

[Source: *https://www.e-magnetica.pl/doku.php/magnetic_materials*]

B-H Relationship

A typical B-H curve for a transformer core is shown in the figure below. It is useful in that it displays multiple concepts regarding magnets. For permanent magnets [or electromagnets—think relays, generators or motors) to be of use, they require an air gap. Then, the curve then displaying their properties is just Quadrant 2, the upper left quadrant shown.[32]

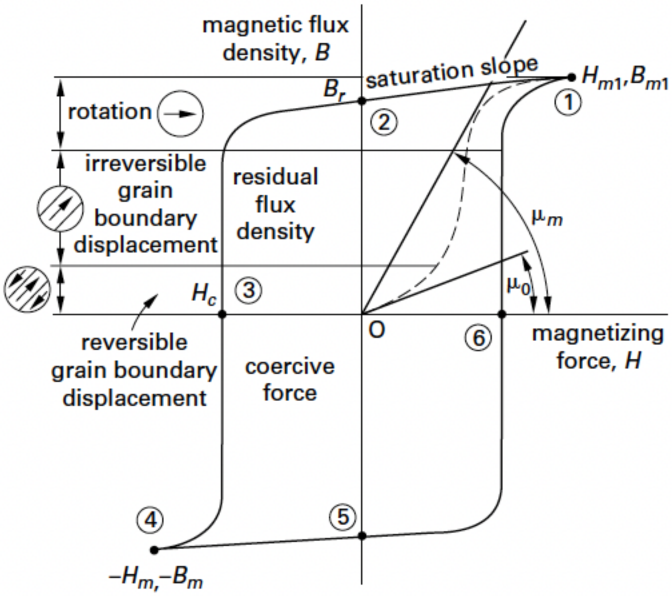


Figure 24: B-H Curve

Permanent Magnet Strength

Modern permanent magnets produce very strong magnetomotive forces and are normally smaller than equivalent electromagnets. Because no energy is required to maintain the magnetic field, permanent magnets allow for increased efficiency and smaller dimensions. The magnetomotive force is measured by the *magnetic potential difference*, U, across an air gap. The total energy, W, is given by the following.

Equation 18: Magnetic Energy in an Air Gap

$$W = \frac{1}{2}U\Phi$$

---

[32] For electromagnets, the flux is produced by the current, not the permanent magnetic flux density, B.

The flux produced, Φ, is equal to BA. And, the magnetomotive force, U, is equal to Hl. Thus, for a volume, V, of a magnet, the energy is the following.

Equation 19: Energy per Volume of Magnet

$$W = \frac{1}{2}\left(BH\right)V$$

Magnetic Attraction

Magnetics attract one another to lower their overall potential energy. This force of attraction is as follows.

Equation 20: Magnet Force of Attraction

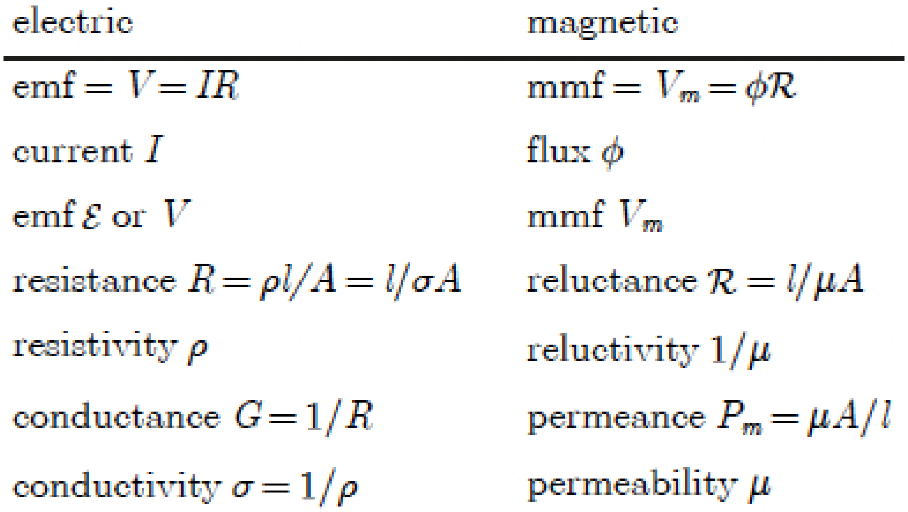
$$F = 400{,}000 B^2 A$$

F is the force of attraction [N], B is the flux density in the air gap [T], and A is the cross section of the poles. The constant 400,000, which accounts for the units, is equal $10^7/8\pi$.

Electric & Magnetic Circuit Analogies

The following table displays a comparison of electric and magnetic circuits. (See also App. G.)

Table 7: Electric vs Magnetic Circuits



electric	magnetic
emf $= V = IR$	mmf $= V_m = \phi\mathcal{R}$
current I	flux ϕ
emf $\mathcal{E}$ or V	mmf V_m
resistance $R = \rho l/A = l/\sigma A$	reluctance $\mathcal{R} = l/\mu A$
resistivity ρ	reluctivity $1/\mu$
conductance $G = 1/R$	permeance $P_m = \mu A/l$
conductivity $\sigma = 1/\rho$	permeability μ

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Items in **bold** are highly recommended for in-depth study. **Always use the latest editions.**

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NOTE

Electrical refers to something related to electricity while “electric” refers to a device or machine that runs on electricity. Nevertheless, the NEC is sometimes referred to as the National Electric Code. This Handbook with its illustrations, commentaries, and guidance is the best source found for those dealing with the NEC frequently in their projects.

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Appendix A: Equivalent Units Of Derived And Common SI Units

Symbol	Equivalent Units			
A	C/s	W/V	V/Ω	$J/(s\cdot V)$
C	$A\cdot s$	J/V	$(N\cdot m)/V$	$V\cdot F$
F	C/V	C^2/J	s/Ω	$(A\cdot s)/V$
F/m	$C/(V\cdot m)$	$C^2/(J\cdot m)$	$C^2/(N\cdot m^2)$	$s/(\Omega\cdot m)$
H	W/A	$(V\cdot s)/A$	$\Omega\cdot s$	$(T\cdot m^2)/A$
Hz	1/s	s^{-1}	cycles/s	$radians/(2\pi\cdot s)$
J	$N\cdot m$	$V\cdot C$	$W\cdot s$	$(kg\cdot m^2)/s^2$
m^2/s^2	J/kg	$(N\cdot m)/kg$	$(V\cdot C)/kg$	$(C\cdot m^2)/(A\cdot s^3)$
N	J/m	$(V\cdot C)/m$	$(W\cdot C)/(A\cdot m)$	$(kg\cdot m)/s^2$
N/A^2	$Wb/(N\cdot m^2)$	$(V\cdot s)/(N\cdot m^2)$	T/N	$1/(A\cdot m)$
Pa	N/m^2	J/m^3	$(W\cdot s)/m^3$	$kg/(m\cdot s^2)$
Ω	V/A	W/A^2	V^2/W	$(kg\cdot m^2)/(A^2\cdot s^3)$
S	A/V	1/Ω	A^2/W	$(A^2\cdot s^3)/(kg\cdot m^2)$
T	Wb/m^2	$N/(A\cdot m)$	$(N\cdot s)/(C\cdot m)$	$kg/(A\cdot s^2)$
V	J/C	W/A	C/F	$(kg\cdot m^2)/(A\cdot s^3)$
V/m	N/C	$W/(A\cdot m)$	$J/(A\cdot m\cdot s)$	$(kg\cdot m)/(A\cdot s^3)$
W	J/s	$V\cdot A$	V^2/Ω	$(kg\cdot m^2)/s^3$
Wb	$V\cdot s$	$H\cdot A$	T/m^2	$(kg\cdot m^2)/(A\cdot s^2)$

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Appendix B: Physical Constants

Table Note 1

Quantity	Symbol	US Customary	SI Units
Charge			
electron	e		-1.6022×10^{-19} C
proton	p		$+1.6022 \times 10^{-19}$ C
Density			
air [STP][32°F, (0°C)]		0.0805 lbm/ft^3	1.29 kg/m^3
air [70°F, (20°C), 1 atm]		0.0749 lbm/ft^3	1.20 kg/m^3
sea water		64 lbm/ft^3	1025 kg/m^3
water [mean]		62.4 lbm/ft^3	1000 kg/m^3
Distance			
Earth radius[2]	⊕	2.09×10^7 ft	6.370×10^6 m
Earth-Moon separation[2]	⊕☾	1.26×10^9 ft	3.84×10^8 m
Earth-Sun separtion[2]	⊕⊙	4.89×10^{11} ft	1.49×10^{11} m
Moon radius[2]	☾	5.71×10^6 ft	1.74×10^6 m
Sun radius[2]	⊙	2.28×10^9 ft	6.96×10^8 m
first Bohr radius	a_0	1.736×10^{-10} ft	5.292×10^{-11} m
Gravitational Acceleration			
Earth [mean]	g	32.174 (32.2) ft/sec^2	9.8067 (9.81) m/s^2
Mass			
atomic mass unit	μ or m_μ $\frac{1}{12}m\left(^{12}C\right)$	3.66×10^{-27} lbm	1.6606×10^{27} kg or 10^{-3} kg mol^{-1} / N$_A$
Earth[2]	⊕	4.11×10^{23} slugs	6.00×10^{24} kg
Earth [customary U.S.][2]	⊕	1.32×10^{25} lbm	-
Moon[2]	☾	1.623×10^{23} lbm	7.36×10^{22} kg
Sun[2]	⊙	4.387×10^{30} lbm	1.99×10^{30} kg
electron rest mass	m_e	2.008×10^{-30} lbm	9.1095×10^{-31} kg

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neutron rest mass	m_n	3.693×10^{-27} lbm	1.6750×10^{-27} kg
proton rest mass	m_p	3.688×10^{-27} lbm	1.6727×10^{-27} kg
Pressure			
atmospheric		14.696 (14.7) lbf/in^2	1.0133×10^{5} Pa
Temperature			
standard		32°F (492°R)	0°C (273 K)
absolute zero		–459.67°F (0°R)	–273.16°C (0 K)
Velocity[3]			
Earth escape		3.67×10^{4} ft/sec	1.12×10^{4} m/s
light (vacuum)	c, c_0	9.84×10^{8} ft/sec	2.9979 (3.00) $\times10^{8}$ m/s
sound [air, STP]	a	1090 ft/sec	331 m/s
sound [air, 70°F, (20°C), 1 atm]		1130 ft/sec	344 ft/s
Volume			
Volume: molal ideal gas (STP)[4]		359 ft^3 / lbmol	22.41 m^3/kmol

Table 1 Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.
2. Symbols shown for the solar system are those used by NASA. See https://science.nasa.gov/resource/solar-system-symbols/.
3. Velocity technically is a vector. It has direction.
4. The unit "lbmol" is an actual unit, not a misspelling.

Appendix C: Fundamental Constants

Quantity	Symbols	US Customary	SI Units
Avogadro's number	N_A, L		6.022×10^{23} mol^{-1}
Bohr magneton	μ_B		9.2732×10^{-24} J/T
Boltzmann constant	κ	5.65×10^{-24} ft-lbf/ R	1.3805×10^{-23} J/T
electron volt: $\left(\frac{e}{C}\right)$ J	eV		1.602×10^{-19} J
Faraday constant, $N_A e$	F		96485 C/mol
fine structure constant, inverse α^{-1}	α α^{-1}		7.297×10^{-3} (≈1/137) 137.035
gravitational constant	g_c	32.174 lbm-ft/lbf-sec^2	
Newtonian gravitational constant	G	3.44×10^{-8} ft^4 / lbf-sec^4	6.672×10^{-11} N·m^2 / kg^2
nuclear magneton	μ_N		5.050×10^{-27} J/T
permeability of a vacuum	μ_0		1.2566×10^{-6} N/A^2 (H/m)
permittivity of a vacuum, electric constant $1/\mu_0 c^2$	ε_0		8.854×10^{-12} C^2 / N·m^2 (F/m)
Planck's constant	h		6.6256×10^{-34} J·s
Planck's constant: h/2π			1.0546×10^{-34} J·s
Rydberg constant	R_∞		1.097×10^{7} m^{-1}
specific gas constant, air	R	53.3 ft-lbf/lbm- R	287 J/kg·K
Stefan-Boltzmann constant		1.71×10^{-9} BTU/ft^2-hr-°R^4	5.670×10^{-8} W/m^2·K^4
triple point, water		32.02 F, 0.0888 psia	0.01109 C, 0.6123 kPa
universal gas constant	R*	1545 ft-lbf/lbmol- R 1.986 BTU/lbmol- R	8314 J/kmol·K

Table Notes

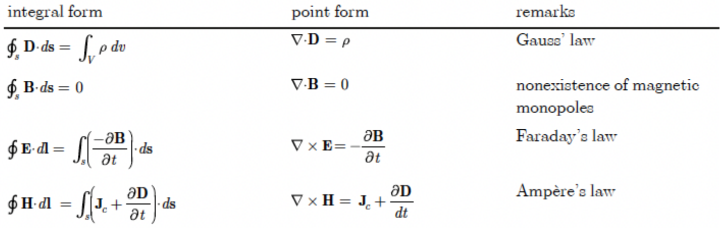
1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/. The unit in Volume of "lbmol" is an actual unit, not a misspelling.

Appendix D: Mathematical Constants, Signs/Symbols, Maxwell's Equations

Quantity	Symbol	Value
Archimedes' constant (pi)	π	3.1415926536
base of natural logs	e	2.7182818285
Euler's constant	C or τ	0.5772156649

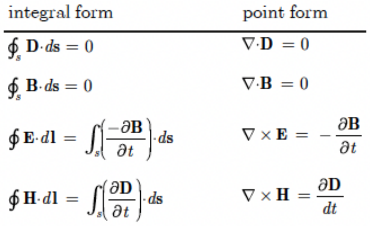
Signs/Symbols	Meaning
$\cdot$	multiplied by
/	divided by
:	ratio
$\gg$	much greater than
$\ll$	much less than
=	equals
$\equiv$	identical with
$\sim$	similar to
$\approx$	approximately equals
$\cong$	approximately equals, congruent
$\rightarrow$, $\doteq$	approaches
$\propto$	proportional, varies as
$\therefore$	therefore

Maxwell's Equations



integral form	point form	remarks
$\oint_s \mathbf{D}\cdot d\mathbf{s} = \int_V \rho\, dv$	$\nabla\cdot\mathbf{D} = \rho$	Gauss' law
$\oint_s \mathbf{B}\cdot d\mathbf{s} = 0$	$\nabla\cdot\mathbf{B} = 0$	nonexistence of magnetic monopoles
$\oint \mathbf{E}\cdot d\mathbf{l} = \int_s\left(\frac{-\partial\mathbf{B}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$	Faraday's law
$\oint \mathbf{H}\cdot d\mathbf{l} = \int_s\left(\mathbf{J}_c + \frac{\partial\mathbf{D}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{H} = \mathbf{J}_c + \frac{\partial\mathbf{D}}{dt}$	Ampère's law

Free-Space Form



integral form	point form
$\oint_s \mathbf{D}\cdot d\mathbf{s} = 0$	$\nabla\cdot\mathbf{D} = 0$
$\oint_s \mathbf{B}\cdot d\mathbf{s} = 0$	$\nabla\cdot\mathbf{B} = 0$
$\oint \mathbf{E}\cdot d\mathbf{l} = \int_s\left(\frac{-\partial\mathbf{B}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
$\oint \mathbf{H}\cdot d\mathbf{l} = \int_s\left(\frac{\partial\mathbf{D}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{H} = \frac{\partial\mathbf{D}}{dt}$

Electromagnetic Field Vector Equations

$$\mathbf{D} = \epsilon\mathbf{E} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon_0(1 + \chi_e)\mathbf{E}$$

$$\mathbf{B} = \mu\mathbf{H} = \mu_0\mathbf{H} + \mu_0\mathbf{M} = \mu_0(1 + \chi_m)\mathbf{H}$$

$$\mathbf{J} = \sigma\mathbf{E} = \rho\mathrm{v}$$

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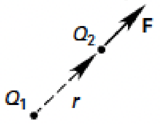
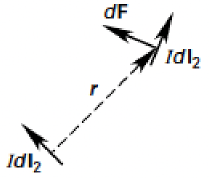
Appendix E: The Greek Alphabet

A	α	alpha	N	υ	nu
B	β	beta	Ξ	ξ	xi
Γ	γ	gamma	O	o	omicron
Δ	δ	delta	Π	π	pi
E	ε	epsilon	P	ρ	rho
Z	ζ	zeta	Σ	σ	sigma
H	η	eta	T	τ	tau
Θ	θ	theta	ϒ	υ	upsilon
I	ι	iota	Φ	ϕ	phi
K	κ	kappa	X	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
M	μ	mu	Ω	ω	omega

Appendix F: SI Prefixes

symbol	prefix	value
a	atto	10^{-18}
f	femto	10^{-15}
p	pico	10^{-12}
n	nano	10^{-9}
μ	micro	10^{-6}
m	milli	10^{-3}
c	centi	10^{-2}
d	deci	10^{-1}
da	deka	10
h	hecto	10^{2}
k	kilo	10^{3}
M	mega	10^{6}
G	giga	10^{9}
T	tera	10^{12}
P	peta	10^{15}
E	exa	10^{18}

Appendix G: Comparison of Electric & Magnetic Equations

equation description	electric version	magnetic version	remarks
experimental force law	Coulomb's law $\mathbf{F} = \left(\frac{Q_1 Q_2}{4\pi\epsilon r^2}\right)\mathbf{r}$ 	force between two current elements $dF = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I_2 d\mathbf{l}_2 \times (I_1 d\mathbf{l}_1 \times r)}{r^2}\right)$ 	The term $Id\mathbf{l}$ in the magnetic column is the equivalent of a "magnetic charge" q_m. The I or the $d\mathbf{l}$ can be the vector. The r is a unit vector pointing from 1 to 2.
field definitions from force law	$\mathbf{F} = Q\mathbf{E}$	$d\mathbf{F} = \mathbf{I}\times\mathbf{B}\,dl$ current element $d\mathbf{F} = \mathbf{J}\times\mathbf{B}\,dV$ distributed current element $d\mathbf{F} = q\,\mathbf{v}\times\mathbf{B}$ moving charge	The V used in this row represents volume, not voltage. The v is the velocity.
general force law	$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$ $d\mathbf{F} = (\rho\mathbf{E} + \mathbf{J}\times\mathbf{B})\,dV$ where $dQ = \rho\,dV$		The V in this row represents the volume, not voltage. The v is the velocity.
definition of scalar and vector potential	$E = -\nabla V$	$\mathbf{B} = \nabla\times\mathbf{A}$	**A** is the magnetic vector potential.
Poisson's equation for the potential function	$\nabla^2 V = -\frac{\rho}{\epsilon}$	$\nabla^2\mathbf{A} = -\mu_0\mathbf{J}$	From a knowledge of the charge distribution, the potential can be found and then the E and B fields determined.
Gauss's law enclosing charge and Ampère's law enclosing current	$\iint \mathbf{D}\cdot d\,\mathbf{A} =$ $\iiint \rho\,dV = Q$ $\nabla\cdot\mathbf{D} = \rho$	$\oint H\cdot d\mathbf{l} = I$ $\nabla\times\mathbf{H} = \mathbf{J}$	The V in this row represents volume.
constitutive relations	$\mathbf{D} = \epsilon\mathbf{E}$ $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$	$\mathbf{B} = \mu\mathbf{H}$ $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$	The second set of equations is always valid. The first set assumes the medium is linear and isotropic.
definitions of relative permittivity and permeability	$\epsilon_r = \frac{\epsilon}{\epsilon_0}$ $\epsilon_0 =$ 8.854×10^{-12} F/m	$\mu_r = \frac{\mu}{\mu_0}$ $\mu_0 = 4\pi\times10^{-7}$ H/m	

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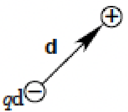
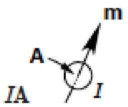
equation description	electric version	magnetic version	remarks
capacitance and inductance of a field cell	$\epsilon_0 = \frac{C}{l}$	$\mu_0 = \frac{L}{l}$	Field cells are a construct designed to represent free space in terms of a parallel plate capacitor and an inductor. This capacitance and inductance exist regardless of the presence of an electric or magnetic field.
capacitance and inductance	$C = \frac{Q}{V}$	$L = \frac{\Lambda}{I}$	Λ is the flux linkage.
energy density of a field	$U = \frac{1}{2}\epsilon E^2$	$U = \frac{1}{2}\mu H^2$	Both energy and momentum are carried by a field.
energy stored by capacitance and inductance	$W = \frac{1}{2}CV^2$	$W = \frac{1}{2}LI^2$	
electromotive and magnetomotive force with sources present	$\oint \mathcal{E} \cdot d\mathbf{l} = \mathcal{E} = V$	$\oint \mathbf{H} \cdot d\mathbf{l} = NI = F_m = V_m$	The $\mathcal{E}$ is the emf, not the permittivity. Without sources present, both line integrals are equal to zero.
dipole moments	$\mathbf{p} = q\mathbf{d}$ 	$\mathbf{m} = I\mathbf{A}$ 	
dipole torque	$\mathbf{T} = \mathbf{p} \times \mathbf{E}$	$\mathbf{T} = \mathbf{m} \times \mathbf{B}$	This torque occurs due to the dipole being immersed in an external **E** or **B** field.
dipole potential energy	$W = -\mathbf{p} \cdot \mathbf{E}$	$W = -\mathbf{m} \cdot \mathbf{B}$	

electric	magnetic
emf $= V = IR$	mmf $= V_m = \phi \mathcal{R}$
current I	flux ϕ
emf $\mathcal{E}$ or V	mmf V_m
resistance $R = \rho l / A = l / \sigma A$	reluctance $\mathcal{R} = l / \mu A$
resistivity ρ	reluctivity $1/\mu$
conductance $G = 1/R$	permeance $P_m = \mu A / l$
conductivity $\sigma = 1/\rho$	permeability μ

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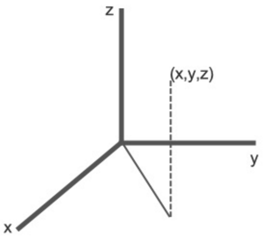
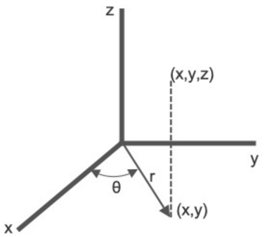
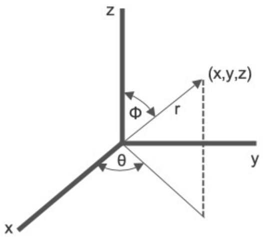
Appendix H: Coordinate Systems and Related Operations

Mathematical Operations	**Rectangular Coordinates**	**Cylindrical Coordinates**	**Spherical Coordinates**
Conversion to Rectangular Coordinants	 $x = x$ $y = y$ $z = z$	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	 $x = r\sin\phi\cos\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\phi$
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\boldsymbol{\theta} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\boldsymbol{\phi} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \theta}\boldsymbol{\theta}$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial\left(rA_r\right)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial\left(r^2 A_r\right)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial\left(A_\phi \sin\phi\right)}{\partial \phi} + \frac{1}{r\sin\phi}\frac{\partial A_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{r} & \boldsymbol{\theta} & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & A_\theta & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r^2\sin\theta}\mathbf{r} & \frac{1}{r^2\sin\theta}\boldsymbol{\phi} & \frac{1}{r}\boldsymbol{\theta} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ A_r & rA_\phi & rA\theta A_\phi \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial r}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{r^2\sin^2\phi}\left(\frac{\partial^2 f}{\partial \theta^2}\right)$