A SunCam Online Continuing Education Course

Power System Protection

Relaying Basics+

by

John A Camara, BS, MS, PE, TF

Nomenclature[1]

A	area	m^2
C	capacitance	F
E	voltage (source), electromotive force	V
I	current	A
pf or *pf*	power factor	-
R	resistance	Ω
V	constant or rms voltage	V
X	reactance	Ω
Z	impedance	Ω
K	constant	-

Greek Symbols

η	efficiency	-
ϕ	impedance angle or torque angle during normal conditions	degrees
ρ	resistivity	Ω·m
θ	torque angle during normal conditions	degrees
σ	conductivity	S/m
μ	permeability	H/m
Φ	magnetic flux	Wb
τ	time delay	s
τ	torque	N·m

[1] Not all the nomenclature, symbols, or subscripts may be used in this course—but they are related and may be found when reviewing the references listed for further information. Further, all the nomenclature, symbols, or subscripts will be found in of many electrical courses (on SunCam, PDH Academy, and also in many texts). For guidance on nomenclature, symbols, and electrical graphics: IEEE 280-2021, IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering, New York: IEEE; and IEEE 315-1975, Graphic Symbols for Electrical and Electronics Diagrams, New York: IEEE, approved 1975, reaffirmed 1993.

Subscripts

0	initial	-
0	standard conditions (if superscript): 1 molar concentration / 25°C / 1 bar≈1 atm	-
adj	adjustable	-
cn	capacity ultimate	-
cs	capacity service	-
f	fault	-
g	ground fault	-
n	nominal	-
o	overload	-
p	phase or pickup	-
pu	per unit	-
r	ratio or relay	-
r	release	-
ref	reference	-
sd	short disconnect	-
tg	time delay ground fault	-

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COURSE REFERENCES

The theoretical information is primarily from one of the author's books, Ref. [A]. The NEC Ref. [B] is always a useful source for electrical engineers. Information useful in many aspects of electric engineering may be found in [C] and [D]. Reference [E] has detailed descriptions of analysis techniques. Reference [F] covers many terms in EE with excellent definitions and explanations. Reference [G] provides excellent electric power engineering background. Reference [H] is the focus of this course, while Ref [I] provide detailed information on the subject. The appendices (A-F) cover information useful in many engineering tasks. Appendix (G) provides a side by side comparison of electric and magnetic equations. Appendix (H) provides useful information on coordinate systems and related operations. Use these texts or their counterparts for indepth information. References in bold are highly recommended. The latest editions are recommended.

This course will focus on basics, that rarely change, that provide the basis for all other knowledge.

POWER SYSTEMS

Structure

The protection and safety of power systems are determined by the structures of the systems. Most power systems consist of the three elements shown in Fig. 1. The *control system* uses equipment to maintain a nominal voltage and frequency, as well as maintain the power system's economic operation and security in interconnected networks. The control system operates continuously on the power system to adjust operating parameters. The *protection system* operates circuit breakers and other devices, such as *relays, that change the structure of the power system to protect it from abnormal situations, such as electrical faults*. The protection system operates when required and is faster than the control system. The advance of computer based electronic protection systems blurs the line between protection and control, incorporating both elements into one system; however, the functions remain the same.

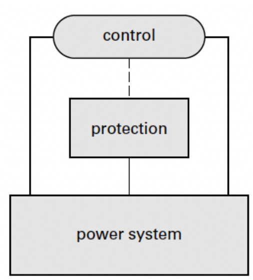

Figure 1: Power System Structure

Grounding

A power system's grounding scheme influences the fault current levels. As a result, the grounding scheme impacts the protection system relaying. In a true ungrounded system, there is zero fault current. In a real system, capacitive coupling of the feeders to ground results in a ground path if a fault occurs. The ungrounded system with a ground fault is shown in Fig. 2.

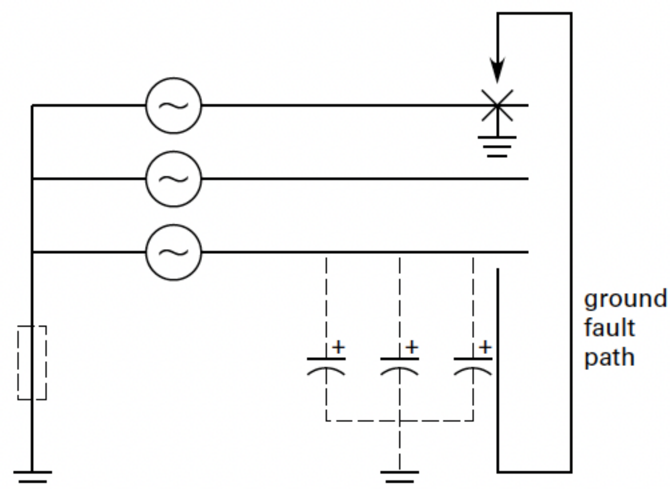

Figure 2: Unground System with a Ground Fault

If the capacitance is large enough, the capacitive ground fault becomes self-sustaining and must be interrupted by circuit breakers. Protective relays (devices used to make or break one or more connections in an electric circuit) must sense the low level of ground-fault current before a ground fault can damage the power system. To do so, impedance is inserted in the dashed box, as illustrated in Fig. 2. The impedance ensures adequate current flow that can then be sensed by the protective system, which can then react before the fault causes excessive damage.

Ungrounded systems provide for high system reliability because one ground does not cause protective systems to react. A second ground fault, or the capacitive ground fault, is required to cause breakers to open and equipment to be removed from service.[2] A disadvantage of ungrounded systems is that when a ground fault occurs on one phase, as shown in Fig. 2, the other two phase voltages become $\sqrt{3}$ larger than their normal values. The phasor representation of a grounded phase is shown in Fig. 3. Phase A voltage changes the reference on the ground resulting in increased

[2] This is the type of system used on submarines. Ground fault detectors are used to discover, find, and correct the ground before a second ground causes an interruption of power.

voltage differences on phases B and C to ground. Insulation levels on the conductors must account for this possible change.

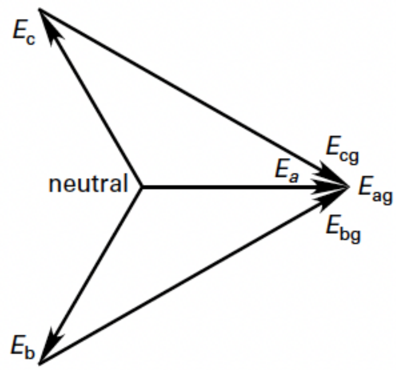

Figure 3: Phasor Diagram of Grounded Phase

On low voltage systems, the insulation level is usually set based on lightning-induced phenomena, which are generally larger than the fault-induced voltage increase. On high voltage systems (e.g., those with voltages greater than 100 kV), fault-induced voltages are more critical. As a result, on high voltage systems, a solidly grounded neutral is used. Such grounded systems are called *effectively grounded*. These systems have high fault currents and the impedance in the dashed box shown in Fig. 2 is set to limit the fault current to a value that circuit breakers can safely interrupt.[3]

The system in Fig. 2 is called an ungrounded system even though the wye connection may be grounded to allow detection of faults. (The conductors in the distribution system are *ungrounded*.) Such a ground is called a *functional earth*. A functional earth is defined as a ground that serves a purpose other than protecting against electrical shock. The conductors supply various loads that are usually encased in metal frames. These frames must be grounded for the safety of personnel.

The ground used to provide this safety is called a *protective earth* (PE).[4]

Configurations

The configuration of buses making up a power system will determine the available protection system options. A *radial system* is a serially connected system with a single source, as shown in Fig. 4. Such a system is normally limited to distribution systems (generally operating at voltages

[3] In lower voltage networks, an alternative to an ungrounded system is the *ground fault neutralizer* (GFN) known as the *Petersen coil*.

[4] International standards call an ungrounded system an IT system. The “I” means isolation and the “T” is for terra (or earth). When the neutral is effectively grounded along with the protective earth, the system is called a TN system. The “T” means terra and the “N” means neutral. When the neutral is grounded, but is physically separate from the protective earth ground, the system is called a TT system, for terra-terra.

of 100 kV or less). It has the advantage of being economical to build and the disadvantage of being less reliable since a problem at any point in the network affects all downstream loads. Radial systems are also easier to protect because a fault current can only flow from the source to the fault, making it less complex to calculate. Because the system is usually remote from the source, the fault current is approximately independent of the generating capacity.

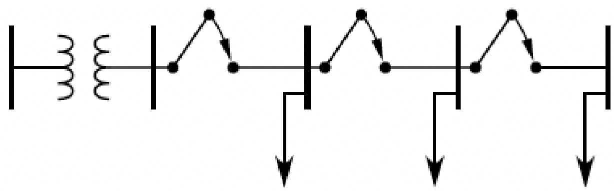

Figure 4: Radial Bus Configuration

A network system, as shown in Fig. 5, is normally used for transmission and sub-transmission systems (generally operating at voltages of 100–200 kV or higher). It has increased the reliability of power delivery to loads, and the loss of a single source of power has minimal impact on reliability. However, network systems are more complicated to protect than radial systems. A fault current can occur from multiple sources and from multiple directions. Because the source may be close to the fault, the fault current may vary widely depending on location.

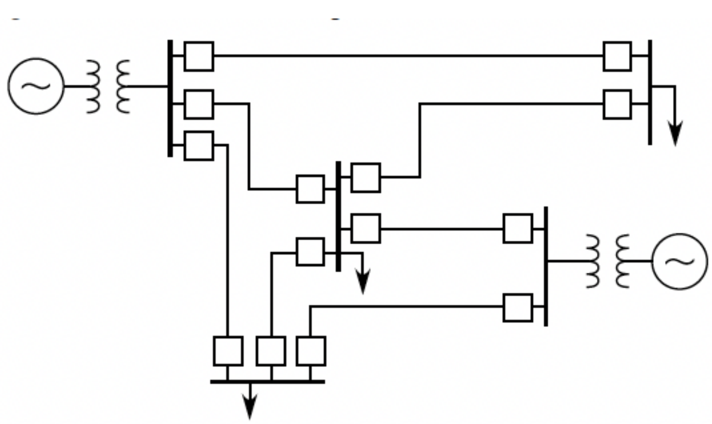

Figure 5: Network Bus Configuration

Each *substation* within a power system is named for its bus and breaker setup. Figure 6 shows a *single bus*, *single breaker substation configuration*.[5] This configuration is simple and cost effective because it uses fewer parts than other configurations. It is also the least flexible configuration and

[5] The IEEE standard practice is to label a breaker with a "52" within its box.

cannot undergo maintenance without de-energizing the bus and associated incoming transmission line.

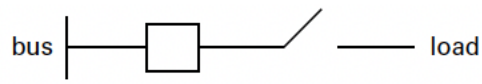

Figure 6: Single Bus, Single Breaker Substation Configuration

A *two bus, single breaker substation configuration* is shown in Fig. 7. Bus 1 and bus 2 represent two separate energized circuits. This configuration allows maintenance of individual breakers without de-energizing either bus. It also lets loads be split between the buses, which increases reliability.

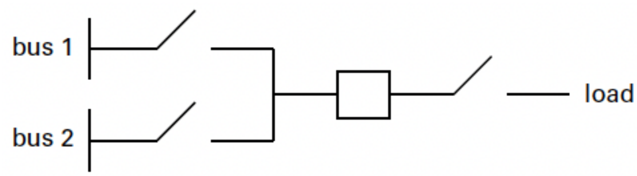

Figure 7: Two Bus, Single Breaker Configuration

A *two bus, two breaker configuration* is shown in Fig. 8. This configuration allows a bus or breaker to be removed for maintenance while maintaining power to the load. A bus fault should trip only one breaker, so that the remaining breaker can maintain power to the load. A load fault will cause both breakers to trip, which is not necessarily ideal for safety; however, this is the most flexible arrangement from the standpoint of operation and maintenance.

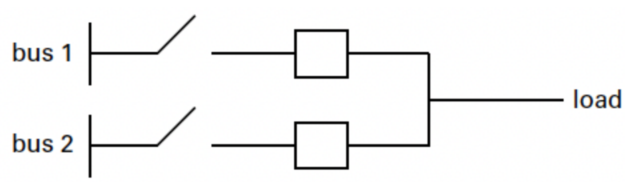

Figure 8: Two Bus, Two Breaker Configuration

A *breaker-and-a-half substation configuration* is shown in Fig. 9. This configuration has the advantages of the two bus, two breaker system, but with one-and-a-half breakers per load instead of two. With fewer breakers needed, this configuration is more economical, and it allows orderly expansion of the substation. As a result, this is the most common configuration for extra high voltage substations.

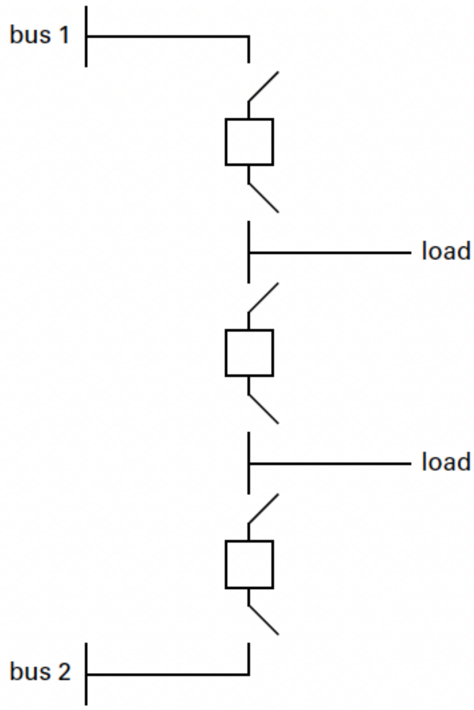

Figure 9: Breaker-and-a-Half Substation Configuration

Figure 10 shows a *ring bus configuration*. This system is highly flexible as long as the ring is maintained so that all the breakers are working. When a breaker is out of service for any reason, the system flexibility is significantly degraded.

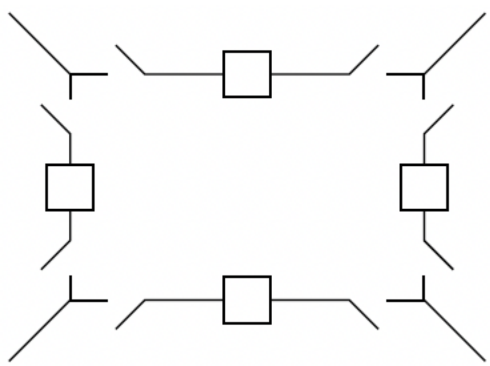

Figure 10: Ring Bus Configuration

RELAYS

Purpose

Relays are designed to determine electrical circuit operating systems and to trip circuit breaks when a fault is detected. They can be electromagnetic or electronic. The various protective functions available on a relay are given by standardized numbers in ANSI/IEEE Standard C37.2, examples of which are given in Table 1.

Table 1: ANSI/IEEE Power System Device Numbers

number	title	function
1	master element	places equipment in or out of operation
2	time delay or closing relay	provides desired amount of time delay
3	interlocking relay	allows or stops a given operating sequence
6	starting circuit breaker	connects device to source of voltage
11	multifunction device	combination of functions in one device
12	overspeed device	operates on machine overspeed
14	underspeed device	operates on machine underspeed
21	distance relay	functions when admittance, impedance, or reactance increases or decreases beyond a certain value[a]
24	volts per hertz relay	operates on a given V/Hz ratio[b]
27	undervoltage relay	operates on machine undervoltage
32	directional power relay	operates on power flow in a given direction[c]
46	phase-balance relay	operates when negative phase sequence exceeds a given value[d]
52	AC circuit breaker	operates a circuit breaker
58	rectification relay	operates when a rectifier fails to conduct or block correctly
59	overvoltage relay	operates on overvoltage condition
64	ground detector relay	operates upon failure of insulation to ground
72	DC circuit breaker	operates a circuit breaker
87	differential protection relay	operates upon a quantitative difference between two or more currents or other electrical quantities

[a]A *distance relay* is useful in determining if a fault exists in a given distribution system.
[b]In variable speed devices, the volts per hertz (V/Hz) ratio is adjusted to maintain the current to a machine constant. A constant torque results in a constant speed.
[c]Reverse power on a generator can damage the machine. This relay detects the angular relationship between the current and voltage, which determines the direction of power flow.
[d]This relays checks for negative phase-sequence currents above a given value. Negative phase-sequence currents indicate unbalance phases meaning that a fault exists in the system.

Prefixes to the numbers indicate the location. For example, RE stands for remote. Numerical prefixes indicate an association with a given unit. For example, 101 is a master element associated with unit 1 of a multiunit plant. *Suffix* letters indicate auxiliary devices and actuating quantities.

For example, CL indicates a device that is energized when the main device is closed, while a suffix of *C* stands for *current*, *P* for *power*, VAR for *reactive power*, and so on.

The goal of any protective system is a quick and accurate diagnosis of an anomaly, a fast response time to the indicated issue, and a minimal disturbance of the entire power system.

Reliability

The purpose of *relaying* is to remove any element that is functioning abnormally from a given power system. In general, the protective system does not prevent equipment damage. For the most part, relays operate after some damage has already occurred. Their purpose is to prevent further damage, minimize danger to personnel, and stabilize the system.

A relay's *reliability* is the probability that it will perform its intended function under a given set of conditions. The conditions can be environmental or electrical. Additionally, the assumption is made that maintenance is accomplished as specified.

A relay can be unreliable in two manners. First, a relay can fail to operate when intended. Secondly, a relay can operate when not expected. Therefore, the definition of protective systems relays must be expanded to include the requirement that they are also dependable and secure. A relay is *dependable* if there is a measure of certainty that it will operate for designed faults. A relay is *secure* if there is a measure of certainty that it will not operate incorrectly. As a system becomes more dependable, it generally becomes less secure.

Zones of Protection

The reliability requirement that relays are dependable (i.e., they operate only as intended) is further defined by the zones in which they operate. A *zone* is a region of the power system and can be closed or open. A *closed zone*, also known as a *differential*, *unit*, or *absolutely selective zone* is one where all power apparatuses entering the zone are monitored at the entry points. In an *open zone*, also known as an *unrestricted*, *non-unit*, or *relatively selective zone*, monitoring does not occur, and the zone is defined by *level of the fault current*.

Current transformers (CTs) whose input is utilized by the relays monitoring the zones are the proximate boundaries of the zones.[6] An example of zones is shown in Fig. 11.

[6] 4Neighboring zones always overlap to ensure system protection. Faults in an overlap region can disconnect both regions, possibly even more of the system, so overlaps are as small as practical.

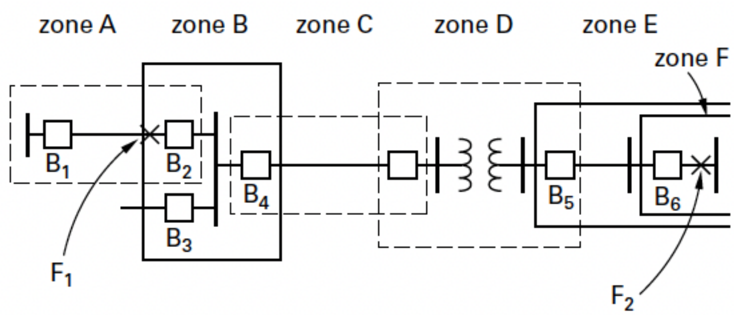

Figure 11: Closed and Open Zones

Coordination in protective systems is designed to meet the general goal of having the breaker, or other protective device, closest to the fault open first, and the breaker farthest from the fault open last. In Fig. 11, zones A, B, C, and D are closed zones. Zones E and F are open zones. Fault 1 occurs in two zones: A and B. As a result, breakers 1 through 4 will open, even though breakers 3 and 4 do not isolate the fault. Fault 2 occurs in an open zone. Breaker 6 is programmed to open first. If it fails to do so, breaker 5 will open after a specified delay. The setup in Fig. 11 represents a form of *coordination*.

Relay Speed

A relay senses the operating parameters of a given system. When those parameters or waveforms are distorted due to a fault, the relay must analyze the distortion to determine the relevant information and make a decision, based on internal logic parameters, regarding the fault. The time it takes for the relay to make the decision and the security of that decision is an inverse relationship. This inverse-time operating characteristic applies to all protection relays.

An *instantaneous relay* operates as soon as the logic circuitry makes the decision. No intentional delay is inserted after the decision. A time-delay relay has an intentional time delay inserted after the decision is made, such as would be done for backup protection. A *high-speed relay* is defined as one that operates in a specified time, such as 50 ms, or three cycles of a 60 Hz system, as given in various electrical standards. An ultra high-speed relay generally indicates a relay that operates in 4 ms or less, or approximately one-quarter of a cycle in a 60 Hz system, though it is not defined by standards.

Protection System Elements: Transducers/Relays/Circuit Breakers/Batteries

A protection system is comprised of three main elements: transducers, relays, and breakers. *Transducers* sense the desired signal and convert it into a form usable by the relays. *Current and voltage instrument transformers are examples of transducers*. *Relays* monitor the input from the transducers and provide an output to operate breakers in the event of a fault. *Breakers* are designed to interrupt power and isolate sections of the power system. They come in a variety of types and interrupting mediums, such as air, oil, gas, solid dielectric, and SF6.

When a fault occurs in a system, the AC voltage available may not be adequate to allow protective system operation. To ensure the necessary trips occur, *battery backups are used to power protective system components*. An uninterruptable power supply (UPS) is often used. A UPS ensures normal system power is provided to components unless that power is interrupted or inadequate. When such an interruption occurs, the batteries in the UPS automatically, and with minimal interruption time, provide the necessary backup power.

Two separate battery systems are sometimes used. One system powers electromechanical relays, and the other system powers solid-state relays. Electromechanical relays can produce severe transients on battery leads and potentially impact sensitive solid-state devices, which is why they are separated.

The general concept of the protection system, simplified, is shown in the following figure.

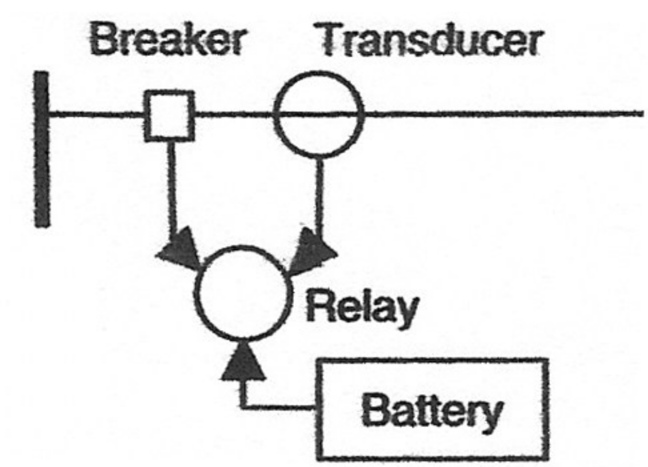

Figure 12: Protection System Main Components

Transducers/Transformers

Though not often called *transducers*, transformers do fall into this category. A transducer is defined as any device or element which converts an input signal into an output signal of a different form. Hence, the transducer in the figure above is often a transformer.

The symbol for a *current transformer* (CT) is shown in the figure below. Current transformers are used as instrument transformers (also known as *metering transformers*) or as transducers for protection circuits (i.e., relaying transformers).[7] They have low power, and ideally, high accuracy. The accuracy is measured in terms of the transformer's ability to lower the current; in other words, they transform the current without changing the expected magnitude or the phase relationship. In Fig. 13, the dots imply that the current leaving the secondary dot and flowing to the load is in phase with the current entering the primary dot.[8]

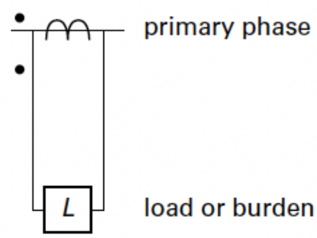

Figure 13: Current Transformer

The *burden* is the amount of power drawn from the circuit connecting the secondary terminals of instrument transformers, usually given as apparent power in voltamperes. The term burden distinguishes power for operation of the device itself from *power sensed*, which is being monitored and is referred to as the *load*. As long as the burden is within the rating of the CT, only acceptably small errors are introduced in the phase relationship between the primary and secondary currents, and the current transformation ratio is exact. For example, a CT with a primary current angle of 30° with respect to the voltage and a ratio of 100:5 will transform a 100 A current at 30° degrees to a 5 A current at 30°. When the burden is too high, or when non-ideal conditions exist, a CT will exhibit both phase angle errors and *transformation errors*. These errors vary with the loading of the transformer and the power factor.

Voltage transformers are also used for metering or relaying. They have low power and, ideally, high accuracy. Accuracy is measured in terms of the ability to transform the voltage without

[7] Metering transformers are designed to operate most accurately between no-load and full-load. Relaying transformers are designed to operate accurately at expected fault conditions.
[8] In terms of voltage polarity, both dots are positive at the same time.

changing the expected magnitude and phase relationship. The change in voltage output from no-load to full-load is called the *voltage regulation*.[9] The standard connections for transformers are shown in figure below. The polarities are called *additive* or *subtractive*. Additive and subtractive polarities are a way to avoid the dot convention. Figure 14(a) shows a voltage transformer with positive polarity using the dot convention. Figure 14(b) shows additive polarity, and Fig. 14(c) shows subtractive polarity. The "H" connection represents the primary phase. The "X" connection represents the secondary phase.[10]

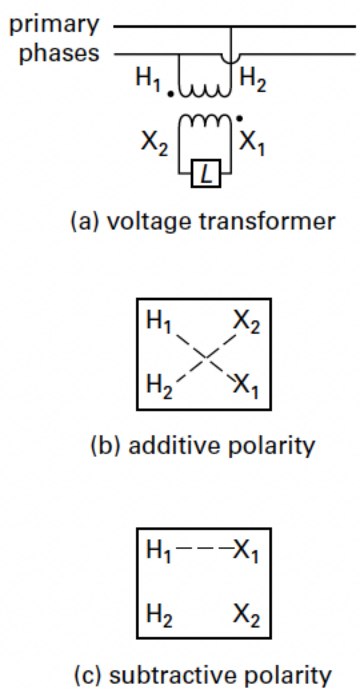

Figure 14: Voltage Transformer

Relays

A relay is defined as a device that is operated by the variation in conditions of one electric circuit and serves to make or break one or more connections in the same or another electric circuit. The varying types will be discussed in the next section.

[9] The phase change can be eliminated with filtering circuits.

[10] Transformer polarity matters because it determines the relative instantaneous direction of current flow between the primary and secondary windings, which is crucial for safety and system functionality. Incorrect polarity causes short circuits (opposing voltages) when transformers are parallel-connected, potentially leading to catastrophic equipment damage, while proper polarity ensures accurate 3-phase banking, metering, and protective relaying.

Circuit Breakers

In regard to this topic, the important information to know about circuit breakers is an understanding of trip curves and the symbology used.[11]

Trip Curve Generalities

A simplified trip curve is shown below. The curves are I^2t curves. This is called the joule energy, which is proportional to energy—energy per unit resistance. The units $A{\cdot}s^2$. While power is energy/time, it also depends on the voltage and resistance of the circuit. So this I^2t term is valid regardless of the voltage and/or resistance of the circuit. It is proportional to the heat, *Q*, generated during a fault. It is plotted on a log-log scale, with multiple of the nominal current (or rating of the breaker) on the x-axis and I^2t on the y-axis. The curve shows how much energy passes through (the let-through energy) before the trip occurs.[12]

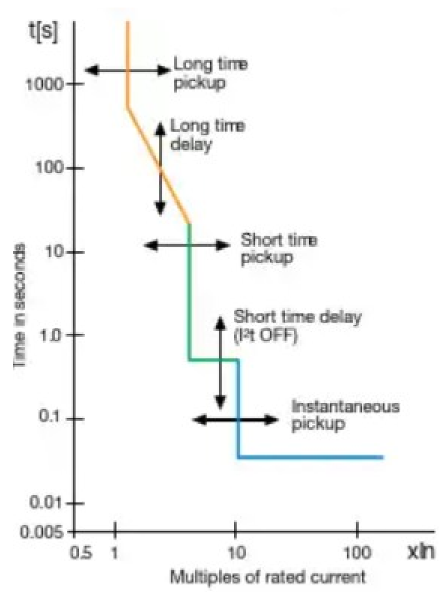

Figure 15: Simplified Trip Curve

Figure 16 shows a more detailed curve with various options (e.g., time delays for coordination). The terminology and symbology is standardized.

[11] What follows is taken from the UFC (Unified Facilities Criteria) [used by the military], various IEEE standards, and information from breaker trip curves (I^2t curves). Rather than being a specific requirement, these are guidelines; they are often average values of the many quantities encountered.

[12] While time-current curves show when a breaker trips, I^2t curves show how much energy made it through before the trip.

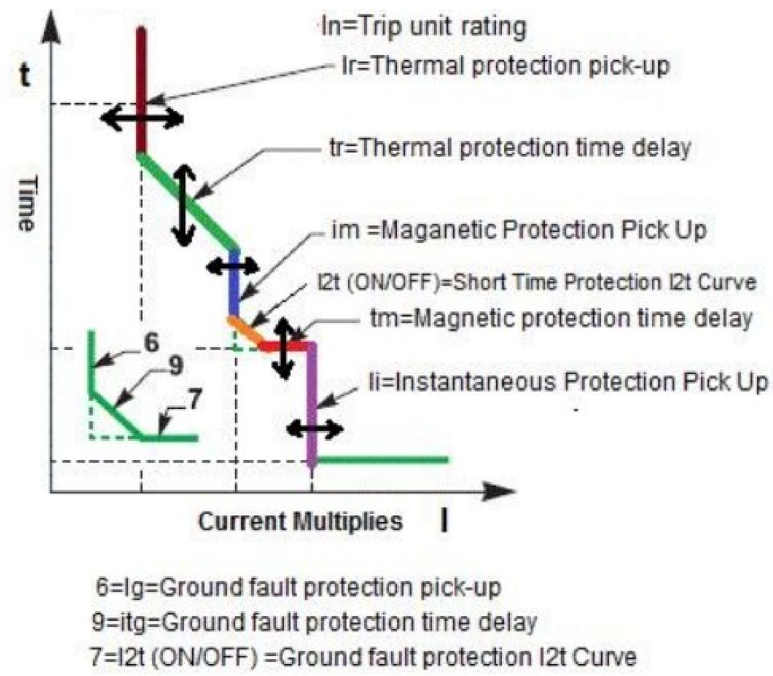

Figure 16: Detailed Trip Curve

Breaker Terminology

Trip Curve Terminology [in terms of both requirements and thumb rules]:

- I_n is Breaker Rating [n ≡ nominal/rated]
    - ***Breaker Frame Size***: 125% of Maximum Load
- I_o is Overload (for I_n x Setpoint) [o ≡ overload]
    - ***Overload***: Set at 125% of Load
- I_r is Long Time Pickup [Delay]: (I_o x Setpoint) [r ≡ release]
    - Allows for ***Inrush***
    - Up to 6-8 Times the Rated Current for the Delay—Diagonal Curve
    - Nominal 1 x I_n
- I_m is Short Time Pickup [Delay]: (I_r x Setpoint) [m ≡ magnetic]
    - Allows for ***Coordination***
    - Up to 10 Times Rated for the Delay—Horizontal Short Time Curve
    - Nominal 1.5 x I_n
- I_i is Instantaneous [i ≡ instantaneous]
    - Allows for ***Short-Circuit Protection***
    - Up to 10 Times Rated—Vertical Long Time Curve
    - Nominal 2 x I_n

Short-Circuit Terminology
I_k is Short Circuit Current [The ‘k’ is from the German *Kurzschluss*]
I_{sd} is Short Circuit Trip [Not Located on Unit] [sd ≡ short disconnect]
I_{cn} is Short Circuit Breaking Capacity [cn ≡ capacity nominal]
I_{cu} & I_{cs} is Breaking Capacity
Testing Differs
cu ≡ capacity ultimate—no damage
cs ≡ capacity service—three times and still serviceable / cs < cu

Trip Thumb Rules
'L' long-time trip (60-600 sec)
'S' short time trip (0.1 to 60 sec)
'I' instantaneous trip (No Delay Intended)
'G' ground fault (20% to 80% of Breaker Rating)

Standards Markings
UL: Underwriters Laboratory
CE: Conformitè Europèenne
CSA: Canadian Standards Association
DIN: Deutsches Institut für Normung [German Institute for Standardization]
RU: Recognized Component [UL Marking for a Component meant to be Use in a Listed Part]

Batteries

A protection system exists to remove faults. It can only do so if it has the power to operate the trip circuits. During a fault such power may not exists, or be too low to be effective. Therefore, a battery or batteries are used. All such batteries are connected via chargers that maintain full battery capacity during nominal operation.

Since electromechanical devices can cause severe transients, which could result in unintended operations in sensitive circuits, electromechanical devices and solid-state equipment are usually connected to separate batteries.

Relay Types

Generally when faults occur, currents increase, voltages drop, harmonics develop, and phase relationships shift. Relays, both analog and digital, can be made to sense and respond to these changes. A relay can be comprised of multiple types embedded in a single device. An explanation of some of these types follows.

Level Detection Relays

Level detection relays, also called *magnitude* and *overcurrent relays*, operate when a fault exceeds a given value.[13] The pick-up setting is the setting at which the relay operates. The properties of a typical level detection relay are shown in Fig. 31.14. The x-axis is a normalized value; that is, a ratio of the fault current to the pick-up current.

The time dial setting and the pick-up setting are selected to give the desired time response in the event of a fault. The time for the relay to function is found by first dividing the fault current by the pick-up setting to obtain the multiple of the pick-up setting. Using this value, starting at the bottom axis, trace a straight line to the value of the time-dial setting selected (see the right axis). From this point, trace a line to the left axis to obtain the operating time. The characteristic curves are plotted on log-log scale.

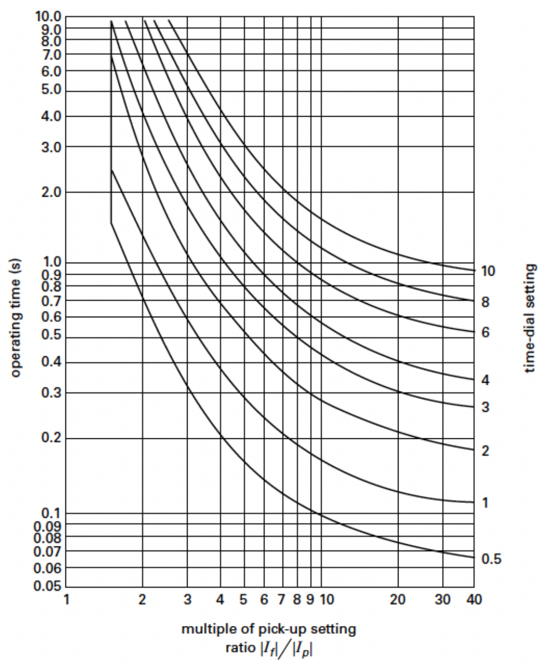

Figure 17: Level Detector Characteristics

Magnitude Comparison

This relay works on the principle that two paralleled networks should have the same or proportional currents within some acceptable tolerance, ∈ . As an example, if the top network A

13 A *fuse* is a simple level detector.

has a current exceeds the current of B by the tolerance value as in Eq. 1, (*and line B is not open*), the relay assumes there is a fault in network A and trip Breaker A.

Equation 1: Magnitude Comparison

$$|I_A| > |I_B + \square|$$

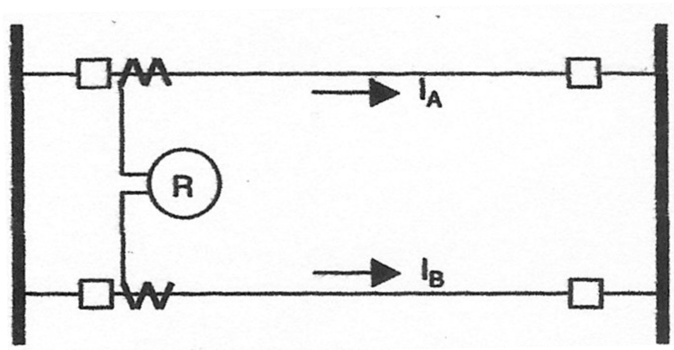

Figure 18: Magnitude Comparison

Differential Relays

Differential comparison relays operate when currents at the input and output of a zone are not identical. This method is very sensitive to changes and is effective in isolating faults. A sample scheme is shown in the figure below. The disadvantage to this scheme is that currents at each end of the zone can be separated by a significant distance, requiring an increased equipment cost for wiring and imposing possible time delays.

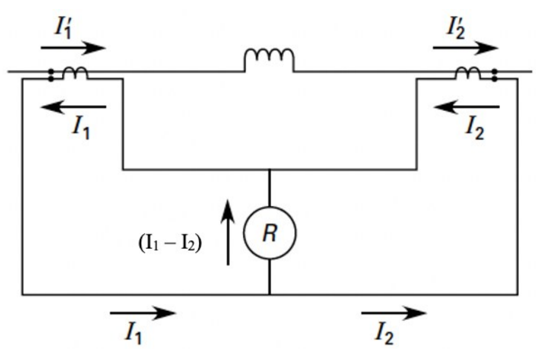

Figure 19: Differential Relay Scheme

Phase Angle Comparison

A *phase angle relay* compares the current and voltage to determine, for example, the direction of power flow. When power flow is normal, the impedance angle of the fault circuit, φ, is 30° for a load power factor of 0.8. When power flow reverses, the phase angle becomes 180°±30°. If a fault occurs, the phase angle of the current becomes either $-\varphi$ (forward faults) or $180°-\varphi$ (reverse faults). For power transmission networks, the *impedance angle of a fault circuit, φ, is near 90°*. A phase angle relay detecting a 90° impedance angle can react under the assumption that a fault has occurred.[14] See the figure below.

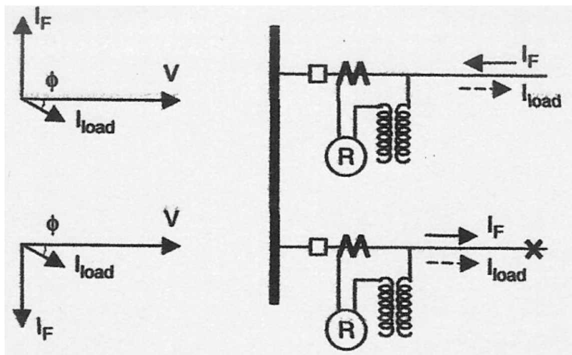

Figure 20: Phase Angle Comparison for Transmission Line Fault

Distance Measurement

A *distance relay* measures the relationship between the voltage and current at a single point. Assuming standard impedance per unit distance, the relay can be adjusted for any desired distance. Since the distance relay depends on the ratio of the voltage to the current, it is also called a *ratio relay* or *impedance relay*. If the relay is not bidirectional, but rather measures faults in only one direction, it is called a *mho relay*.

Pilot Relay

Pilot relaying involves the operation of a relays based on information obtained from a location remote to the relay. The information is carried via communication lines using a power line carrier, microwave, telephone circuits, fiber optics, or pilot cable/wire.

A tripping wire scheme is shown below. (A blocking scheme can also be used. The only difference is the relative location of the restraining and operating circuits.)

[14] The fault current exists on both circuits though the fault is only on one. Both circuits show the 90° phase angle. This occurs because the resistance is near zero and the circuit is primarily inductive. A faulted transmission line represents the ratio of inductive reactance to resistance, *X/R*, determining the DC component decay and asymmetry of the fault current. The higher the reactance, the slower the decay.

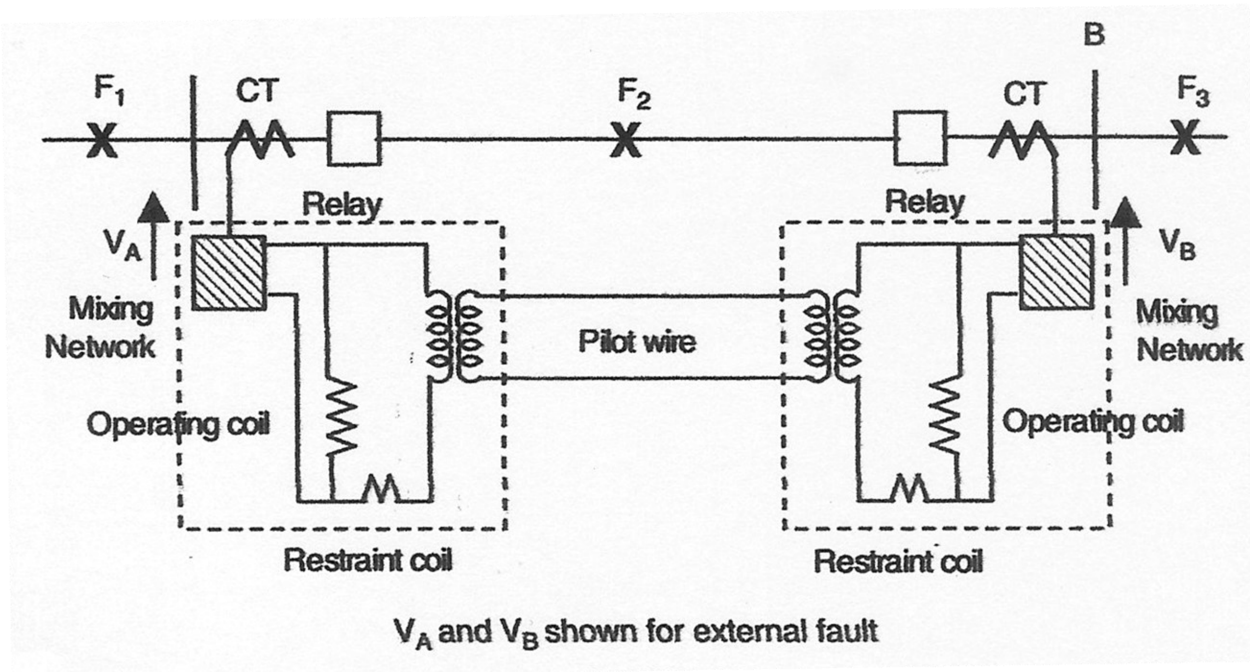

Figure 21: Tripping Pilot Scheme

In the tripping pilot scheme, the secondary currents from the three CTs are fed into a mixing network whose output is a single voltage. (This is disadvantage over some other schema in that one cannot tell which phase caused the fault.) The operating circuit is more sensitive than the restraining circuit.

For a fault at F1 (as well as for F3), the current entering B and leaving A will be equal and in phase. Thus V_A and V_B are also equal and in phase. There are no currents circulating in the pilot wires or in the operating circuits within the relays. However, the operating circuit voltage does result in current in the restraining coils the relay will not operate.

If a fault occurs at F2, the current entering bus A will be 180° out of phase with the current entering bus B. (The magnitudes may differ due to line impedances. The key is the shift in phase.) This results in circulating currents in the pilot wires. Some current flows in the restraining circuit but the operating circuit dominates and the relays will operate.

If the pilot wires are open, no trip can occur, if shorted a false trip might occur. These can be overcome by a DC monitoring circuit for the pilot wires and a fault detector on the same wires, respectively.

Harmonic Relay

The general assumption is that an alternating voltage or current is purely sinusoidal and has only one frequency, called the *fundamental frequency*, but this is not always the case. Nonideal waveforms are generated by nonlinear components, such as inductors, light ballasts, switching loads, and heavily inductive machines like alternators or generators. Electronic switching circuits that control the power by using only a portion of the sinusoidal signal are especially conducive to the creation of complex waveforms. The resulting complex waveform is said to contain harmonics, and the waveform can be built from a combination of the harmonics, as shown in the equation below.

Equation 2: Harmonic Equation

$$X_{\text{rms}} = \sqrt{X_{\text{DC}}^2 + \sum_{n=1}^{\infty} X_n^2}$$

The X represents a sine or cosine function. The first term of Eq. 2 represents any DC component, and the second represents the harmonics, including the fundamental harmonic.

Harmonics are multiples of the fundamental frequency: $2f$, $3f$, $4f$, $5f$, $6f$, and so on. The term $1f$ is the fundamental harmonic. So for a fundamental frequency of 60 Hz, the harmonics are 60 Hz, 120 Hz, 180 Hz, 240 Hz, 300 Hz, and so on, and 60 Hz is the fundamental harmonic. Harmonics are classified by their name, frequency, and sequence. A *positive sequence harmonic* rotates in the same direction (forward) as the fundamental frequency—4th, 7th, 10th, and so on. A *negative sequence harmonic* rotates in the opposite direction of the fundamental frequency— 2nd, 5th, 8th, and so on. A *zero sequence harmonic* (*triplen*) has a zero rotational sequence—3rd, 6th, 9th, and so on.

Positive sequence harmonics are additive, resulting in potential overheating of conductors, power lines, and transformers. Negative sequence harmonics tend to cancel, resulting in weaker magnetic fields and less torque in motors. Zero sequence harmonics circulate between the phase and neutral and don't cancel one another out; the result is that a neutral wire could have three times the phase current amplitude at the fundamental frequency. The impacts are summarized in the table below.

Table 2: Harmonic Sequence Summary

title	1st	2nd	3rd	4th	5th	6th
frequency (Hz)	60	120	180	240	300	360
sequence	+	−	0	+	−	0

All these abnormal (from the fundamental) conditions can be detected by sensing harmonic current through filters in electromagnetic or solid-state relays, or by calculation in a digital relay.[15] Harmonic relays are similar to Fig. 21 in operating principles and can be realized in any relay format: electromechanical, solid-state, or digital.

Frequency Sensing Relay

In the US, the standard frequency is 60 Hz. Frequency relays use sampling and digital computer techniques can be used to count zero crossings of the frequency. Any deviation from the desired frequency can use the relay output to correct back to the baseline.

Relay Designs

The description of relays, and the equations involved, derive from the first electromechanical relays. But the construction of the relay can use these equations/concepts regardless of the type of implementation: solid-state, digital, and…. As such principles are useful in understanding all relays, the general principles are universal will be covered. Details may be found in the manufacturer's literature.

Universal Relay Equation

The *universal relay equation* is shown below. Developed from analog relay devices, *it can be used with solid-state relays or programmed into computer relays*. The term θ is the torque angle in an analog system during normal conditions and is related directly to the *impedance angle*. The term φ is the *torque angle* in an analog system at the referenced (potentially, the fault) condition, which is also related to the *impedance angle*. By manipulating the constants in either an analog or digital protective system, a variety of relays can be realized. Figure 22 shows the relay types that can occur.[16,17]

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[15] Hum is a periodic signal arising from power lines. It occurs at the power supply frequency or its harmonics (60 Hz or 120 Hz in the United States). In transformer iron cores with loose laminations, the frequency of the hum is twice the supply frequency and is related to the alignment of magnetic domains in the material. Hum can be eliminated by proper construction, filtering, and shielding. It is generally not caused by harmonics, more from a phenomena called *magnetostriction*.

[16] The term Zr is the impedance of the transmission line at the maximum response of the relay; that is, the maximum distance from the relay that will result in a response. The angle ($\theta + \varphi$) between a line connecting Zr and the actual impedance, Z, when less than 90°, means the point lies inside the circle and a trip will result. When the angle is greater than 90°, the point lies outside the circle and the trip will be blocked.

[17] The universal relay equation is often shown as the cosine function. It is expressed using the sine function to account for the phase angle difference relative to the maximum torque angle, often showing how magnetic fluxes lag voltage. So, embedded in φ is the torque angle $\tau = 90° - \phi$. The term $VI\cos(\theta - \tau)$) is generally preferred in power system

Equation 3: Universal Relay Equation

$$\tau = \tau_{\text{magnetic}} - \tau_{\text{spring}} = K_1 I^2 + K_2 V^2 + K_3 IV \sin\left(\theta + \varphi\right) - \tau_{\text{spring}}$$

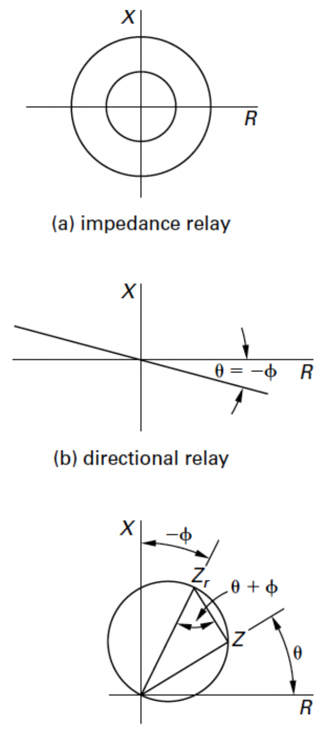

Figure 22: Universal Relay Characteristics

While the R-X plane used in Fig. 22 appears to use negative resistance, this is not the case. A relay can be located in any number of locations within a system. The power flows, under normal conditions, toward the load. If a fault occurs upstream of the relay's location, power flow reverses, and the power will then flow from the load to the fault point. Recall that *motors exposed to a fault in the system become generators feeding power to the fault from their inertial energy*. *Transformers do the same from the inductive energy stored in their magnetic fields*.

The various zones of the R-X plane are shown in Fig. 23. Current and voltage in the system can be plotted on the plane to indicate phase relationships. Impedance values plot as points. Changes to the voltage-current relationships can then be analyzed as conditions shift due to faults.

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engineering literature to make the maximum torque angle immediately identifiable in the equation. And, the torque angle τ is represented in Fig. 22 as ϕ. Thus, caution is warranted during study to ensure the terminology being utilized.

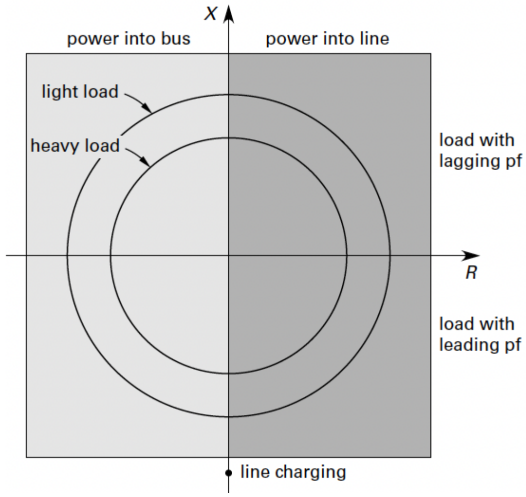

Figure 23: R-X Diagram

Example 1[18]

[Source: Ref H, Example 2.3, p. 36, Modified by Author]

Consider the choice of K_3 equal to zero in Eq. 3, and that the control spring torque τ_S is negligible. When the relay is on the verge of operation (i.e., at its balance point) $\tau = 0$, and if the voltage induced torque is arranged to be the opposite direction to that produced by the current (i.e., replacing K_2 by $-K_2$), then

$$\tau = 0 = K_1 I^2 - K_2 V^2 = \frac{K_1 I^2 - K_2 V^2}{I^2} = K_1 - K_2\left(\frac{V^2}{I^2}\right) = K_1 - K_2\left(Z^2\right) = 0$$

$$K_1 - K_2\left(Z^2\right) = 0$$

$$K_2\left(Z^2\right) = K_1$$

$$Z^2 = \frac{K_1}{K_2}$$

$$|Z| = \frac{V}{I} = \sqrt{\frac{K_1}{K_2}}$$

[18] The universal relay torque equation describes the torque T (or τ) produced in an electromagnetic relay based on current I and voltage V inputs. While typically expressed in terms of the cosine function (representing real power), it can be expressed in terms of the sine function to represent reactive power (X) or to change the relay's maximum torque angle.

This is a circle in the *R*-*X* plane, as shown in Fig. 22(a).[19] This is known as an *impedance* or *ohm relay*. The torque is greater than this pickup value when the ratio of voltage to current (or impedance) lies inside the operative circle.

By adding a current-carrying coil on the structure carrying a current proportional to the voltage, the torque equation at the balance point is

$$\tau = 0 = K_1 I^2 - K_2 \left(V + K_4 I \right)^2$$

or, in the *R*-*X* plane, is

$$\left| Z = K_4 \right| = \sqrt{\frac{K_1}{K_2}}$$

which is the equation of a circle with its center offset by a constant.

Now, consider the choice of K_1, K_2, and τ_S all equal to zero in Eq. 3. These choices produce a balance point of[20]

$$0 = IV \sin\left(\theta + \varphi \right)$$

Now, divide by I^2, and assuming the *I* is not zero, the balance point equation is

$$0 = Z \sin\left(\theta + \varphi \right)$$

This is the equation of a straight line in the *R*-*X* plane passing through the origin at an angle of $-\varphi$ to the *R* axis as shown in Fig. 22(b). This is characteristic of a directional relay.

Finally, by setting K_1 and τ_S equal to zero, and reversing the sign of the torque produced by the *VI* term, the torque equation balance point becomes

$$\tau = 0 = K_2 V^2 - K_3 VI \sin\left(\theta + \varphi \right)$$

or by dividing through by I^2, and assuming that *I* is not zero, the balance point equation is

[19] This derives from impedance, Z, being R + X, and a circle being $R^2 + X^2 = r^2$.
[20] The constant divides out.

$$|Z| = \frac{K_3}{K_2}\sin(\theta + \varphi)$$

This is an equation of a circle passing through the origin in the *R*-*X* plane, with a diameter of K_3/K_2. The diameter passing through the origin makes an angle of −φ with the X axis, as shown in Fig. 22(c). This is known as an admittance or mho relay characteristic.

Solid-State Relays

Many relays are electronic, or solid-state. Such relays offer numerous advantages: rugged design, ease of adjustment and programmability, accurate response, and high reliability with little or no maintenance. A solid-state level relay is shown in the figure below. A level relay is also known as an *overcurrent relay*. The signal sensed is passed through the resistor, *R*, rectified by the bridge network, amplified and filtered by the *R*-*C* circuit, and then compared to a reference voltage by an operational amplifier, *A*. If the signal exceeds the reference value, the output of the op amp increases, resulting in a protective signal after a self-imposed time delay occurs.

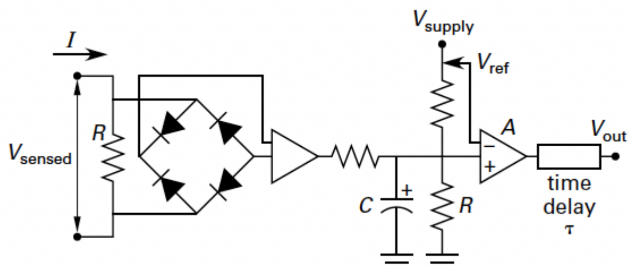

Figure 24: Solid-State Overcurrent Relay

PROTECTIVE DEVICES

A wide variety of protective devices are available. The table below lists these devices in order of complexity and cost.

Table 3: Protective Devices by Cost and Complexity

protective device	cost and complexity
fuse	low
sectionalizer	↑
recloser	
instantaneous overcurrent	
inverse time-delay overcurrent	
directional overcurrent distance	↓
pilot	high

Transmission Lines

Fuses are simple and inexpensive and common in protective systems. They come in a series of standard sizes, each with a time-current characteristic curve.

A *sectionalizer* is a self-contained device used in conjunction with other devices, such as reclosers and circuit breakers, to permanently isolate a faulted portion of a distribution system.

A *sectionalizer* does not interrupt a fault current. Instead, it counts the number of times a fault is seen (i.e., sensed) and opens when the count reaches a pre-programmed level. Only then is the circuit deenergized, usually by a recloser or circuit breaker. Sectionalizers are an economical way to isolate faults without the additional step of coordinating trip levels.

A *recloser* senses a fault and interrupts power to a system. After a preset time, the recloser attempts to reenergize the system by closing. If other protective devices have isolated the fault, the recloser remains closed.

An *overcurrent device* operates when a fault exceeds a given value.

Pilot protection refers to a communication channel between the ends of a long transmission line that enables the system to instantaneously clear faults over 100% of the line. Such communication channels include the power line itself using a *power line carrier*, *microwaves*, *fiber optics*, or *communication cables*.

Non-pilot protection refers to a system that responds only to the parameters at one end of a transmission line. If a fault occurs on the line away from the protection, a time-delay is incurred automatically and the fault is not interrupted instantaneously at both ends of the line. Such a system is also called a *graded*, *relatively selective*, or *non-unit system*.

Electrical Component Protection: Crowbar Circuits

Circuits in all electrical components need some type of protection to prevent faults from damaging equipment. A variety of circuits provide protection against overvoltage conditions.

One common protective device is a *crowbar circuit*, which is used with a circuit attached to a power supply to prevent damage due to overvoltage from the power supply. By putting a short circuit or a very low resistance path across the voltage output, the crowbar circuit places a high overload on the actuating element of a circuit breaker or other protective device. Once triggered, a crowbar circuit brings the voltage between the terminals of the device to near zero, thus protecting downstream electrical components.[21] Even though the voltage is now too low to trigger it, the crowbar circuit will not return to normal operation until power is removed.

Figure 31.20 shows an example of a crowbar circuit. In this case, overvoltage on the output line is compared with the reference voltage provided by the resistor, R_{ref}.[22] When the voltage, V_{ref}, is greater than the breakdown voltage of the gated zener diode, the gated zener diode is triggered, and the triac's voltage is set by the zener's voltage. This voltage is at the appropriate level for the triac to turn on, resulting in a large current between the output and return lines due to the very low impedance of the triac.

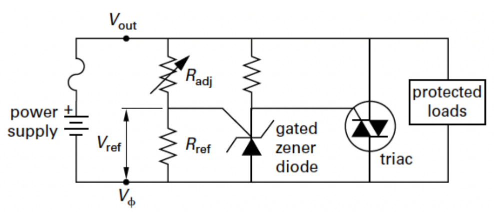

Figure 25: Crowbar Circuit Example

If the zener diode were used alone, the voltage between the terminals would be set at a level higher than desired. Instead, the zener diode turns on the triac, which provides an impedance path that is a short circuit between the terminals, bringing the voltage of the downstream circuitry well below the triggering voltage of the zener diode itself. In fact, the voltage is near zero; that is, near the normal voltage of the return line to the power supply. As a result, the downstream loads do not experience the overvoltage from the power supply and are protected.

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21 An actual crowbar (or large gauge wire, generically called a jumper) can be used during maintenance on large pieces of electrical equipment or transmission system segments. A crowbar placed across the output terminals of the power supply creates a short circuit and prevents voltage in downstream elements, which provides protection to people working on the system.

22 While a battery is shown as the power supply in the figure, often the source is an AC/DC converter, which does have high-voltage failure modes.

IEEE STANDARDS

Buff Book

IEEE Standard 242, *IEEE Recommended Practice for Protection and Coordination of Industrial and Commercial Power Systems* (*IEEE Buff Book*)[23], describes the industry best practices for the selection, setup, and coordination of components of the protection system for industrial and commercial power networks. The information, often extracted from other codes, standards, and texts covers reasonable electrical abnormalities and how they affect the system as a whole. The IEEE Buff Book contains chapters on system protection principles; short-circuit calculations; instrument transformers; selection of relays; low- and high-voltage fuses; low voltage circuit breakers; ground fault, conductor, motor, and transformer protection; bus and switchgear protection; overcurrent coordination; and maintenance, testing, and calibration.

Gold Book

IEEE Standard 493, *IEEE Recommended Practice for the Design of Reliable Industrial and Commercial Power System*s (*IEEE Gold Book*) describes the best practices for performing a comprehensive reliability analysis. The information it contains is derived primarily from extensive surveys of electrical equipment in power plants; other codes, standards, and industry texts are not referenced. The IEEE Gold Book is an accurate historical reference on power reliability and the associated costs.

The chapters in the IEEE Gold Book are on industrial and commercial power systems concepts, planning, and design; evaluating and improving reliability on existing systems; preventative maintenance; emergency and standby power; voltage sag analysis; 7 × 24 continuous power facilities; reliability and maintainability verification; and equipment reliability data.

Yellow Book

IEEE Standard 902, *IEEE Guide for Maintenance, Operation, and Safety of Industrial and Commercial Power Systems* (*IEEE Yellow Book*) consists of information, often extracted from other codes, standards, and texts on safety during operation and maintenance of all sizes and types of industrial and commercial power systems. The IEEE Yellow Book provides guidelines for the

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[23] The IEEE Color Books are a series of 13+ renowned IEEE Standards Association recommended practices for industrial and commercial power systems, focusing on design, safety, maintenance, and operation. Developed by the IEEE IAS I&CPS department, they cover topics like grounding, protection, and reliability, and are now being transitioned into the *IEEE 3000 Standards Collection*™.

establishment of procedures, recommended organizational structure, and maintenance strategies. It contains chapters on operating diagrams; system management; control responsibilities and clearing procedures; maintenance strategies; electrical safety; safety programs; maintaining safe facilities; and safe work practices, tools, and test equipment.

Blue Book

IEEE Standard 1015, *IEEE Recommended Practice for Applying Low Voltage Circuit Breakers Used in Industrial and Commercial Power Systems* (*IEEE Blue Book*) provides information necessary for the proper selection of low-voltage circuit breakers. The information, often extracted from other codes, standards, and texts, helps determine the type of breaker, appropriate ratings, requisite trips, acceptance testing, and maintenance needed. The IEEE Blue Book contains chapters on definitions, acronyms, and abbreviations; rating and testing; coordination; fused and special-purpose breakers; and acceptance and maintenance requirements.

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Items in **bold** are highly recommended for in-depth study. **Always use the latest editions.**

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NOTE

Electrical refers to something related to electricity while "electric" refers to a device or machine that runs on electricity. Nevertheless, the NEC is sometimes referred to as the National Electric Code. This Handbook with its illustrations, commentaries, and guidance is the best source found for those dealing with the NEC frequently in their projects.

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Appendix A: Equivalent Units Of Derived And Common SI Units

Symbol	Equivalent Units			
A	C/s	W/V	V/Ω	$J/(s \cdot V)$
C	$A \cdot s$	J/V	$(N \cdot m)/V$	$V \cdot F$
F	C/V	C^2/J	s/Ω	$(A \cdot s)/V$
F/m	$C/(V \cdot m)$	$C^2/(J \cdot m)$	$C^2/(N \cdot m^2)$	$s/(\Omega \cdot m)$
H	W/A	$(V \cdot s)/A$	$\Omega \cdot s$	$(T \cdot m^2)/A$
Hz	1/s	s^{-1}	cycles/s	$radians/(2\pi \cdot s)$
J	$N \cdot m$	$V \cdot C$	$W \cdot s$	$(kg \cdot m^2)/s^2$
m^2/s^2	J/kg	$(N \cdot m)/kg$	$(V \cdot C)/kg$	$(C \cdot m^2)/(A \cdot s^3)$
N	J/m	$(V \cdot C)/m$	$(W \cdot C)/(A \cdot m)$	$(kg \cdot m)/s^2$
N/A^2	$Wb/(N \cdot m^2)$	$(V \cdot s)/(N \cdot m^2)$	T/N	$1/(A \cdot m)$
Pa	N/m^2	J/m^3	$(W \cdot s)/m^3$	$kg/(m \cdot s^2)$
Ω	V/A	W/A^2	V^2/W	$(kg \cdot m^2)/(A^2 \cdot s^3)$
S	A/V	1/Ω	A^2/W	$(A^2 \cdot s^3)/(kg \cdot m^2)$
T	Wb/m^2	$N/(A \cdot m)$	$(N \cdot s)/(C \cdot m)$	$kg/(A \cdot s^2)$
V	J/C	W/A	C/F	$(kg \cdot m^2)/(A \cdot s^3)$
V/m	N/C	$W/(A \cdot m)$	$J/(A \cdot m \cdot s)$	$(kg \cdot m)/(A \cdot s^3)$
W	J/s	$V \cdot A$	V^2/Ω	$(kg \cdot m^2)/s^3$
Wb	$V \cdot s$	$H \cdot A$	T/m^2	$(kg \cdot m^2)/(A \cdot s^2)$

Appendix B: Physical Constants

Table Note 1

Quantity	Symbol	US Customary	SI Units
Charge			
electron	e		$-1.6022 \square 10^{-19}$ C
proton	p		$+1.6022 \square 10^{-19}$ C
Density			
air [STP][32°F, (0°C)]		0.0805 lbm/ft^3	1.29 kg/m^3
air [70°F, (20°C), 1 atm]		0.0749 lbm/ft^3	1.20 kg/m^3
sea water		64 lbm/ft^3	1025 kg/m^3
water [mean]		62.4 lbm/ft^3	1000 kg/m^3
Distance			
Earth radius[2]	⊕	$2.09 \square 10^{7}$ ft	$6.370 \square 10^{6}$ m
Earth-Moon separation[2]	⊕☾	$1.26 \square 10^{9}$ ft	$3.84 \square 10^{8}$ m
Earth-Sun separtion[2]	⊕⊙	$4.89 \square 10^{11}$ ft	$1.49 \square 10^{11}$ m
Moon radius[2]	☾	$5.71 \square 10^{6}$ ft	$1.74 \square 10^{6}$ m
Sun radius[2]	⊙	$2.28 \square 10^{9}$ ft	$6.96 \square 10^{8}$ m
first Bohr radius	a_0	$1.736 \square 10^{-10}$ ft	$5.292 \square 10^{-11}$ m
Gravitational Acceleration			
Earth [mean]	g	32.174 (32.2) ft/sec^2	9.8067 (9.81) m/s^2
Mass			
atomic mass unit	μ or m_μ $\frac{1}{12}m\left({}^{12}C\right)$	$3.66 \square 10^{-27}$ lbm	$1.6606 \square 10^{27}$ kg or 10^{-3} kg mol^{-1} / N_A
Earth[2]	⊕	$4.11 \square 10^{23}$ slugs	$6.00 \square 10^{24}$ kg
Earth [customary U.S.][2]	⊕	$1.32 \square 10^{25}$ lbm	-
Moon[2]	☾	$1.623 \square 10^{23}$ lbm	$7.36 \square 10^{22}$ kg
Sun[2]	⊙	$4.387 \square 10^{30}$ lbm	$1.99 \square 10^{30}$ kg
electron rest mass	m_e	$2.008 \square 10^{-30}$ lbm	$9.1095 \square 10^{-31}$ kg

neutron rest mass	m_n	$3.693\square 10^{-27}$ lbm	$1.6750\square 10^{-27}$ kg
proton rest mass	m_p	$3.688\square 10^{-27}$ lbm	$1.6727\square 10^{-27}$ kg
Pressure			
atmospheric		14.696 (14.7) lbf/in^2	$1.0133\square 10^{5}$ Pa
Temperature			
standard		32˚F (492˚R)	0˚C (273 K)
absolute zero		–459.67˚F (0˚R)	–273.16˚C (0 K)
Velocity[3]			
Earth escape		$3.67\square 10^{4}$ ft/sec	$1.12\square 10^{4}$ m/s
light (vacuum)	c, c_0	$9.84\square 10^{8}$ ft/sec	2.9979 (3.00) $\square 10^{8}$ m/s
sound [air, STP]	a	1090 ft/sec	331 m/s
sound [air, 70°F, (20°C), 1 atm]		1130 ft/sec	344 ft/s
Volume			
Volume: molal ideal gas (STP)[4]		359 ft^3 / lbmol	22.41 m^3/kmol

Table 1 Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/.
2. Symbols shown for the solar system are those used by NASA. See https://science.nasa.gov/resource/solar-system-symbols/.
3. Velocity technically is a vector. It has direction.
4. The unit “lbmol” is an actual unit, not a misspelling.

Appendix C: Fundamental Constants

Quantity	Symbols	US Customary	SI Units
Avogadro's number	N_A, L		6.022×10^{23} mol^{-1}
Bohr magneton	μ_B		9.2732×10^{-24} J/T
Boltzmann constant	κ	5.65×10^{-24} ft-lbf/ R	1.3805×10^{-23} J/T
electron volt: $\left(\frac{e}{C}\right)$ J	eV		1.602×10^{-19} J
Faraday constant, $N_A e$	F		96485 C/mol
fine structure constant, inverse α^{-1}	α α^{-1}		7.297×10^{-3} (≈1/137) 137.035
gravitational constant	g_c	32.174 lbm-ft/lbf-sec^2	
Newtonian gravitational constant	G	3.44×10^{-8} ft^4 / lbf-sec^4	6.672×10^{-11} N $\cdot$ m^2 / kg^2
nuclear magneton	μ_N		5.050×10^{-27} J/T
permeability of a vacuum	μ_0		1.2566×10^{-6} N/A^2 (H/m)
permittivity of a vacuum, electric constant $1/\mu_0 c^2$	ε_0		8.854×10^{-12} C^2 / N $\cdot$ m^2 (F/m)
Planck's constant	h		6.6256×10^{-34} J $\cdot$ s
Planck's constant: h/2π			1.0546×10^{-34} J $\cdot$ s
Rydberg constant	R_∞		1.097×10^{7} m^{-1}
specific gas constant, air	R	53.3 ft-lbf/lbm- R	287 J/kg $\cdot$ K
Stefan-Boltzmann constant		1.71×10^{-9} BTU/ft^2-hr-°R^4	5.670×10^{-8} W/m^2 $\cdot$ K^4
triple point, water		32.02 F, 0.0888 psia	0.01109 C, 0.6123 kPa
universal gas constant	R*	1545 ft-lbf/lbmol- R 1.986 BTU/lbmol- R	8314 J/kmol $\cdot$ K

Table Notes

1. Units come from a variety of sources, but primarily from the Handbook of Chemistry and Physics, The Standard Handbook for Aeronautical and Astronautical Engineers, and the Electrical Engineering Reference Manual for the PE Exam. See also the NIST website at https://pml.nist.gov/cuu/Constants/. The unit in Volume of "lbmol" is an actual unit, not a misspelling.

Appendix D: Mathematical Constants, Signs/Symbols, Maxwell's Equations

Quantity	Symbol	Value
Archimedes' constant (pi)	π	3.1415926536
base of natural logs	e	2.7182818285
Euler's constant	C or τ	0.5772156649

Signs/Symbols	Meaning
$\cdot$	multiplied by
/	divided by
:	ratio
$\gg$	much greater than
$\ll$	much less than
$=$	equals
$\equiv$	identical with
$\sim$	similar to
$\approx$	approximately equals
$\cong$	approximately equals, congruent
$\rightarrow$, $\doteq$	approaches
$\propto$	proportional, varies as
$\therefore$	therefore

Maxwell's Equations

integral form	point form	remarks
$\oint_s \mathbf{D}\cdot d\mathbf{s} = \int_V \rho\, dv$	$\nabla\cdot\mathbf{D} = \rho$	Gauss' law
$\oint_s \mathbf{B}\cdot d\mathbf{s} = 0$	$\nabla\cdot\mathbf{B} = 0$	nonexistence of magnetic monopoles
$\oint \mathbf{E}\cdot d\mathbf{l} = \int_s \left(\frac{-\partial \mathbf{B}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	Faraday's law
$\oint \mathbf{H}\cdot d\mathbf{l} = \int_s \left(\mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{dt}$	Ampère's law

Free-Space Form

integral form	point form
$\oint_s \mathbf{D}\cdot d\mathbf{s} = 0$	$\nabla\cdot\mathbf{D} = 0$
$\oint_s \mathbf{B}\cdot d\mathbf{s} = 0$	$\nabla\cdot\mathbf{B} = 0$
$\oint \mathbf{E}\cdot d\mathbf{l} = \int_s \left(\frac{-\partial \mathbf{B}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\oint \mathbf{H}\cdot d\mathbf{l} = \int_s \left(\frac{\partial \mathbf{D}}{\partial t}\right)\cdot d\mathbf{s}$	$\nabla\times\mathbf{H} = \frac{\partial \mathbf{D}}{dt}$

Electromagnetic Field Vector Equations

$$\mathbf{D} = \epsilon\mathbf{E} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon_0(1+\chi_e)\mathbf{E}$$
$$\mathbf{B} = \mu\mathbf{H} = \mu_0\mathbf{H} + \mu_0\mathbf{M} = \mu_0(1+\chi_m)\mathbf{H}$$
$$\mathbf{J} = \sigma\mathbf{E} = \rho\mathrm{v}$$

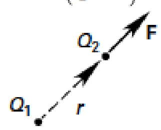
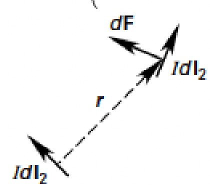
Appendix E: The Greek Alphabet

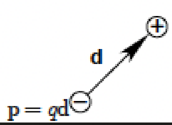
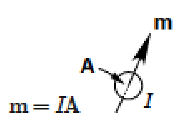
Α	α	alpha	Ν	ν	nu
Β	β	beta	Ξ	ξ	xi
Γ	γ	gamma	Ο	o	omicron
Δ	δ	delta	Π	π	pi
Ε	ε	epsilon	Ρ	ρ	rho
Ζ	ζ	zeta	Σ	σ	sigma
Η	η	eta	Τ	τ	tau
Θ	θ	theta	ϒ	υ	upsilon
Ι	ι	iota	Φ	ϕ	phi
Κ	κ	kappa	Χ	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
Μ	μ	mu	Ω	ω	omega

Appendix F: SI Prefixes

symbol	prefix	value
a	atto	10^{-18}
f	femto	10^{-15}
p	pico	10^{-12}
n	nano	10^{-9}
μ	micro	10^{-6}
m	milli	10^{-3}
c	centi	10^{-2}
d	deci	10^{-1}
da	deka	10
h	hecto	10^{2}
k	kilo	10^{3}
M	mega	10^{6}
G	giga	10^{9}
T	tera	10^{12}
P	peta	10^{15}
E	exa	10^{18}

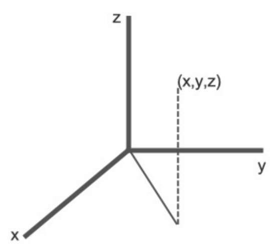
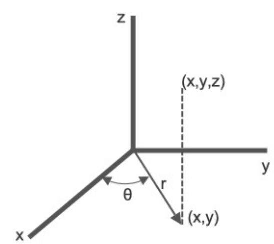
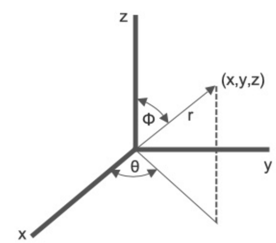
Appendix G: Comparison of Electric & Magnetic Equations

equation description	electric version	magnetic version	remarks
experimental force law	Coulomb's law $\mathbf{F} = \left(\frac{Q_1 Q_2}{4\pi\epsilon r^2}\right)\mathbf{r}$ 	force between two current elements $dF = \left(\frac{\mu_0}{4\pi}\right)\left(\frac{I_2 d\mathbf{l}_2 \times (I_1 d\mathbf{l}_1 \times r)}{r^2}\right)$ 	The term Idl in the magnetic column is the equivalent of a "magnetic charge" q_m. The I or the dl can be the vector. The r is a unit vector pointing from 1 to 2.
field definitions from force law	$\mathbf{F} = Q\mathbf{E}$	$d\mathbf{F} = \mathbf{I}\times\mathbf{B}\,dl$ current element $d\mathbf{F} = \mathbf{J}\times\mathbf{B}\,dV$ distributed current element $d\mathbf{F} = q\,\mathbf{v}\times\mathbf{B}$ moving charge	The V used in this row represents volume, not voltage. The v is the velocity.
general force law	$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$ $d\mathbf{F} = (\rho\mathbf{E} + \mathbf{J}\times\mathbf{B})\,dV$ where $dQ = \rho\,dV$		The V in this row represents the volume, not voltage. The v is the velocity.
definition of scalar and vector potential	$E = -\nabla V$	$\mathbf{B} = \nabla \times \mathbf{A}$	**A** is the magnetic vector potential.
Poisson's equation for the potential function	$\nabla^2 V = -\frac{\rho}{\epsilon}$	$\nabla^2\mathbf{A} = -\mu_0\mathbf{J}$	From a knowledge of the charge distribution, the potential can be found and then the **E** and **B** fields determined.
Gauss's law enclosing charge and Ampère's law enclosing current	$\iint \mathbf{D}\cdot d\mathbf{A} =$ $\iiint \rho\,dV = Q$ $\nabla\cdot\mathbf{D} = \rho$	$\oint H\cdot dl = I$ $\nabla \times \mathbf{H} = \mathbf{J}$	The V in this row represents volume.
constitutive relations	$\mathbf{D} = \epsilon\mathbf{E}$ $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$	$\mathbf{B} = \mu\mathbf{H}$ $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$	The second set of equations is always valid. The first set assumes the medium is linear and isotropic.
definitions of relative permittivity and permeability	$\epsilon_r = \frac{\epsilon}{\epsilon_0}$ $\epsilon_0 =$ 8.854×10^{-12} F/m	$\mu_r = \frac{\mu}{\mu_0}$ $\mu_0 = 4\pi \times 10^{-7}$ H/m	

equation description	electric version	magnetic version	remarks
capacitance and inductance of a field cell	$\epsilon_0 = \frac{C}{l}$	$\mu_0 = \frac{L}{l}$	Field cells are a construct designed to represent free space in terms of a parallel plate capacitor and an inductor. This capacitance and inductance exist regardless of the presence of an electric or magnetic field.
capacitance and inductance	$C = \frac{Q}{V}$	$L = \frac{\Lambda}{I}$	Λ is the flux linkage.
energy density of a field	$U = \frac{1}{2}\epsilon E^2$	$U = \frac{1}{2}\mu H^2$	Both energy and momentum are carried by a field.
energy stored by capacitance and inductance	$W = \frac{1}{2}CV^2$	$W = \frac{1}{2}LI^2$	
electromotive and magnetomotive force with sources present	$\oint \mathcal{E} \cdot d\mathbf{l} = \mathcal{E} = V$	$\oint \mathbf{H} \cdot d\mathbf{l} = NI = F_m = V_m$	The $\mathcal{E}$ is the emf, not the permittivity. Without sources present, both line integrals are equal to zero.
dipole moments			
dipole torque	$\mathbf{T} = \mathbf{p} \times \mathbf{E}$	$\mathbf{T} = \mathbf{m} \times \mathbf{B}$	This torque occurs due to the dipole being immersed in an external **E** or **B** field.
dipole potential energy	$W = -\mathbf{p} \cdot \mathbf{E}$	$W = -\mathbf{m} \cdot \mathbf{B}$	

electric	magnetic
emf $= V = IR$	mmf $= V_m = \phi \mathcal{R}$
current I	flux ϕ
emf $\mathcal{E}$ or V	mmf V_m
resistance $R = \rho l/A = l/\sigma A$	reluctance $\mathcal{R} = l/\mu A$
resistivity ρ	reluctivity $1/\mu$
conductance $G = 1/R$	permeance $P_m = \mu A/l$
conductivity $\sigma = 1/\rho$	permeability μ

Appendix H: Coordinate Systems and Related Operations

Mathematical Operations	Rectangular Coordinates	Cylindrical Coordinates	Spherical Coordinates
Conversion to Rectangular Coordinants	 $x = x$ $y = y$ $z = z$	 $x = r\cos\theta$ $y = r\sin\theta$ $z = z$	 $x = r\sin\phi\cos\theta$ $y = r\sin\phi\sin\theta$ $z = r\cos\phi$
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\boldsymbol{\theta} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\boldsymbol{\phi} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \theta}\boldsymbol{\theta}$
Divergence	$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$	$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\phi}\frac{\partial (A_\phi \sin\phi)}{\partial \phi} + \frac{1}{r\sin\phi}\frac{\partial A_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r}\mathbf{r} & \boldsymbol{\theta} & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_r & A_\theta & A_z \end{vmatrix}$	$\nabla \times \mathbf{A} = \begin{vmatrix} \frac{1}{r^2 \sin\theta}\mathbf{r} & \frac{1}{r^2 \sin\theta}\boldsymbol{\phi} & \frac{1}{r}\boldsymbol{\theta} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ A_r & rA_\phi & rA\theta A_\phi \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial r}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}$	$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{r^2 \sin^2\phi}\left(\frac{\partial^2 f}{\partial \theta^2}\right)$

